9:05 - 9: 55
34-101

Name SOLUTIONS

Unified Quiz M5

April 29, 2009

<u>M - PORTION</u>

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit <u>cannot</u> be given without intermediate results.
- Show the logical path of your work. Explain <u>clearly</u> your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the final answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units throughout. Answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators are allowed.***
- ***Print-outs of all M&S Handouts, particularly "HO-M-15", along with 2 sides of pages of handwritten material are allowed.***

EXAM SCORING

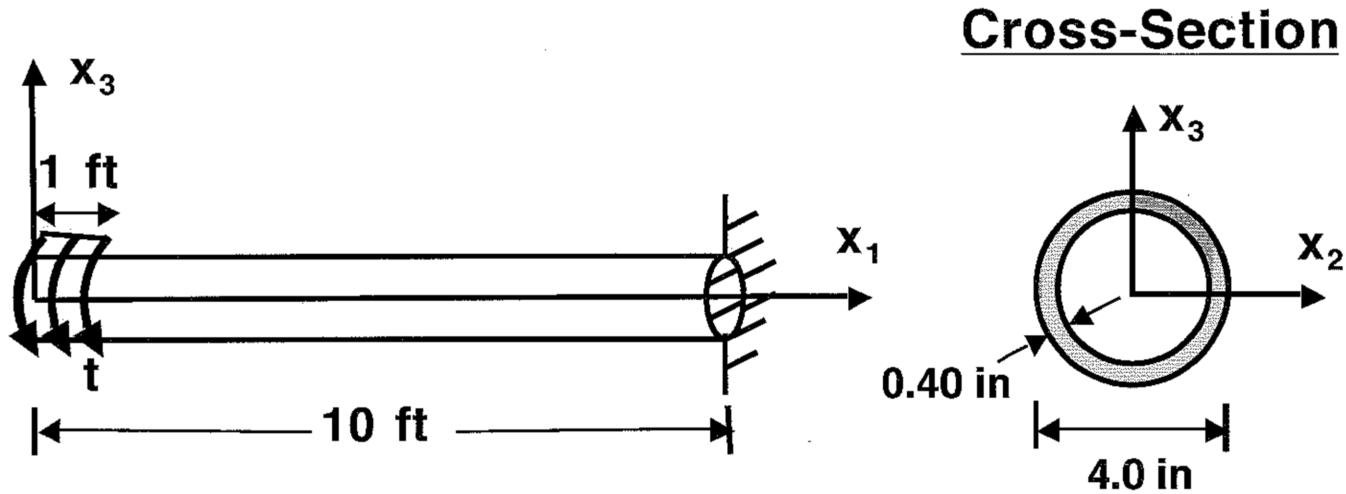
#1M (1/3)	
#2M (1/3)	
#3M (1/3)	
FINAL SCORE	

Unified Quiz M5 - Materials and Structures Portion
April 29, 2009

Name SOLUTIONS

PROBLEM #1M (1/3)

A shaft configuration has been designed as a means to transmit a constant distributed torque applied over 1 foot at a free end, to a device at the other end. This device can be represented as a clamped support. The shaft is 10 feet long and is a tube with an outer diameter of 4 inches and a wall thickness of 0.40 inches. Two materials are being considered for use. The first is steel which has a Young's modulus of 30 Msi, a Poisson's ratio of 0.30, and a yield stress of 50 ksi. The second is titanium which has a Young's modulus of 15 Msi, a Poisson's ratio of 0.30, and a yield stress of 210 ksi.



(a) A critical consideration in the design is the rotation of the free end at the critical applied torque. Determine the ratio of that rotation for the two materials being considered.

