Unified Propulsion Quiz

April 21, 2006

Closed Book – no notes other than the equation sheet provided with the exam

- **Put your name on each page of the exam.**
- **Read all questions carefully.**
- **Do all work for each problem on the pages provided.**
- **Show intermediate results.**
- **Explain your work --- don't just write equations.**
- **Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.**
- **Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.**
- **Show appropriate units with your final answers.**
- **Box your final answers.**

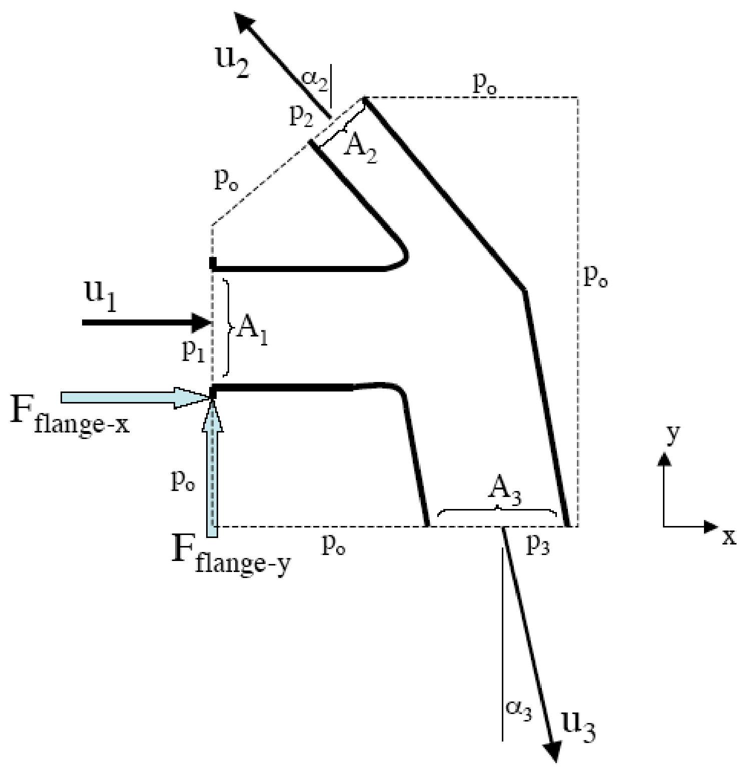
Exam Scoring

#1 (40%)	
#2 (25%)	
#3 (35%)	
Total	

Unified Engineering Propulsion Quiz — Number: SOLUTIONS____

Spring 2006

1. (40 points, partial credit given, L.O.'s A & B) Below is a schematic of a nozzle that vectors thrust in two directions. Write equations for the x- and y-components of force on the nozzle flange in terms of the parameters given.



a) (**18 points**) What are the x- and y-components of the momentum flux for this control volume?

$$\underbrace{\sum \vec{F} - \vec{F}_o}_{\text{sum of forces on C.V.}} = \underbrace{\int_V \left[\frac{\partial(\rho\vec{u})}{\partial t}\right] dV}_{\text{change in momentum of mass contained in C.V. over time}} + \underbrace{\int_s \vec{u}(\rho\vec{u}) \cdot \vec{n} ds}_{\text{change in momentum flux across surface of C.V.}}$$

Assume steady flow and no acceleration of the reference frame (2nd and 3rd terms are zero). Then the x- and y-components are:

$$\sum F_x = \int_s u_x(\rho\vec{u}) \cdot \vec{n} ds \quad and \quad \sum F_y = \int_s u_y(\rho\vec{u}) \cdot \vec{n} ds$$

x-momentum flux = $u_1(-u_1)\rho A + (-u_2\sin\alpha_2)(u_2)\rho A_2 + (u_3\sin\alpha_3)(u_3\cos\alpha_3)\rho A_3$

y-momentum flux = $0 + (u_2\cos\alpha_2)(u_2)\rho A_2 + (-u_3\cos\alpha_3)(u_3\cos\alpha_3)\rho A_3$

b) (**12 points**)What are the x- and y-components of the pressure forces on this control volume?

x-pressure force = $(p_1-p_o)A_1 + (p_2-p_o)A_2\sin\alpha_2 + 0$

y-pressure force = $0 - (p_2-p_o)A_2\cos\alpha_2 + (p_3-p_o)A_3$

c) (**10 points**) Write equations for the x- and y-components of the forces that hold the nozzle in place.

