

## Unified Propulsion Quiz

April 24, 2003

One 8 1/2" x 11" sheet (two sides) of notes  
Calculators allowed.

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the pages provided.
- Show intermediate results.
- Explain your work --- don't just write equations.
- Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

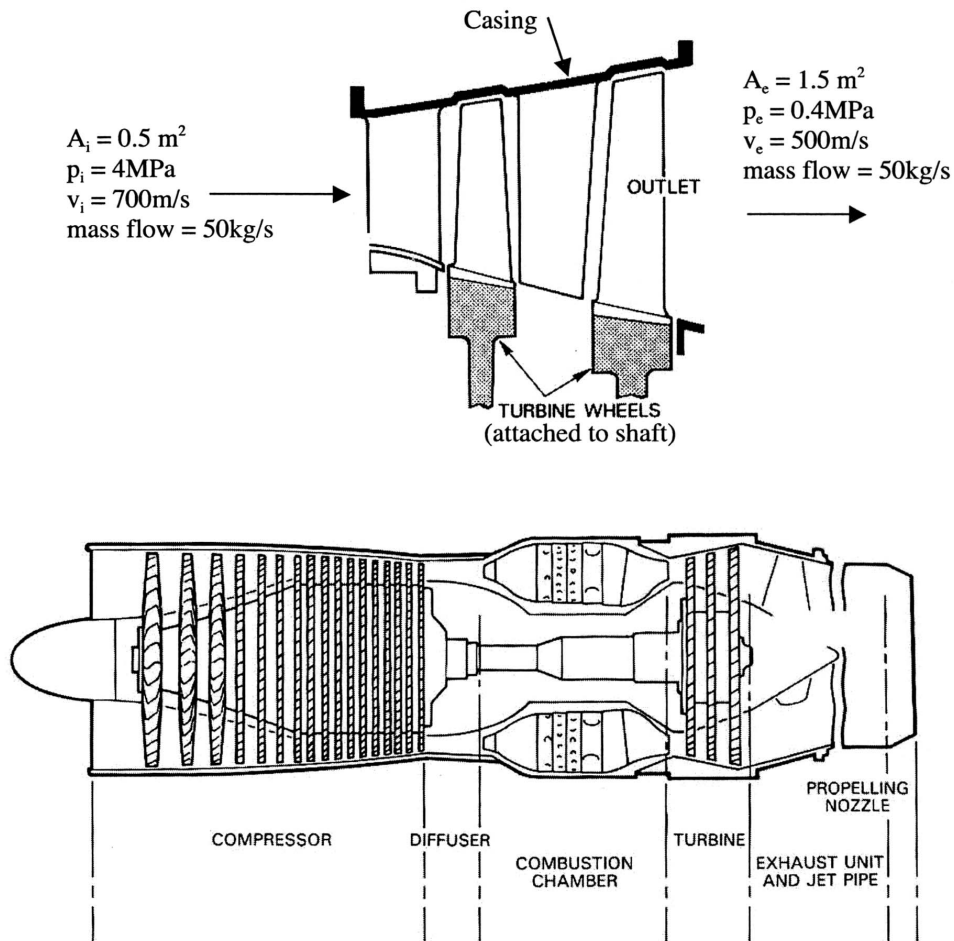
### Exam Scoring

#1 (10%)	
#2 (25%)	
#3 (20%)	
#4 (20%)	
#5 (25%)	
Total	

1. (10 points, partial credit given, L.O. A) In a few sentences describe how jet propulsion works.

Forces are related to changes in momentum (Newton's Second Law). In the case of a rocket, mass stored in the vehicle is thrown backwards giving it momentum. The resultant force propels the vehicle forward. (This is like throwing a heavy stone from a boat floating in the water—the boat moves in the opposite direction. In the case of a jet engine, mass (the air) is taken in with low momentum and it is thrown backwards with higher momentum. The force associated with the change in momentum of the air passing through the engine propels the vehicle forward.

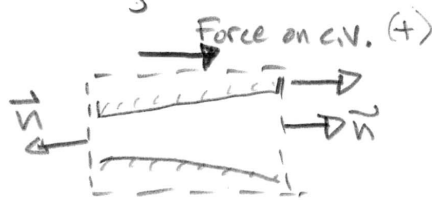
2. (25 points, partial credit given, L.O. B) Below is a schematic of a turbine within a turbojet engine. Parameters describing the flow into and out of the turbine are also given. The changes in flow properties result in a force that is carried by the casing around the blades and the main shaft. What is the magnitude of this force? In what direction does it act?



