

PRODUCTION OF THRUST

Newton's 2nd Law (e.g. $\sum \mathbf{F} = d/dt (m\mathbf{v})$)

for a control volume of fixed mass with steady flow in and out and no acceleration of the frame of reference relative to inertial coordinates:

$$\sum \mathbf{F} = \sum_s \bar{\mathbf{u}}(\rho \bar{\mathbf{u}}) \cdot \bar{\mathbf{n}} ds$$

Sum of external forces
on control volume, e.g.

Pressure forces

Shear forces

Body forces

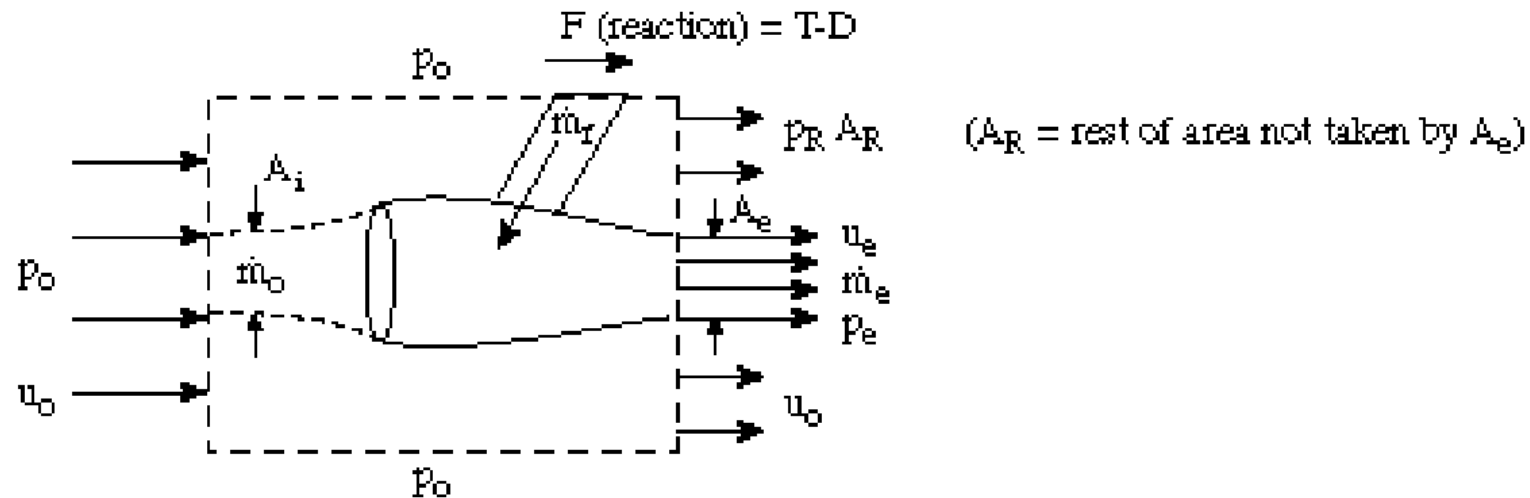
Reaction forces

Net flux of momentum through
surface of control volume

For x-component of vectors:

$$\sum F_x = \sum_s u_x \rho \bar{\mathbf{u}} \cdot \bar{\mathbf{n}} ds$$

PRODUCTION OF THRUST



$$T = D + \text{Pressure Forces} = \int_s \rho u_x (\bar{u} \cdot \bar{n}) dA$$

$$T = D + \int_{A_e} P_e A_e + P_o A_e - \int_{A_R} (P_R - P_o) dA_R = \int_s \rho u_x (\bar{u} \cdot \bar{n}) dA$$

$$\int_s \rho u_x \bar{u} \cdot \bar{n} dA = \rho_e u_e A_e u_e - \rho_o u_o A_o u_o + \int_{C_s \cap A_o \cap A_e} \rho u_x \bar{u} \cdot \bar{n} dA$$

$$= \dot{m}_e u_e - \dot{m}_o u_o + \int_{C_s \cap A_o \cap A_e} \rho u_x \bar{u} \cdot \bar{n} dA$$

$$T = D + (P_e - P_o) A_e - \int_{A_R} (P_R - P_o) dA_R = \dot{m}_e u_e - \dot{m}_o u_o + \int_{C_s \cap A_e \cap A_o} \rho u_x \bar{u} \cdot \bar{n} dA$$

PRODUCTION OF THRUST

Everything that relates to flow through the engine is conventionally called thrust. Everything that relates to the flow on the outside of the engine is conventionally call drag. Therefore, gathering only those terms that relate to the fluid that passes through the engine, we have:

$$T = \dot{m}_e u_e - \dot{m}_o u_o + (P_e - P_o) A_e$$

The **thrust** is largely composed of the **net change in momentum of the air entering and leaving the engine**, with a typically small adjustment for the differences in pressure between the inlet and the exit.