$F_{flange-x} = u_1(-u_1)\rho A + (-u_2\sin\alpha_2)(u_2)\rho A_2 + (u_3\sin\alpha_3)(u_3\cos\alpha_3)\rho A_3 -(p_1-p_o)A_1 - (p_2-p_o)A_2\sin\alpha_2$

$F_{flange-y} = (u_2\cos\alpha_2)(u_2)\rho A_2 + (-u_3\cos\alpha_3)(u_3\cos\alpha_3)\rho A_3 + (p_2-p_o)A_2\cos\alpha_2 - (p_3-p_o)A_3$

2. (25 points, partial credit given, L.O.'s C & F) A rocket nozzle is initially designed to be ideally expanded at launch such that $p_{exit} = p_o$ and $A_{exit} = A_{exit\text{-}ideal}$ You are considering shortening the nozzle so that $A_{exit} < A_{exit\text{-}ideal}$. Assume the propellant flow rate is constant. <u>What impacts might this have on vehicle performance and why?</u> In your answer, **you must make reference to principal figures of merit for the propulsion system and equations describing rocket propulsion and performance.** Discuss the trade-offs—which parameters will increase, which will stay the same, which will decrease?

The principal figures of merit for the rocket propulsion system are thrust, specific impulse (Isp) and weight.

Reducing the nozzle exit area leads to an under-expanded flow.

- Exit velocity decreases
- Exit pressure increases
- Nozzle weight decreases

In terms of the static thrust,

$$T = \dot{m}u_e + (p_e - p_o)A_e,$$

maximum thrust is for an ideally-expanded flow: although the pressure forces and exit area change, the decrease in forces due to net momentum flux will outweigh them, leading to decreased overall thrust. Therefore, the specific impulse,

$$Isp = \frac{T}{\dot{m}_p g}$$

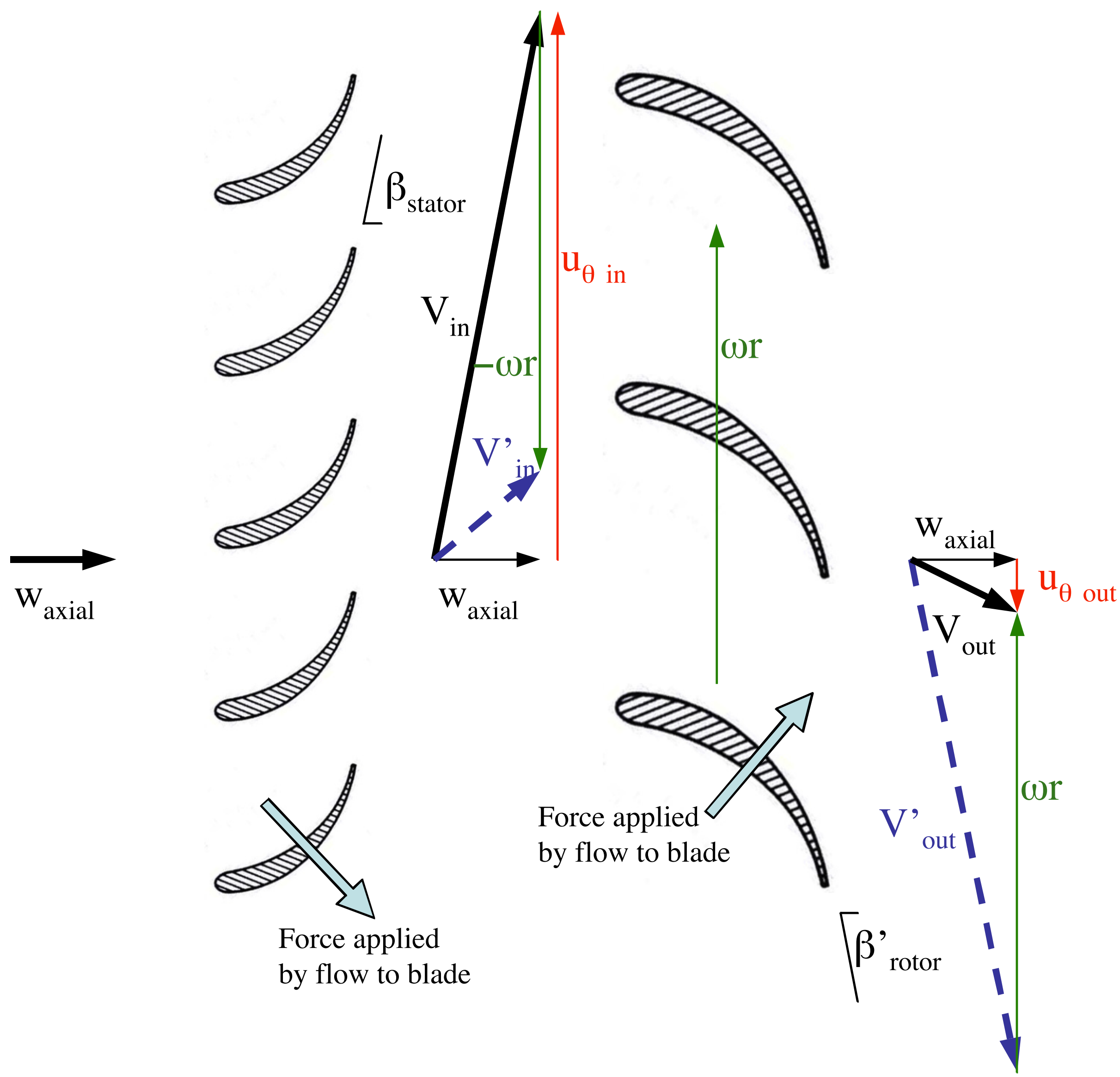
will decrease, because less thrust will be produced for each unit of propellant weight flow rate.

