10:05 - 10: 55
34-101

Name SOLUTIONS

Unified Quiz FM3

March 18, 2009

M - PORTION

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of all M&S Handouts, particularly "HO-M-12", "HO-M-13", and "HO-M-14", along with 2 sides of pages of handwritten material are allowed.***

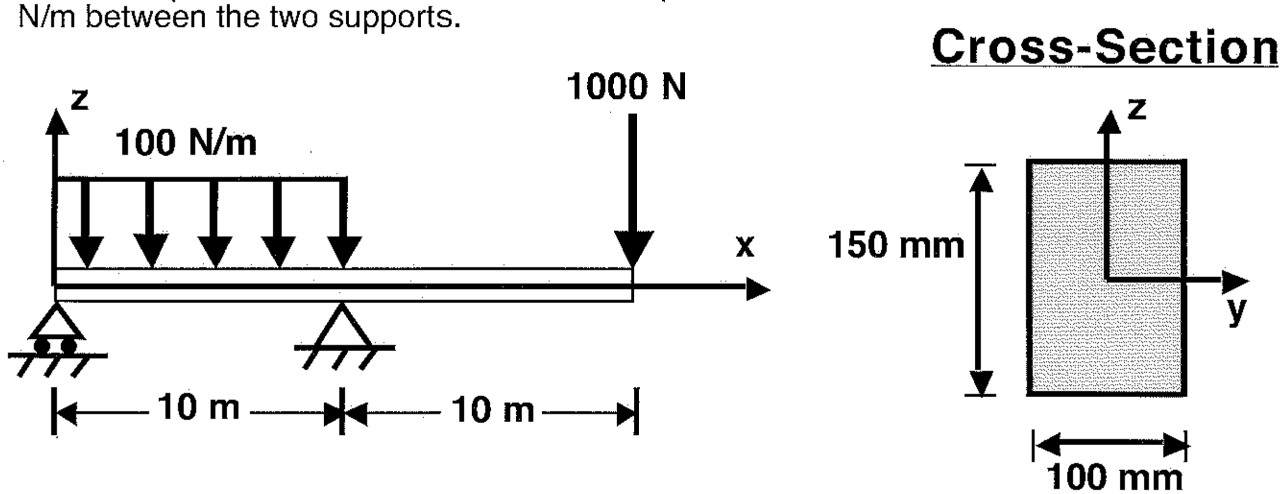
EXAM SCORING

#1M (1/2)	
#2M (1/2)	
FINAL SCORE	

 Name SOLUTIONS

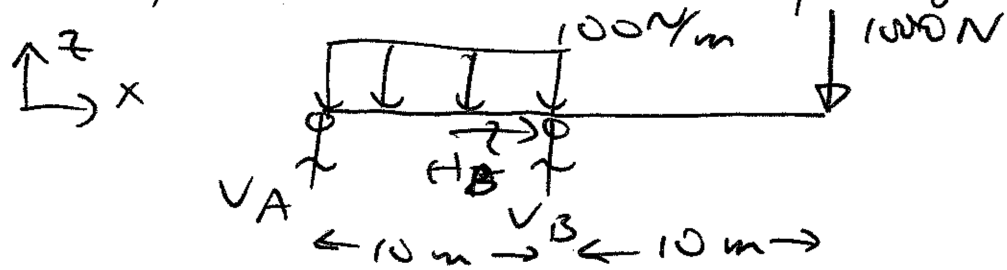
PROBLEM #1M (1/2)

A titanium beam (E = 100 GPa, ν = 0.3) is supported by a roller support at one end and by a pin at its mid-span point. The beam is a total of 20 meters long and has a solid rectangular cross-section with a height of 150 mm and width of 100 mm. The beam has a downward point load of 1000 Newtons at the tip, and a distributed downward load of 100 N/m between the two supports.



(a) Sketch the shear force and bending moment resultant distributions as a function of position along the beam. Be sure to note the key values of each and their locations.

