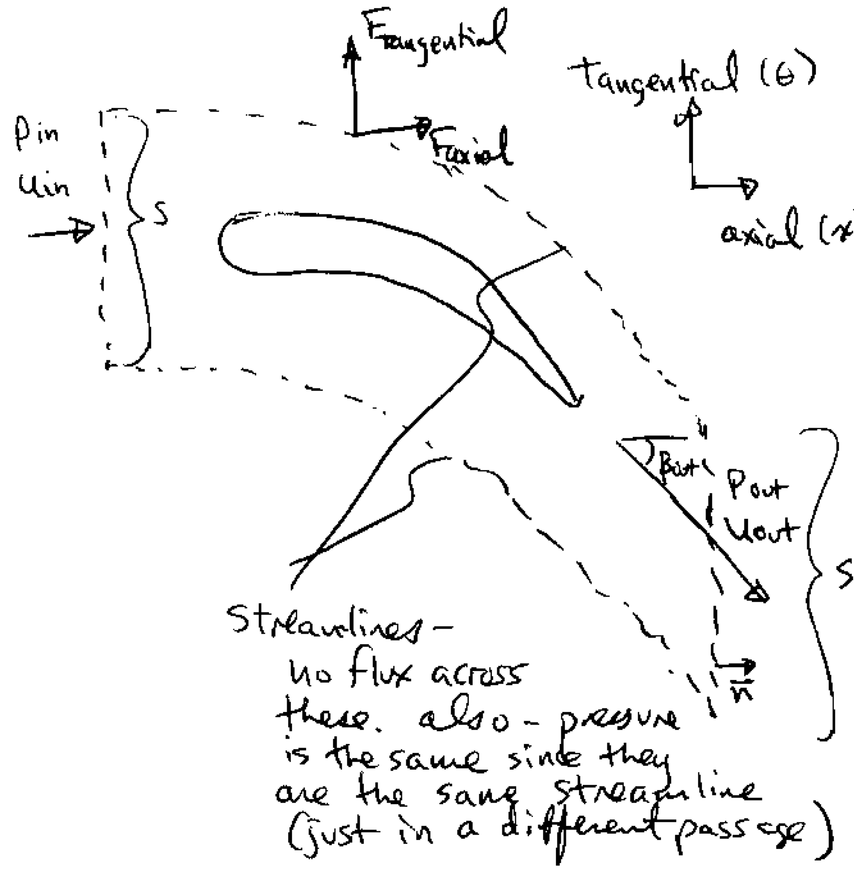
P3 SOLUTIONS BY WAITZ

2006



a) Axial flux of tangential momentum is equal to the mass flow rate times the tangential momentum per unit mass (i.e. the tang. vel.)

$$= \dot{m} u_\theta$$

$$= \int_A \rho u_\theta (\vec{u}\cdot\vec{n})\, dA$$

AT INLET $= 0$

AT OUTLET $= (-U_{out}\sin\beta_{out})\cdot(U_{out}\cos\beta_{out})\rho S$

MAGNITUDE INCREASES ACROSS BLADE ROW (MORE SWIRL)

a) $$\boxed{\int \rho u_\theta (\vec{u}\cdot\vec{n})\, dA = -U_{out}^2 \rho S \sin\beta_{out}\cos\beta_{out}}$$

b) MASS FLOW IN $= \rho u_{in} S$ MASS FLOW OUT $= \rho U_{out}\cos\beta_{out} S$

$$U_{out} = \frac{U_{in}}{\cos\beta_{out}} \implies \text{flow accelerates through nozzle}$$

$\implies$ $\boxed{p_{out} < p_{in} \text{ SINCE INCOMPRESSIBLE}}$

c) $$\sum F_x = F_{axial} + \text{pressure forces}_x = \int_A \rho u_x \vec{u}\cdot\vec{n}\, dA$$

BY SYMMETRY, PRESSURE FORCES ON TOP & BOTTOM STREAMLINES ARE THE SAME AND THUS BALANCE — ∴ ONLY NEED TO CONSIDER PRESSURE FORCES ON LEFT & RIGHT OF C.V.

$$F_{axial} + p_{in}S - p_{out}S = \rho u_{in}(\cancel{-u_{in}})S + \rho u_{out}\cos\beta_{out}(U_{out}\cos\beta_{out})S$$

$U_{out} = \dfrac{U_{in}}{\cos\beta_{out}}$ SO

$$= 0$$

xial $= (P_{OUT} - P_{IN})\, S$

$P_{OUT} < P_{IN}$ SO < 0 FORCE IS IN ← DIRECTION

$F_\theta = F_{TANGENTIAL} + \cancel{\text{PRESSURE FORCES}_y}^{\,0} = \int_A \rho u_\theta \vec{u}\cdot\vec{n}\, dA$

$F_{TANGENTIAL} = 0 - U_{OUT} \sin\beta_{OUT} \cdot U_{OUT} \cos\beta_{OUT}\, \rho S < 0$

SO FORCE ON FLOW IS IN ↓ DIRECTION

SO OVERALL FORCE ON FLUID IS ↙

FORCE ON BLADES IS EQUAL AND OPPOSITE ↗