To determine whether vehicle performance will improve it is necessary to understand whether the reduction in nozzle weight outweighs the change in Isp. This would be evaluated by integrating the rocket equation over a particular mission.

$$u_{bo} = -u_e \ln\left(\frac{m_{bo}}{m_o}\right) + \int_0^{t_{bo}} \frac{(p_e - p_o(t))A_e - D(t)}{m_v(t)} dt - gt_{bo}$$

3. (35 points, partial credit given, L.O.'s B & G) A set of blade rows is shown.

a) **(15 points)** Draw and label the velocity triangles for these blade rows <u>on top of the axial velocity vectors that are given</u> (i.e. show the velocities in the relative and absolute frames). Assume *r* is a constant.



b) **(5 points)** Is this a compressor or a turbine? Why?

This is a turbine since the tangential component of velocity in the stationary frame decreases in the direction of rotor motion across the moving blade row.

Power $=\omega(\text{Torque})=\omega r$ (mass flow)$(u_{\theta out}-u_{\theta in})$.

c) **(5 points)** Which blade rows experience torques from the flow? Why? Draw the direction of the force applied by the flow to the blades in each of the blade rows. In what direction does the torque on the first blade row act?

Torque is equal to the change in axial flux of angular momentum.

Torque =(mass flow)$(ru_{\theta out}-ru_{\theta in})$

Both blade rows experience a torque since the tangential velocity changes across each of the blade rows (however, it is only the moving blade that does work changing the total enthalpy of the flow).

The directions of the forces applied by the flow to the blades are shown in the figure on the previous page.

The torque applied to the stator by the flow is in the same direction as the axial velocity.

d) **(5 points)** Explain in words how a multistage axial compressor or turbine works.

The rotating blade rows do work on the flow (compressor) or have work done on them by the flow (turbine). This changes the total enthalpy of the flow—increasing (compressor) or decreasing (turbine) the energy in the flow. The stationary blade rows are nozzles that convert energy in the flow from one form to another. Since the axial velocity is relatively constant, a stator that adds swirl converts internal energy to swirling kinetic energy. A stator that removes swirl does the opposite. The purpose of the alternating blade rows in a compressor is to add energy (in the form of swirling kinetic energy) by doing work on the flow by applying a torque on the spinning blades that changes the axial flux of angular momentum. Then the swirling component of the kinetic energy is converted to internal energy so that the axial velocity through the machine remains roughly constant (or else viscous forces would increase too much). The opposite processes hold for a turbine.

e) **(5 points)** If the blade speed is increased by approximately 30%, does the angle of attack of the flow into the rotor increase or decrease?

The angle of attack decreases (see figure below). The magnitude of the power extracted from the flow may increase or decrease depending on the relative contributions of the increase in blade speed and the decrease in the tangential velocity difference $P = \dot{m}\omega r\left(u_{\theta out} - u_{\theta in}\right)$.