→ Draw the free Body Diagram:



→ Do equilibrium:

$$\sum F_x = 0 \; \xrightarrow{+} \; \Rightarrow H_B = 0$$

$$\sum F_z = 0 \; \uparrow + \; \Rightarrow V_A + V_B - (100\,N/m)(10\,m) - 1000\,N = 0$$

$$\Rightarrow V_A + V_B = 2000\,N$$

$$\sum M_A = 0 \; \circlearrowleft + \; \Rightarrow -\int_0^{10\,m} (100\,N/m)\,x\,dx + V_B(10\,m) - (1000\,N)(20\,m) = 0$$

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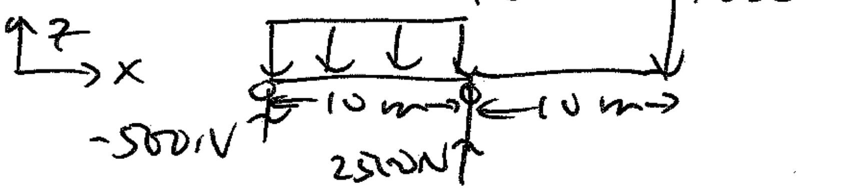
PROBLEM #1M (continued)

$$\Rightarrow 100\,N/m \left.\frac{x^2}{2}\right]_0^{10m}\left(\frac{1}{10m}\right) + 2000\,N = V_B$$

giving $V_B = 2000N + 500N = 2500N$

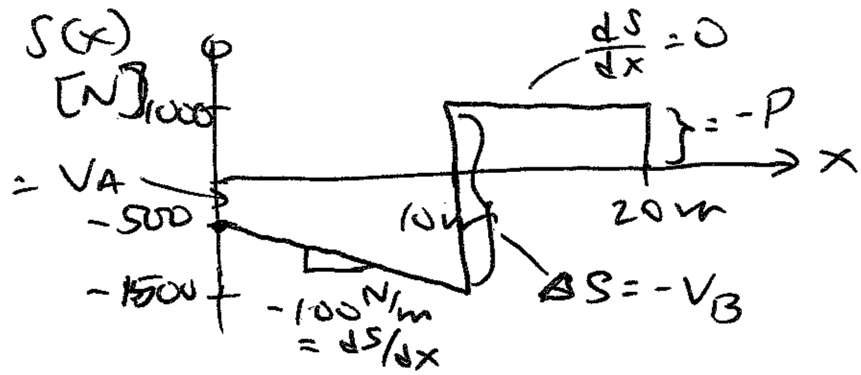
with $V_A + V_B = 2000N \Rightarrow V_A = -500\,N$

→ Redraw FBD:



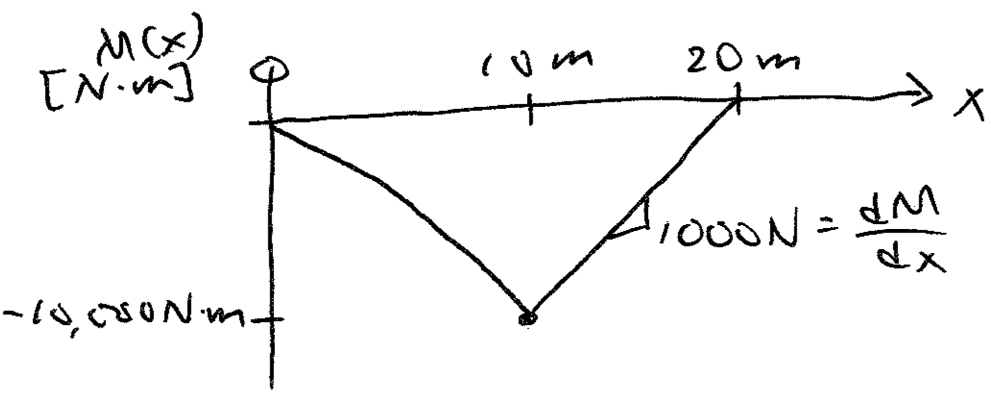
→ use $\frac{dS(x)}{dx} = q(x)$ and that $S(0) = V_A$

$S(20m) = 1000\,N$



→ use $\frac{dM(x)}{dx} = S(x)$ and that $M(0) = 0$

$M(20m) = 0$



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PROBLEM #1M (continued)

(b) Determine the x-location of the maximum shear stress (i.e. σ_{xz}).

$$\rightarrow \quad \sigma_{xz} = -\frac{SQ}{Ib}$$

→ Q, I, and b do not vary with x.