(BLANK PAGE FOR EXTRA WORK)

$$\sum F_x = \iint_S \rho \mathbf{u} \cdot \mathbf{\hat{n}} \, ds$$

$$\begin{aligned} A_i &= 0.5 \text{ m}^2 \\ p_i &= 4 \text{ MPa} \\ v_i &= 700 \text{ m/s} \\ \dot{m} &= 50 \text{ kg/s} \end{aligned}$$



$$\begin{aligned} A_e &= 1.5 \text{ m}^2 \\ p_e &= 0.4 \text{ MPa} \\ v_e &= 500 \text{ m/s} \\ \dot{m} &= 50 \text{ kg/s} \end{aligned}$$

$$F + p_i A_i - p_e A_e = v_i \underbrace{(-v_i)}_{\mathbf{\hat{u}} \cdot \mathbf{\hat{n}}} \rho A_i + v_e (v_e) \rho A_e$$

$$F = p_e A_e - p_i A_i + \dot{m}(v_e - v_i)$$

$$= (0.4 \times 10^6)(1.5) - (4 \times 10^6)(0.5) + 50(500 - 700)$$

$$F_{\text{on mass in control volume}} = -1410 \text{ kN}$$



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3. (20 points, partial credit given, L.O. C & D) Assume you are an engineer designing a gas turbine engine for a new airplane. You are considering increasing the bypass ratio of the engine. Assume the thrust and thermal efficiency do not change. How might this affect aircraft range? How might this affect aircraft maneuverability? In your answer, **you must make reference to specific figures of merit and equations describing gas turbine engine propulsion and aircraft performance.** Discuss the trade-offs—which parameters will increase, which will stay the same, which will decrease?

Increasing the bypass ratio while maintaining constant thrust results in the following changes to important figures of merit:

Increased propulsive efficiency, therefore increased overall efficiency

Increased drag (engine and therefore airplane)

Increased weight (engine and therefore airplane)

Considering aircraft range:

$$R = \frac{h}{g} \eta_o \frac{L}{D} \ln \frac{W_i}{W_f}$$

Handwritten notes and equations:

- $T \approx \dot{m} (u_e - u_o)$  with  $\uparrow$  above  $T$  and  $\downarrow$  above  $u_o$
- $\eta_p = \frac{2}{1 + \frac{u_e}{u_o}}$  with  $\uparrow$  above  $\eta_p$  and  $\uparrow$  above  $\frac{u_e}{u_o}$
- $\therefore \eta_o = \eta_{th} \eta_p \uparrow$

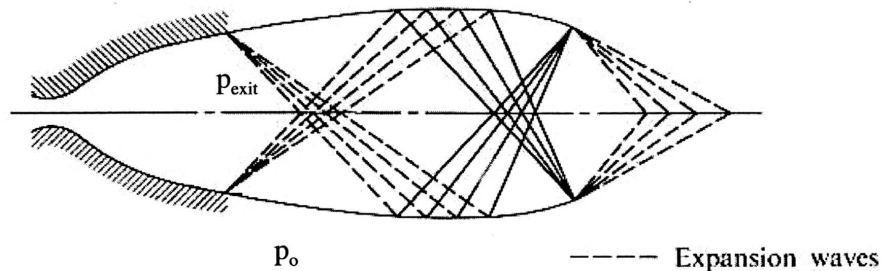
One expects that the L/D ratio will be reduced due to increased drag on the engine and that the weight of the aircraft will increase. The net effect on range depends on whether these detrimental effects outweigh the benefits of the increased engine efficiency.

Considering aircraft maneuverability:

$$TV - DV = W \frac{dh}{dt} + \frac{d}{dt} \left( \frac{1}{2} \frac{W}{g} V^2 \right)$$

One again expects the difference between power available and power required to be reduced due to the increased drag, and also the aircraft weight will increase. All of these things suggest that maneuverability will be compromised unless the increase in propulsive efficiency outweighs these such that for fixed range a lighter fuel load can be carried thereby reducing the aircraft weight, then maneuverability might be improved.

4. (20 points, partial credit given, L.O. C & F) A rocket nozzle is initially designed to be underexpanded such that  $p_{\text{exit}} > p_o$ . Additional nozzle length can be added to further expand the flow. Assume the propellant flow rate is constant. What impacts might this have on vehicle performance and why? In your answer, **you must make reference to specific figures of merit and equations describing rocket propulsion and performance.** Discuss the trade-offs— which parameters will increase, which will stay the same, which will decrease?