EFFICIENCY

We have related the thrust of a propulsion system to the net changes in momentum, pressure forces, etc. Now we will look at how efficiently the propulsion system converts one form of energy to another on its way to producing thrust:

$$\text{overall efficiency} = \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{propulsive power}}{\text{fuel power}}$$

$$\text{propulsive power} = \text{thrust} \cdot \text{flight velocity} = Tu_o$$

$$\text{fuel power} = \text{fuel mass flow rate} \cdot \text{fuel energy per unit mass} = \dot{m}_f h$$

Thus,

$$\eta_{\text{overall}} = \frac{Tu_o}{\dot{m}_f h}$$

EFFICIENCY

It is often convenient to break the overall efficiency into: thermal efficiency and propulsive efficiency where

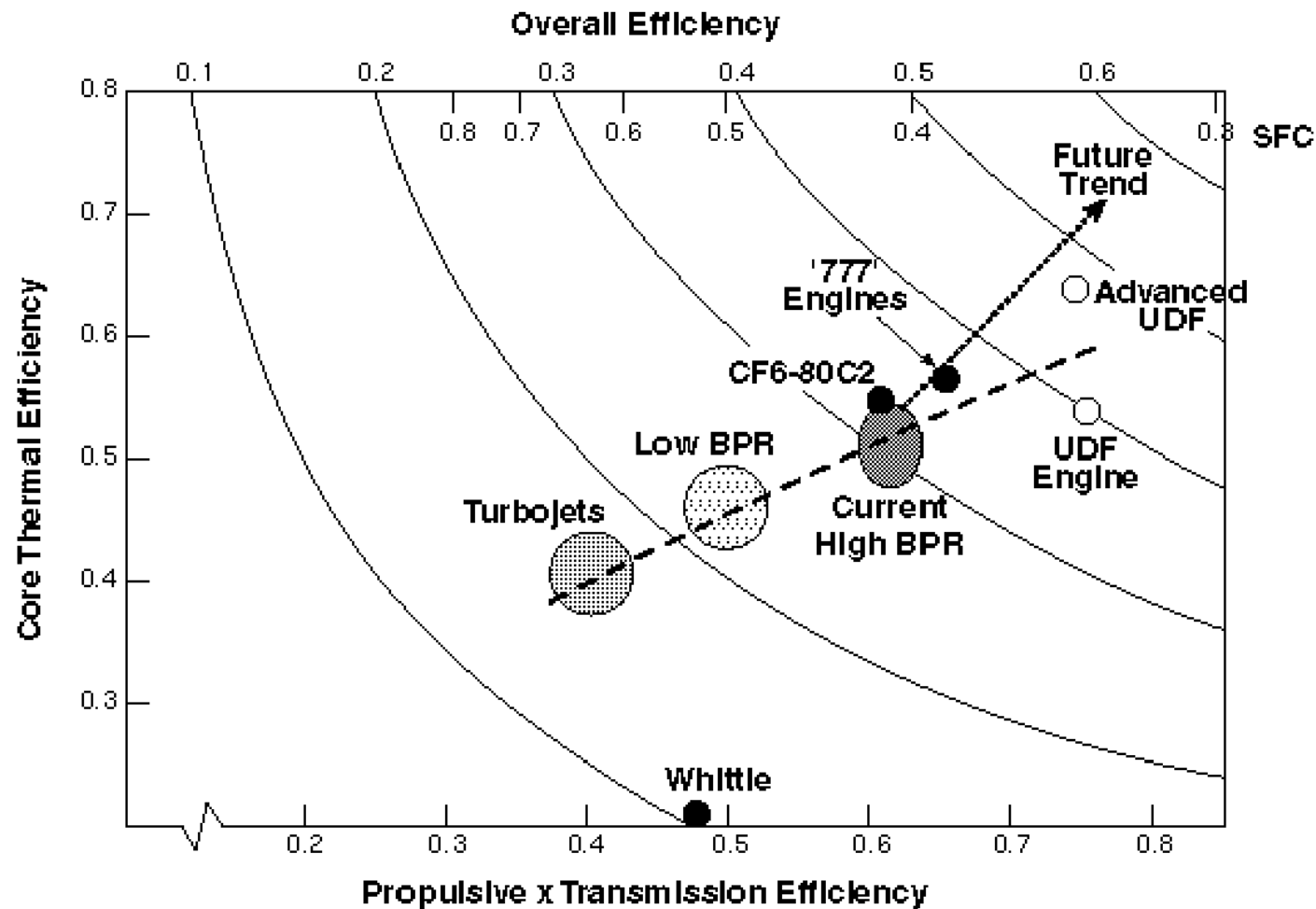
$$\eta_{\text{thermal}} = \frac{\text{rate of production of propellant k.e.}}{\text{fuel power}} = \frac{\frac{\dot{m}_e u_e^2}{2} + \frac{\dot{m}_o u_o^2}{2}}{\dot{m}_f h}$$

$$\eta_{\text{prop}} = \frac{\text{propulsive power}}{\text{rate of production of propellant k.e.}} = \frac{T u_o}{\frac{\dot{m}_e u_e^2}{2} + \frac{\dot{m}_o u_o^2}{2}}$$

Such that,

$$\eta_{\text{overall}} = \eta_{\text{thermal}} \eta_{\text{propulsive}}$$

OVERALL PROPULSION SYSTEM EFFICIENCY



Trends in *thermal efficiency* are driven by increasing compression ratios and corresponding increases in turbine inlet temperature. Whereas trends in *propulsive efficiency* are due to generally higher bypass ratio engines

(After Koff, 1991)

