P5 SOLUTIONS BY WAITZ

2006

a) $T = \dot{m} u_e + A_e(p_e - p_o)$; $I_{sp} = \frac{T}{\dot{m} g}$

$$\dot{m} = \frac{\frac{p_c \sqrt{\gamma}}{\sqrt{RT_c}}}{\left[1 - \frac{\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} A^* = 1.52 \text{ kg/s}$$

$$T_e = \frac{T_c}{1 + \frac{\gamma-1}{2} M_e^2} = \frac{T_c}{1 + \frac{\gamma-1}{2}} = 3478 \text{ K}$$

$$p_e = \frac{p_c}{\left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\gamma/\gamma-1}} = \frac{p_c}{\left[1 + \frac{\gamma-1}{2}\right]^{\gamma/\gamma-1}} = 2.7 \text{ MPa}$$

$$u_e = M_e \sqrt{\gamma R T_e} = \sqrt{\gamma R T_e} = 1165 \text{ m/s}$$

$$\boxed{T = 3087 \text{ N}, \quad I_{sp} = 207 \text{ s}}$$

b) FOR IDEALLY EXPANDED, SOLVE FOR M_e SUCH THAT $p_e = p_{atm}$

$$p_e = p_{atm} = \frac{p_c}{\left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\gamma/\gamma-1}} \Longrightarrow M_e = 3.12$$

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{1 + \frac{\gamma-1}{2} M_e^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \Longrightarrow \boxed{A_e = 0.00295 \text{ m}^2}$$

$$\boxed{T = 3783 \text{ N}, \quad I_{sp} = 253 \text{ s}}$$