START WITH INTEGRAL FORM OF MOMENTUM EQN. FOR CONTROL VOLUME OF FIXED MASS:

$$\sum \vec{F} - \vec{F}_o = \int_V \frac{\partial}{\partial t}(\rho \vec{u})\, dV + \int_S \vec{u}(\rho \vec{u})\cdot \vec{n}\, dA$$

FOR STEADY FLOW WITH NO ACCELERATION OF REFERENCE FRAME

$$\sum \vec{F} = \int_S \vec{u}(\rho \vec{u})\cdot \vec{n}\, dA$$

INTERESTED ONLY IN AXIAL DIRECTION

$$\sum F_x = \int_S u_x(\rho \vec{u})\cdot \vec{n}\, dA$$

$$F_s + \underbrace{(p_o - p_N)A_N}_{\substack{\text{FORCE ACTS ON STRUCTURE – EVERYWHERE ELSE } p = p_o \text{ EXCEPT} \\ \text{INSIDE NOZZLE (AND ALONG CENTERLINE, BUT NO COMPONENT} \\ \text{OF PRESSURE IN X-DIR ON CENTERLINE)}}} = \underbrace{-\rho u_o u_o A_o}_{\text{FLUX AT INLET}} + \underbrace{\rho u_N u_N A_N}_{\text{FLUX AT NOZZLE EXIT}} + \underbrace{\rho u_o u_o (A_e - A_s - A_N)}_{\text{FLUX THROUGH AREA NOT OCCUPIED BY NOZZLE EXIT OR NOZZLE STRUCTURE}}$$

CONSERVATION OF MASS: $\rho u_o A_o = \rho u_N A_N + \rho u_o (A_e - A_s - A_N)$

$$\therefore (A_e - A_s - A_N) = \frac{\rho u_o A_o - \rho u_N A_N}{\rho u_o}$$ } SUBSTITUTE IN MOM. EQN.

$$F_s + (p_o - p_N)A_N = -\rho u_o \cancel{u_o} A_o + \rho u_N u_N A_N + \rho u_o \cancel{u_o} A_o - \rho u_o u_N A_N$$

$$= \rho u_N u_N A_N - \rho u_o u_N A_N$$

USE BERNOULLI'S EQN TO GET RID OF PRESSURE TERMS

$$p_o + \tfrac{1}{2}\rho u_o^2 = p_N + \tfrac{1}{2}\rho u_N^2 \Longrightarrow p_o - p_N = \tfrac{1}{2}\rho u_o^2 - \tfrac{1}{2}\rho u_N^2$$

$$F_s = \rho u_N^2 A_N - \rho u_N u_o A_N + \tfrac{1}{2}\rho u_o^2 A_N - \tfrac{1}{2}\rho u_N^2 A_N$$

$$= \tfrac{1}{2}\rho u_N^2 A_N \left[\frac{u_o^2}{u_N^2} - \frac{2u_o}{u_N} + 1\right]$$

a)

$$\boxed{F_s = \tfrac{1}{2}\rho u_N^2 A_N \left[\frac{u_o}{u_N} - 1\right]^2 \geq 0}$$

b) THE NOZZLE IS IN TENSION. AS THE AIR ACCELERATES AROUND THE NOZZLE LIP THE PRESSURE DROPS AND A SUCTION FORCE IS PRODUCED.