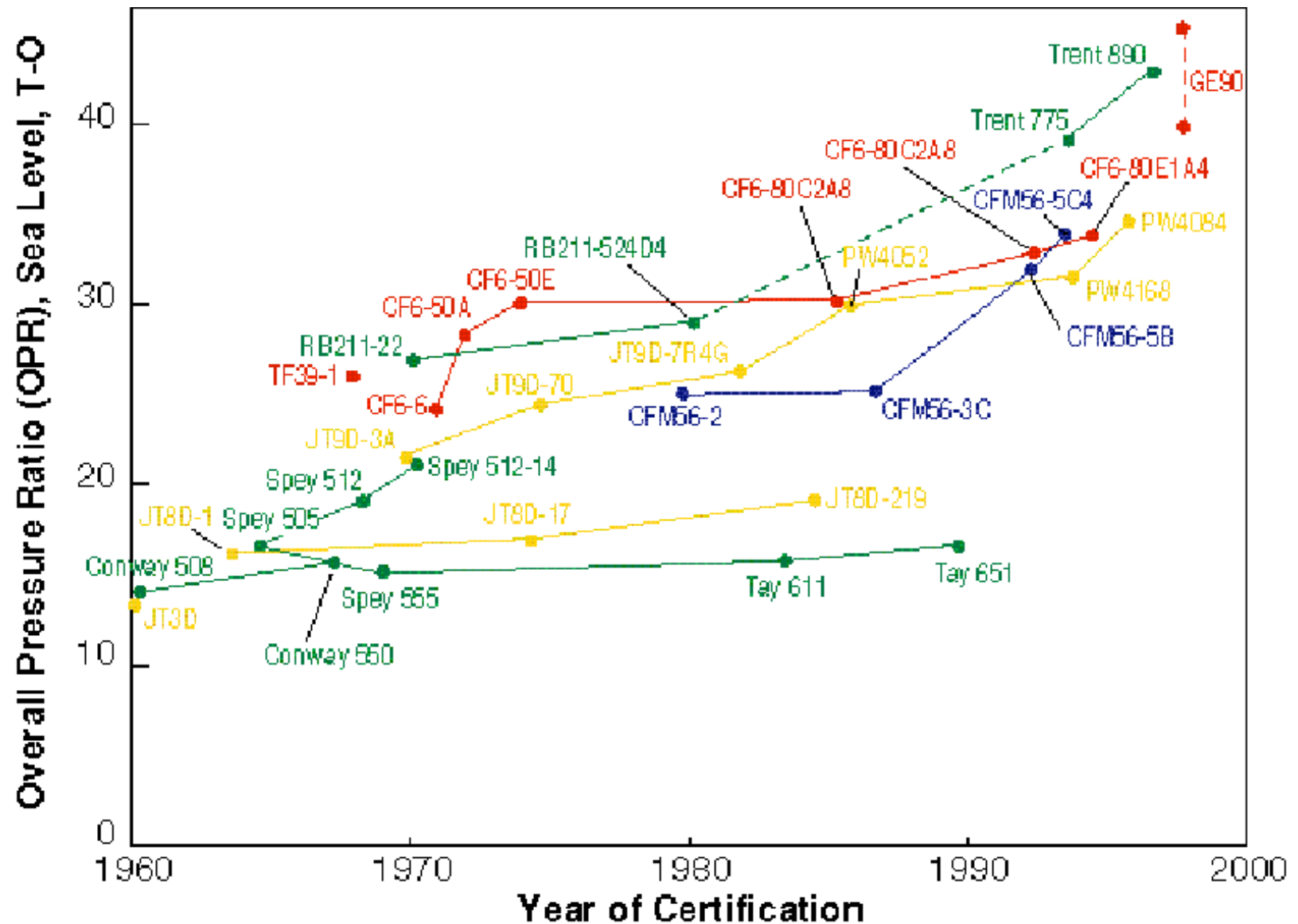
EFFICIENCY

The thermal efficiency is the same as that used in thermodynamics. For an ideal Brayton cycle it is a **function of the temperature ratio across the compressor**

$$\eta_{th, ideal Brayton cycle} = \frac{W_{net}}{Q_{in}} = 1 - \frac{T_{atm}}{T_{comp exit}} = 1 - \frac{1}{(PR)^{\frac{\gamma-1}{\gamma}}}$$

