P6 SOLUTIONS — WAITZ

a) $T = \dot{m} u_e + A_e(p_e - p_0)$

$$\dot{m} = \frac{p_c\sqrt{\gamma}}{\sqrt{RT_c}} \frac{M}{\left[1-\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} A = \frac{\frac{p_c\sqrt{\gamma}}{\sqrt{RT_c}}}{\left[1-\frac{\gamma-1}{2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \cdot A^*$$

$$\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{1+\frac{\gamma-1}{2}M_e^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$p_e = \frac{p_c}{\left[1+\frac{\gamma-1}{2}M_e^2\right]^{\gamma/\gamma-1}} \qquad T_e = \frac{T_c}{1+\frac{\gamma-1}{2}M_e^2} \qquad u_e = M_e\sqrt{\gamma RT_e}$$

I PUT THESE INTO A SPREADSHEET AND FOUND

$\frac{A_e}{A^*} = 13.333 \Rightarrow M_e = 4.04 \Rightarrow u_e = 2448.9$ m/s

$\Rightarrow p_e = 27{,}545$ Pa

$\dot{m} = 1.98$ kg/s

$\therefore T = 4257$ N

$I_{sp} = \frac{T}{\dot{m}g} = 219.5$ s] TAKE-OFF

b) FOR IDEALLY EXPANDED $T = \dot{m} u_e \Rightarrow$

$I_{sp} = \frac{u_e}{g} = 249.9$ s] ideal exp.

THIS NOZZLE IS OVEREXPANDED AT LAUNCH $p_e < p_0$

c) FOR IDEALLY EXPANDED, D=0, Vertical launch
g = const.

$$u_{burnout} = g\left[I_{sp_{ideal}} \ln\left(\frac{m_{initial}}{m_{final}}\right) - t_{burnout}\right]$$

$$h_{burnout} = g\left[-t_{b_0}\frac{I_{spideal}\ln\left(\frac{m_i}{m_f}\right)}{\left(\frac{m_i}{m_f}-1\right)} + t_{b_0} I_{pideal} - \frac{1}{2}t_{b_0}^2\right]$$

$$t_{b_0} = \frac{m_{propellant}}{\dot{m}} = \text{[illegible]} = 15.15\,s$$

$u_{burnout} = 3246$ m/s

$h_{burnout} = $ [illegible] $= 18837$ m

GLIDE PHASE: $\frac{1}{2} m_{final} u_{final}^2 = m_{final} g \Delta h$

$h_{glide} = \Delta h = $ [illegible] 537692 m

$h_{TOTAL} = $ [illegible] $= 556530$ m

d) $m_i = 41$ kg, $m_{final} = 11$ kg (still 30 kg fuel)

AR = 14.667 $\Rightarrow$ Me = 4.13

TAKE OFF THRUST [illegible] HIGHER, $I_{spideal}$ also higher

BUT WEIGHT PENALTY.

$T = \dot{m} u_e + (p_e - p_0) A_e$

NET EFFECT IS BAD $h_{TOTAL} = 506305$ m