→ Governing equation is: $\frac{d\phi}{dx_1} = \frac{T(x_1)}{GJ}$

→ The torque loading $T(x_1)$ will not change with a change in material

→ The boundary conditions will not change with a change in material

→ J is due to the geometrical configuration and thus will not change with a change in material

→ G and J are constant with respect to x_1

Thus: $\phi_{tip} \propto \frac{1}{GJ} \int T(x_1)$ @ $x = 0$ ft with boundary conditions

PROBLEM #1M (continued)

So with $T(x,)$, boundary conditions, and J the same regardless of material, a constant, C, can be used to say:

$$\phi_{tip} = \frac{C}{G} \qquad (1)$$

Thus:

$$\frac{\phi_{tip\,steel}}{\phi_{tip\,Titanium}} = \frac{C/G_{steel}}{C/G_{titanium}} = \frac{G_{titanium}}{G_{steel}}$$

→ The shear modulus can be determined via:

$$G = \frac{E}{2(1+\nu)}$$

for both materials: $\nu = 0.3$

So the ratio of shear modulus is the ratio of the extensional (Young's modulus):

$$\frac{\phi_{tip\,steel}}{\phi_{tip\,titanium}} = \frac{G_{titanium}}{G_{steel}} = \frac{E_{titanium}}{E_{steel}} = \frac{15\,Msi}{30\,Msi}$$

$$\Rightarrow \boxed{\frac{\phi_{tip\,steel}}{\phi_{tip\,Titanium}} = \frac{1}{2}}$$

 Name SOLUTIONS

PROBLEM #1M (continued)

(b) The maximum shear stress in the shaft is also an important consideration in the design. Determine the ratio of that for the two materials being considered.

→ Governing equation is: $\tau_{res} = \frac{Tr}{J}$

→ As noted in (a): T does not change with material, and J does not change with material

→ The maximum shear stress occurs for the maximum value of r. This is a geometrical consideration and thus does not change between the two materials

Thus: $$\boxed{\frac{\tau_{max\ steel}}{\tau_{max\ titanium}} = 1}$$

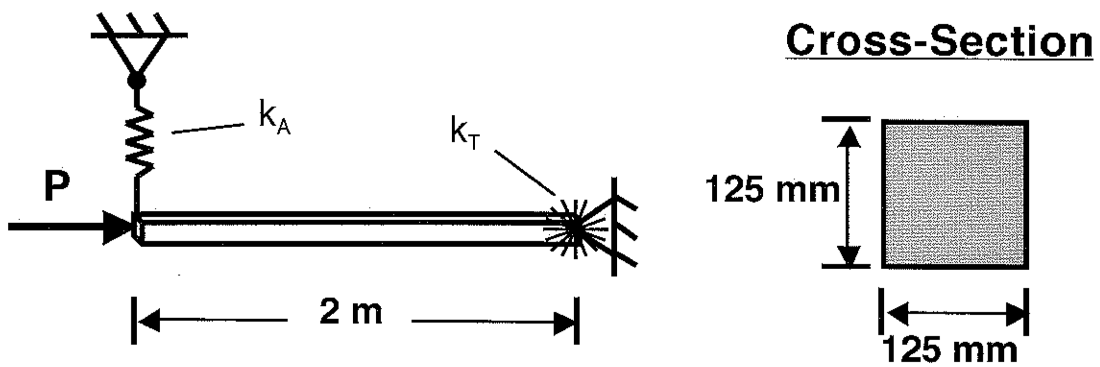
same value for all materials

Unified Quiz M5 - Materials and Structures Portion
April 29, 2009

Name SOLUTIONS

PROBLEM #2M (1/3)

A component of a load transfer device in an industrial machinery system has a square cross-section of 125 mm to a side and is 2 meters long. This piece can be modeled as a component that is attached to a pin support via a torsional spring of stiffness k_T at one end, and has a compressive load applied at the other end with a linear spring of stiffness k_A providing transverse support. The component is made of machinery-grade steel with a modulus of 200 GPa and an ultimate stress of 400 MPa.



Set up the equation(s) needed to determine the maximum load of this component assuming that manufacturing, alignment, and loading are "perfect". This includes contributions due to ***any*** deformation prior to instability. Describe how you would use the resulting equation(s) to determine the response but **DO NOT SOLVE**. Use figures if/as appropriate.