Higher temperature ratio = higher pressure ratio = higher thermal efficiency

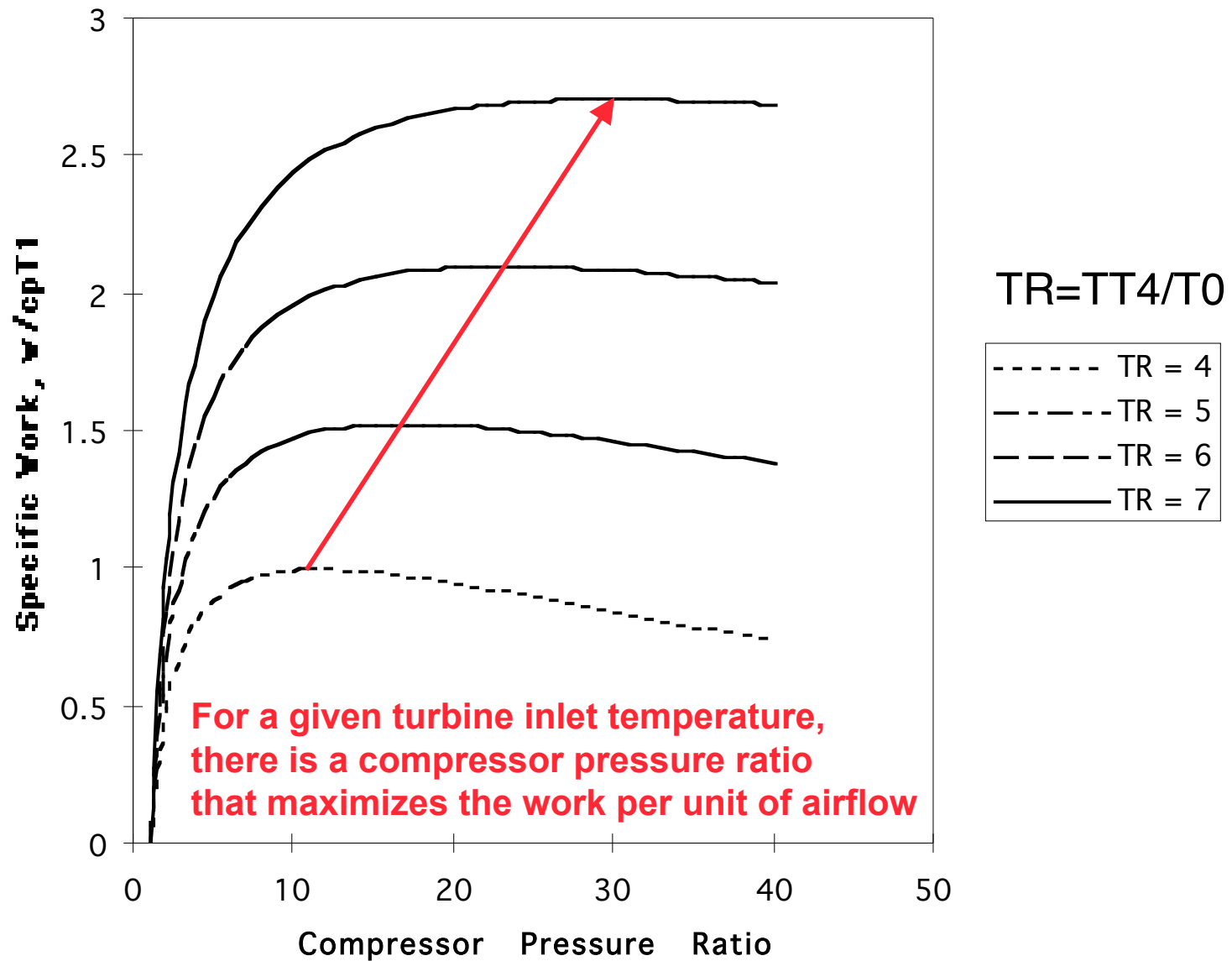
PRESSURE RATIO TRENDS



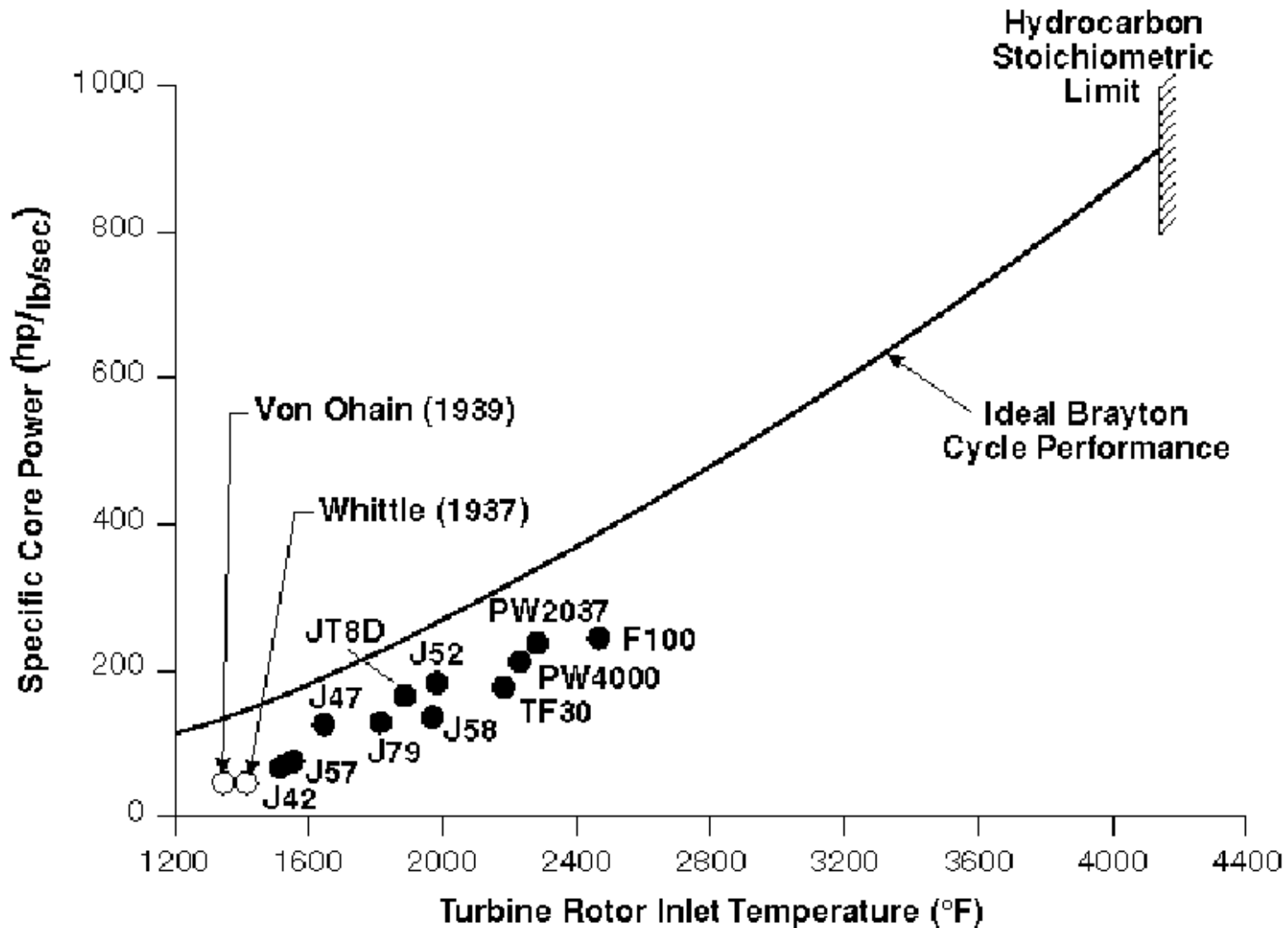
TURBINE INLET TEMPERATURE

- **Desire for higher turbine inlet temperature is driven by desire for high specific work**
- **Specific work is work per unit of airflow**
- **High work per unit of airflow = smaller engine, lower weight, etc.**