Adding additional nozzle length to further expand the flow has the following effects:

- Exit velocity increases
- Exit pressure decreases
- Exit area increases
- Nozzle weight increases

$$T = \dot{m} u_e + A_e (p_e - p_o)$$

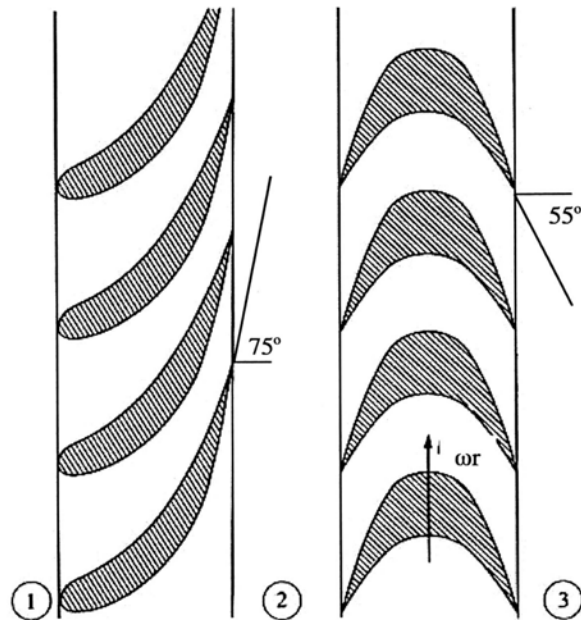
$$I_{sp} = \frac{T}{\dot{m} g}$$

Maximum thrust is for an ideally expanded flow, so although the pressure forces are smaller, the increase in forces due to net momentum flux will outweigh this, leading to increased overall thrust. At the same time the specific impulse will increase, because more thrust will be produced for each unit of propellant flow rate.

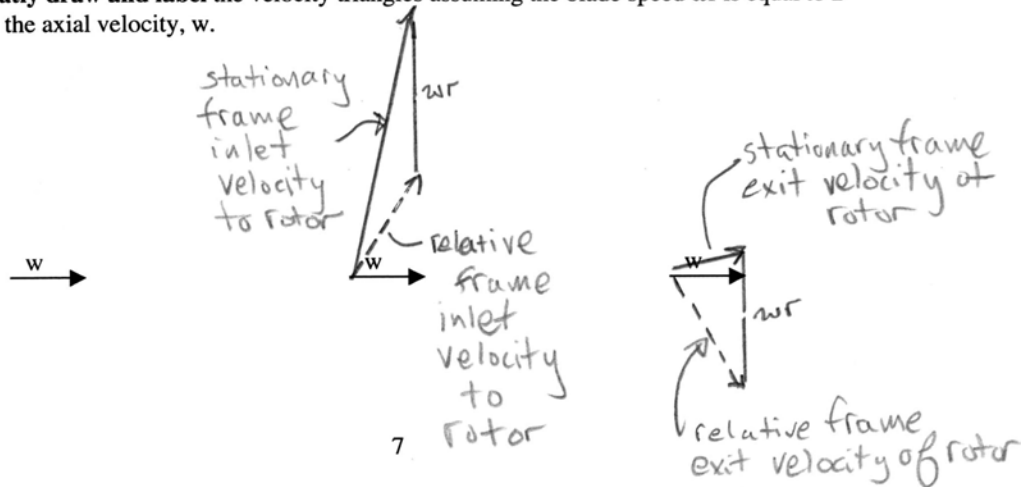
To determine whether vehicle performance will improve it is necessary to understand whether the improvement in how efficiently the propellant is used outweighs the costs of the additional nozzle weight. This would be evaluated using the rocket equation over a particular mission.

$$u = g \left[ I_{sp} \ln \left( \frac{m_{v_0}}{m_v} \right) - t \right]$$

5. (25 points, partial credit given, L.O. I & J) Two rows of turbomachine blades are shown below. At inlet to the first blade row, the flow is purely axial. Assume that the axial component of velocity ( $w$ ) then remains constant through the stage.



a) Neatly draw and label the velocity triangles assuming the blade speed  $\omega r$  is equal to 2 times the axial velocity,  $w$ .



b) Is this a compressor or a turbine? How do you know?

This is a turbine. The stationary frame tangential velocity in the direction of rotor motion is reduced across the moving blade row.

c) On which blade row(s) is there a torque applied? Why?

Torque is applied blade rows since there is a change in angular momentum across each of them. However, power is extracted only from the moving blade row.

$$T = \dot{m} r (\underbrace{u_{\theta \text{ out}} - u_{\theta \text{ in}}}_{\text{stationary frame velocities}})$$
$$(P = \omega T)$$

d) Describe in words the energy exchange processes in each of the two blade rows shown in the figure.

In the first blade row, fluid internal energy is converted to swirling kinetic energy by accelerating the flow through a nozzle. No additional energy is added or removed from the flow. In the second blade row, swirling kinetic energy is extracted from the flow reducing the overall level of energy in the flow and transferring it to the spinning rotor blades.