→ Failure/maximum load can occur due to buckling/instability or compressive ultimate

→ first buckling/instability

· In the "perfect" case as noted here, there are no deformations perpendicular to the load prior to the critical load, so there is only a need to solve for the critical load, P_{cr}

· Start with the general solution to the governing relationship for out-of-plane deflection (u_3) of a column:

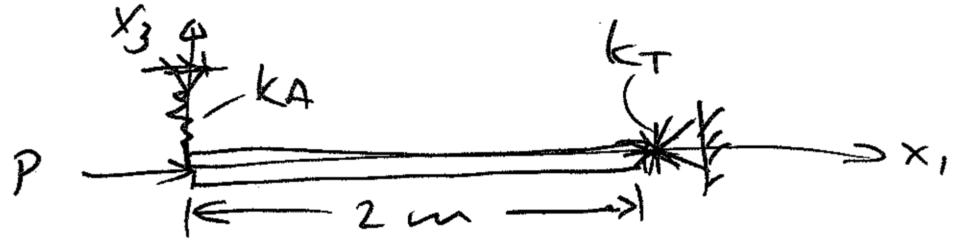
$$u_3 = A \sin\sqrt{\frac{P}{EI}}\,x_1 + B\cos\sqrt{\frac{P}{EI}}\,x_1 + C + Dx_1$$

where x_1 is the dimension along the column.

 Name SOLUTIONS

PROBLEM #2M (continued)

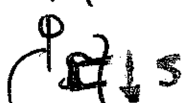
- Place $x_1 = 0$ at the end with the load:



- To find the constants and the eigenvalue (P_{cr}), need to use the boundary conditions:

@ $x_1 = 0$ with the vertical spring:

$$M = 0 = EI \frac{d^2u_3}{dx_1^2} \quad (a)$$



$F_{spring} = (-k_A u_3)$

$$\Sigma F = 0 \uparrow + \Rightarrow F_{spring} + S = 0$$

$$\Rightarrow S = k_A u_3 = EI \frac{d^3u_3}{dx_1^3} \quad (b)$$

@ $x_1 = 2m$, the pin implies: $u_3 = 0$ (c)

and for the torsional spring:

$$M = -k_T \frac{du_3}{dx_1} \Rightarrow -k_T \frac{du_3}{dx_1} = EI \frac{d^2u_3}{dx_1^2} \quad (d)$$

slope

- Now find the derivatives of the general solution so that the B.C.'s can be used to set up the equations to be solved

<u>Note</u>: for ease of writing use $\lambda = \sqrt{\frac{P}{EI}}$

Name SOLUTIONS

PROBLEM #2M (continued)

So: $u_3 = A\sin\lambda x_1 + B\cos\lambda x_1 + C + Dx_1$

$$\frac{du_3}{dx_1} = A\lambda\cos\lambda x_1 - B\lambda\sin\lambda x_1 + D$$

$$\frac{d^2u_3}{dx_1^2} = -A\lambda^2\sin\lambda x_1 - B\lambda^2\cos\lambda x_1$$

$$\frac{d^3u_3}{dx_1^3} = -A\lambda^3\cos\lambda x_1 + B\lambda^3\sin\lambda x_1$$

- Apply the B.C.'s one at a time:

(a) $\Rightarrow \quad 0 = -A\lambda^2\sin(\lambda 0) - B\lambda^2\cos(\lambda 0)$

giving: $B\lambda^2 = 0$

(b) $\Rightarrow \frac{k_A}{EI}\left(A\sin(\lambda 0) + B\cos(\lambda 0) + C\right) = -A\lambda^3\cos(\lambda 0) + B\lambda^3\sin(\lambda 0)$

giving: $(B+C)\frac{k_A}{EI} = -A\lambda^3 \Rightarrow A + (B+C)\frac{k_A}{\lambda^3 EI} = 0$