BRAYTON CYCLE SPECIFIC WORK



TURBINE INLET TEMPERATURE TRENDS



(After Koff, 1991)

PROPULSIVE EFFICIENCY

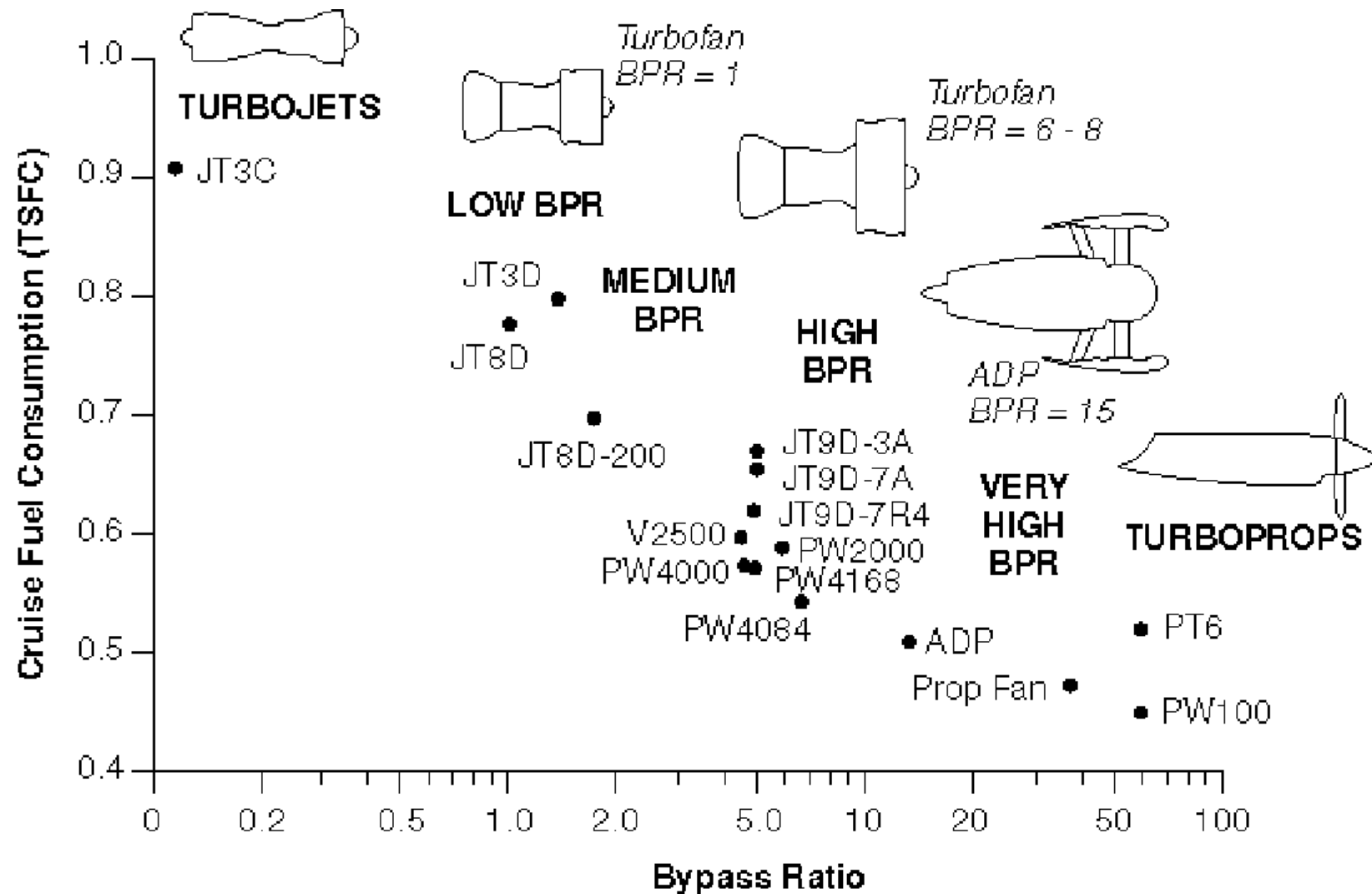
We can use our expression for thrust to rewrite the equation for propulsive efficiency in a more convenient form

$$T = \dot{m}(u_e - u_o) \quad (\text{since } \dot{m}_e = \dot{m}_o)$$

Then,

$$\eta_p = \frac{\dot{m} u_o (u_e - u_o)}{\frac{\dot{m}}{2} (u_e^2 - u_o^2)} = \frac{2u_o}{u_o + u_e} = \frac{2}{1 + \frac{u_e}{u_o}}$$

TRENDS IN BYPASS RATIO



OTHER EXPRESSIONS FOR OVERALL EFFICIENCY

Specific Impulse (I or I_{sp}):

$$I = \frac{\text{thrust}}{\text{fuel weight flow rate}} \quad (\text{units: seconds})$$

Thrust Specific Fuel Consumption (SFC or TSFC):

$$\text{SFC} = \frac{\text{mass flow rate of fuel}}{\text{thrust}}$$

(units: lbf/hr/lbf or kg/s/N)

IMPLICATIONS FOR ENGINE DESIGN

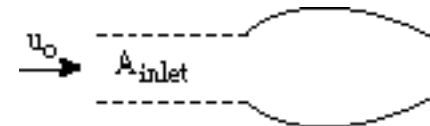
Considering jointly the expressions for thrust and propulsive efficiency,

$$F = \dot{m}(u_e - u_o) \quad \eta_{\text{prop}} = \frac{2}{1 + \frac{u_e}{u_o}}$$