⇒ Maximum $|\sigma_{xz}|$ occurs at maximum $|S(x)|$

$$\Rightarrow \quad \boxed{\text{max at } x = 10 \text{ m (just before)}}$$

(c) Determine the x-location of the maximum axial stress (i.e. σ_{xx}).

$$\rightarrow \quad \sigma_{xx} = -\frac{Mz}{I}$$

→ I and z do not vary with x

⇒ Maximum $|\sigma_{xx}|$ occurs at maximum $|M(x)|$

$$\Rightarrow \quad \boxed{\text{max at } x = 10 \text{ m}}$$

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PROBLEM #1M (continued)

(d) How do the answers to parts (a), (b), and (c) change if steel (E = 200 GPa, ν = 0.3) is used rather than titanium? *Explain your answer* ***clearly.***

→ The configuration is statically determinate

⇒ reactions and thus stress resultants are determined only by equilibrium and material behavior does not enter in

⇒ stresses directly tied to stress resultants and thus do not depend on material behavior

⇒ DO NOT CHANGE

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PROBLEM #1M (continued)

(e) How does the maximum deflection of the beam and its location change when the beam is made of steel rather than titanium? *Explain your answer **clearly**.*

→ The deflection is related to the moment via:

$$\frac{d^2w}{dx^2} = \frac{M}{EI}$$

→ The moment M(x), resultant does not change with material since the configuration is statically determinate. The boundary conditions (w = 0 @ x = 0; w = 0 @ x = 10 m) also stay the same. The cross-section and thus I, stays the same. Thus, the modulus of the material enters into this.

$$\Rightarrow \quad w \propto 1/E$$

$$\text{so:} \quad \frac{w_{Ti}}{w_{steel}} = \frac{1/E_{Ti}}{1/E_{steel}} = \frac{1/100\,\text{GPa}}{1/200\,\text{GPa}} = 2$$

⇒ $\boxed{\text{deflection decreases to } 1/2 \text{ of Titanium value with maximum location unchanged}}$

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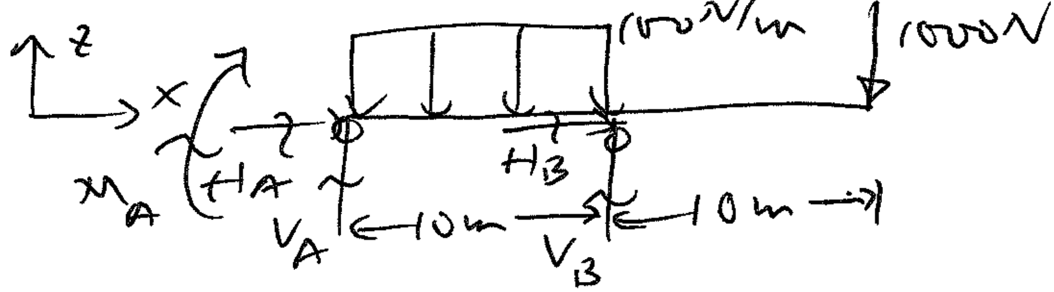
PROBLEM #1M (continued)

(f) The roller support at x=0 is replaced by a clamped support. Would the procedure for determining the answers to part (a) change? Be sure to explain ***clearly***. Use figures, ratios, etc. as appropriate. *Obtaining final values or operative quantified equations is* ***not necessary***.

→ Draw the new Free Body Diagram!



→ There are now more reactions than degrees of freedom (and thus equations of equilibrium)

⇒ Statically Indeterminate

⇒ It is now necessary to include the entire behavior of the beam and the constitutive response (deflection behavior) and solve these simultaneously to determine the reactions

⇒ Procedure does change

Unified Quiz FM3 - Materials and Structures Portion
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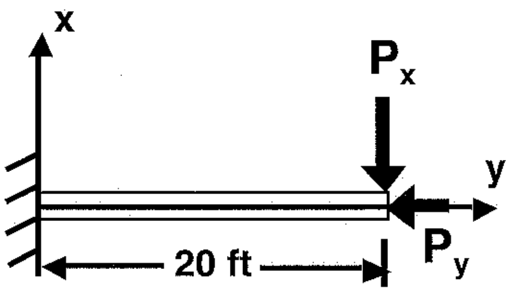
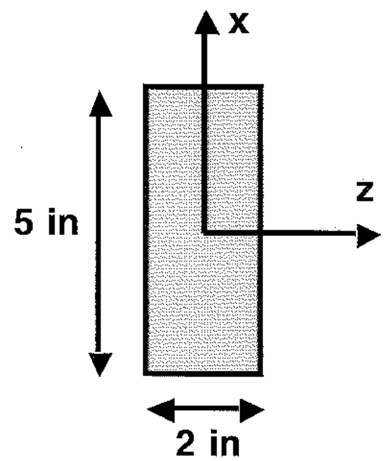
PROBLEM #2M (1/2)