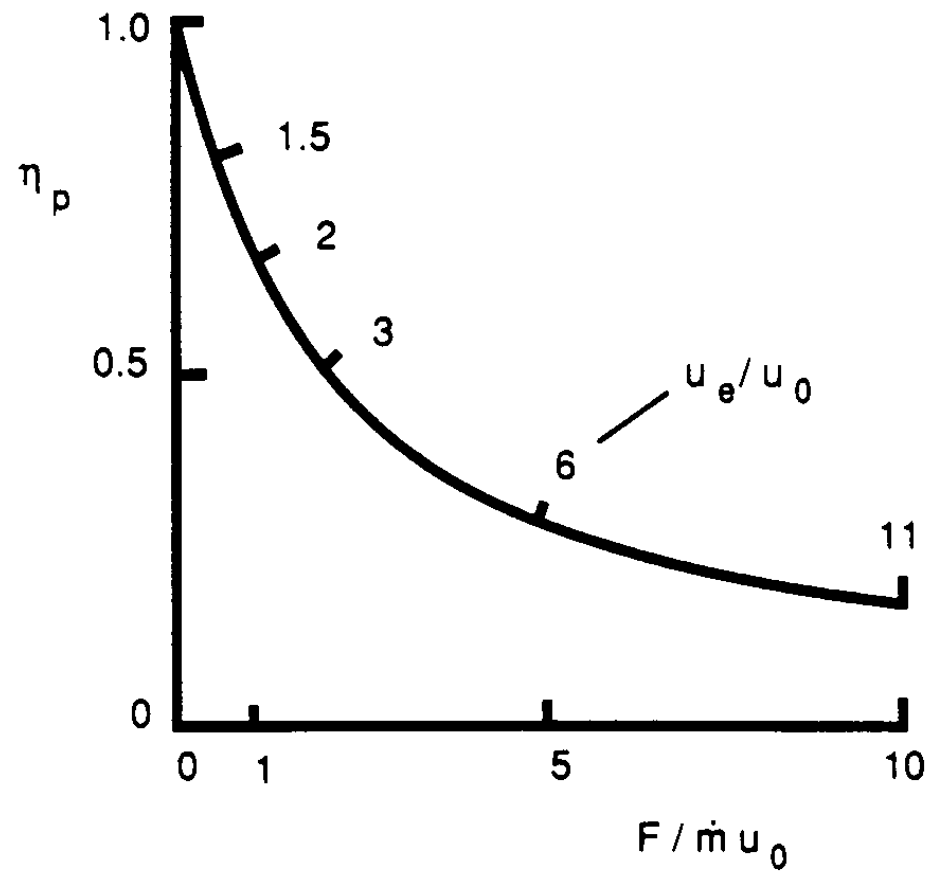
As $\frac{u_e}{u_o} \uparrow$ $\frac{F}{\dot{m}} \uparrow$ $\eta_{\text{prop}} \downarrow$

As $\frac{u_e}{u_o} \downarrow 1$ $\frac{F}{\dot{m}} \downarrow$ $\eta_{\text{prop}} \uparrow$

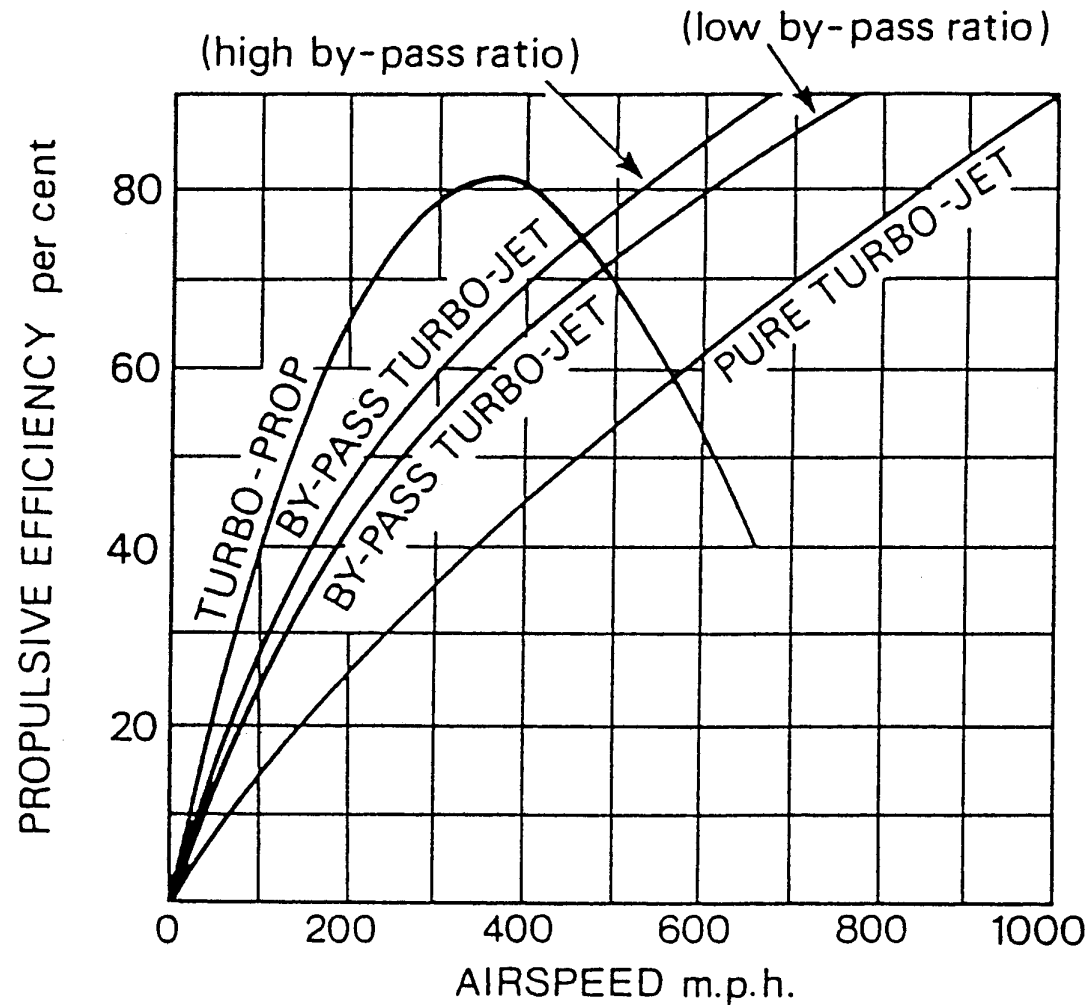
Also, as $\frac{F}{\dot{m}u_o} \uparrow$ $A_{\text{inlet}} \downarrow$ Drag $\downarrow$



PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

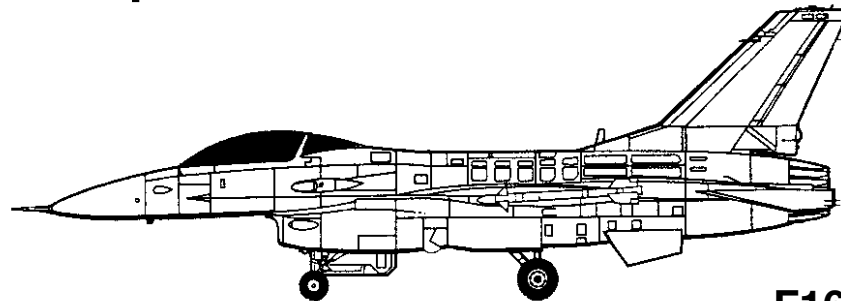


PROPULSIVE EFFICIENCY VS. FLIGHT SPEED



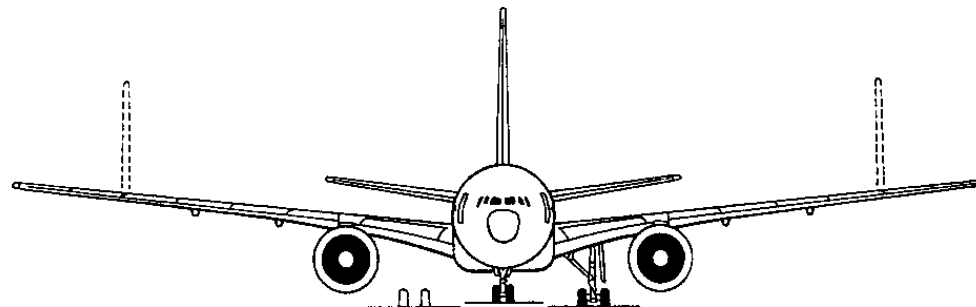
PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

For fighter aircraft that need high thrust/weight and fly at high speed, it is typical to employ engines with smaller inlet areas and higher thrust per unit mass flow



F16-C, *Janes*, 1998-9

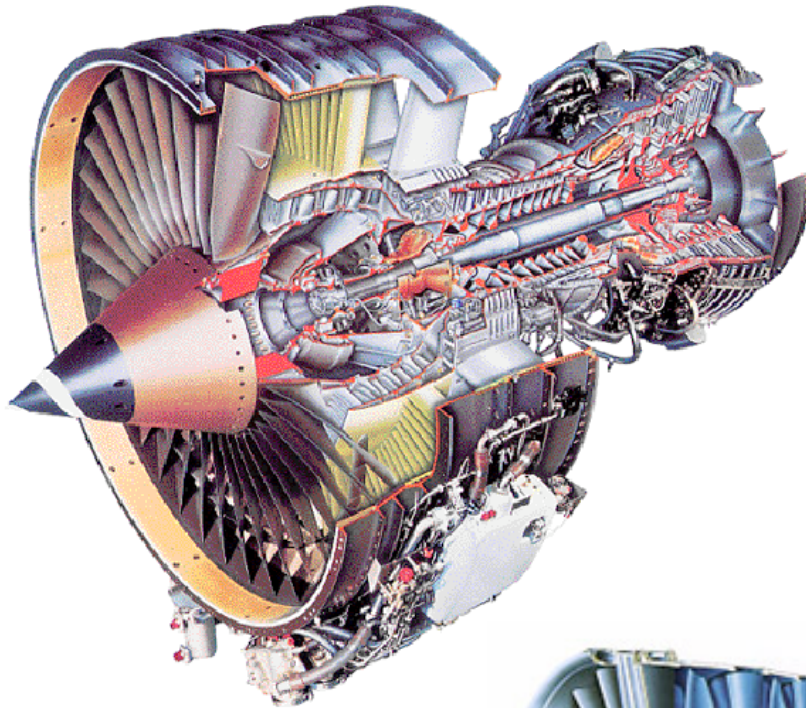
However, transport aircraft that require higher efficiency and fly at lower speeds usually employ engines with relatively larger inlet areas and lower thrust per unit mass flow