A 20-foot long aluminum beam (E = 10 Msi, ν = 0.3) is cantilevered in the x-y plane with a clamped support at its root. The beam has a rectangular cross-section 2 inches across and 5 inches deep. The structural configuration, as illustrated in the accompanying figure, is loaded by a vertical tip load of magnitude P_x, causing bending deformation in the x-direction. This results in a maximum axial stress, σ_{yy}, of 20 ksi.



Cross-Section



The structural configuration is *subsequently* ***also*** subjected to a horizontal tip load along its primary axis, P_y, of 100,000 pounds.

(a) Determine how the horizontal tip load affects the maximum values of the axial stress, σ_{yy}, in both tension and compression Quantify as best as you can. **Clearly** explain any modeling assumptions and associated limitations.

→ To first order, the two separate models (beam and rod) can be superposed (linear modeling).

→ One must note the axes are different than in the normal models (i.e. long direction is along y-axis) and account for such. This is only notation

→ Axial stress due to the axial load via the rod model:

$$\sigma_{YY} = \frac{P_Y}{A}$$

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PROBLEM #2M (continued)

$$A = (2\,in)(5\,in) = 10\,in^2$$

$$P_y = -100{,}000\,lbs$$

$$\Rightarrow \sigma_{yy_{rod}} = \frac{-100{,}000\,lbs}{10\,in^2} = -10\,ksi$$

→ Maximum axial stress due to bending in tension and compression $= \pm 20\,ksi$

→ Total axial stress = rod stress + beam stress

$\Rightarrow$ Max tensile axial stress $= -10\,ksi + 20\,ksi$
$= +10\,ksi$
$\Rightarrow$ reduced by 50%

Max compressive axial stress $= -10\,ksi - 20\,ksi$
$= -30\,ksi$
$\Rightarrow$ increased by 50%

PROBLEM #2M (continued)

(b) You can double one cross-sectional dimension, while keeping the total area constant, in order to reduce the maximum deflection. Which dimension would you change to make this the most effective? **Clearly** explain your reasoning. Also indicate how this change would affect the total axial stress, σ_{yy}, for this two-load configuration.

→ deflection related to loading and geometry via: $\frac{d^2 w}{dy^2} = \frac{M}{EI}$

→ all else stays constant except I

$\Rightarrow w \propto 1/I$

→ for rectangular cross-section: $I = \frac{bh^3}{12}$

→ $A = bh = \text{constant}$

$\Rightarrow$ to increase I, $\boxed{\text{increase } h}$ by factor of 2 $\Rightarrow$ b decreased by factor of 2

giving $I' = \frac{b'(h')^3}{12} = \frac{(b/2)(2h)^3}{12} = 4\frac{bh^3}{12}$

$'$ = new

$\Rightarrow I$ increase by 4

→ $\sigma_{yy\,bending} = -\frac{Mx}{I}$

x increases by factor of 2
I increases by factor of 4

$\Rightarrow \sigma'_{yy\,bending} = -\frac{M(2x)}{4I} = \frac{1}{2}\left(-\frac{Mx}{I}\right) = \frac{1}{2}\left(\sigma_{yy\,bending}\right)$

$= \frac{1}{2}(\pm 20\,\text{ksi}) = \pm 10\,\text{ksi}$

→ $\sigma'_{yy\,rod} = \frac{P}{A} = \text{constant} = -10\,\text{ksi}$

$\Rightarrow$ $\boxed{\sigma'_{yy\,total} = 0\,\text{ksi}, -20\,\text{ksi}}$

 Name SOLUTIONS

PROBLEM #2M (continued)

(c) Describe how you would check results to determine whether your modeling is applicable.

The main thing is to check for $\boxed{\text{consistency}}$. This means checking to see that results are "consistent" with the assumptions and limitations. For example, the full stress and strain states could be calculated and compared to the baseline assumptions. The judgement would then be based on "how good" these are (i.e. "how consistent") -- it is not a clear Yes/No.