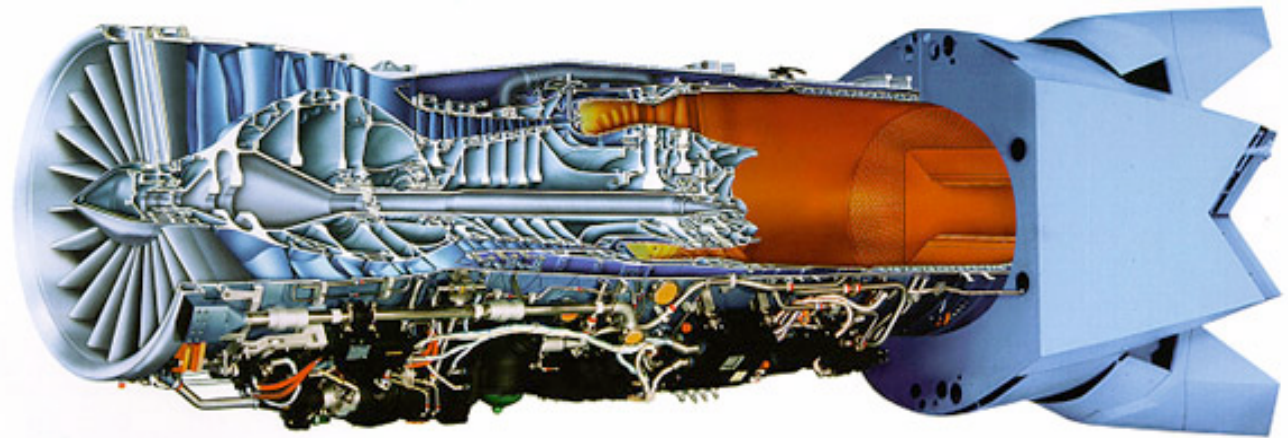
B777-200, *Janes*, 1998-9

COMMERCIAL AND MILITARY ENGINES

(Approx. same thrust, approx. correct relative sizes)

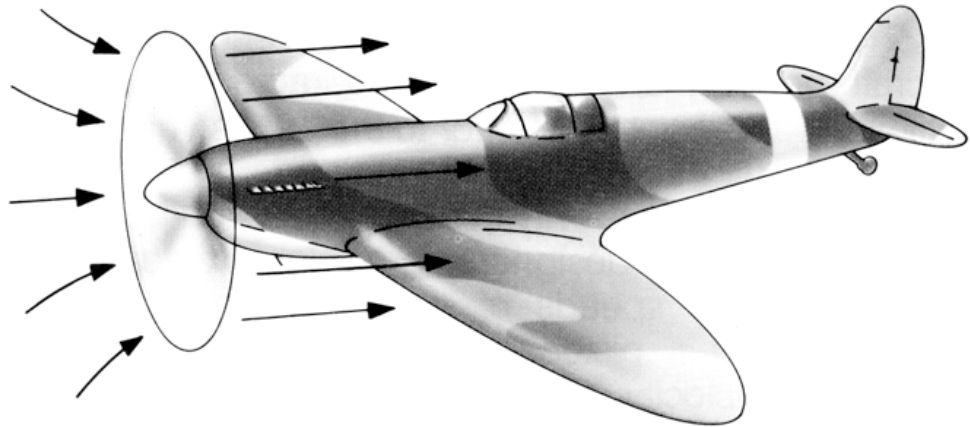


GE CFM56 for Boeing 737

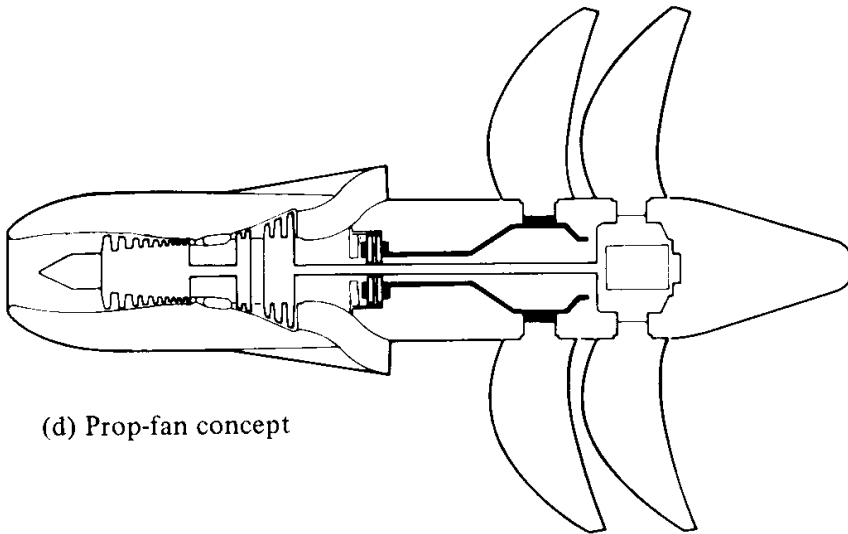


PW JSF119 for Joint Strike Fighter

PROPULSIVE EFFICIENCY



At low flight velocities, the highest propulsive efficiency is typically obtained with a propeller or an unducted fan



(d) Prop-fan concept

AIRCRAFT AND ENGINE DESIGN ISSUES

- Thrust $\approx$ (mass flow) x (change in velocity across engine)
- Range $\sim$ fuel efficiency (commercial and military)
 - Thermal efficiency  High pressures and temperatures
 - Propulsive efficiency  Large mass flow with small velocity change
- Maneuverability (military)
 - High thrust-per-weight, small compact engine  High energy conversion per unit volume (high temperatures and pressures)
- Supersonic flight (military)
 - Low drag, small compact engine  Small mass flow with large velocity change

AN EXAMPLE OF CYCLE PERFORMANCE IMPROVEMENT