(c) $\Rightarrow A\sin\lambda L + B\cos\lambda L + C + DL = 0$

(d) $\Rightarrow -\frac{k_T}{EI}\left(A\lambda\cos\lambda L - B\lambda\sin\lambda L + D\right) = -A\lambda^2\sin\lambda L - B\lambda^2\cos\lambda L$

giving: $\frac{k_T}{\lambda^2 EI}\left(A\lambda\cos\lambda L - B\lambda\sin\lambda L + D\right) = A\sin\lambda L + B\cos\lambda L$

$\Rightarrow A\left(\frac{k_T}{\lambda EI}\cos\lambda L - \sin\lambda L\right) + B\left(-\frac{k_T}{\lambda EI}\sin\lambda L - \cos\lambda L\right) + D\frac{k_T}{\lambda^2 EI} = 0$

- Assemble these into matrix form:

 Name SOLUTIONS

PROBLEM #2M (continued)

$$\boxed{\begin{bmatrix} 0 & \lambda^2 & 0 & 0 \\ 1 & \frac{k_A}{\lambda^3 EI} & \frac{k_A}{\lambda^3 EI} & 0 \\ \sin\lambda L & \cos\lambda L & 1 & L \\ \left(\frac{k_T}{\lambda EI}\cos\lambda L - \sin\lambda L\right) & \left(-\frac{k_T}{\lambda EI}\sin\lambda L - \cos\lambda L\right) & 0 & \frac{k_T}{\lambda^2 EI} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \underset{\sim}{0}}$$

→ Solve this to find the eigenvalues (lowest = P_{cr})

with: $L = 2\,m$

$$I = \frac{bh^3}{12} = \frac{(125\,mm)(125\,mm)^3}{12} = 2.03 \times 10^{-5}\,m$$

$$\lambda = \sqrt{\frac{P}{EI}}$$

$E = 200\,GPa$

→ For compressive ultimate:

$$\frac{P}{A} = \sigma_{ult} = 400\,MPa$$

$$A = (.125\,m)(.125\,m) = 0.01563\,m^2$$

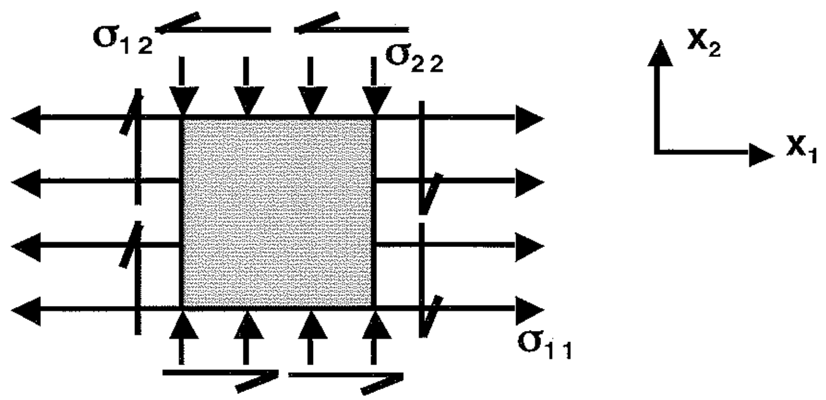
$$\Rightarrow \boxed{P_{comp\,ult} = 6.25 \times 10^6\,N}$$

→ Compare the two loads and the failure is at the lowest value of P

Unified Quiz M5 - Materials and Structures Portion
April 29, 2009 **Name** SOLUTIONS

PROBLEM #3M (1/3)

A joint structure is to be designed to withstand in-plane stresses using either the basic strength approach or the damage tolerance approach. The loading is such that the stresses along each axis have one in tension and one in compression, with a ratio in magnitude of 3:1, tension to compression. There is also a negative shear stress with the shear stress being half in magnitude to the tensile stress. Aluminum and steel are being considered for this piece. The particular aluminum has a modulus of 70 GPa, a Poisson's ratio of 0.30, a value of the tensile yield stress of 200 MPa, a value of the tensile ultimate strength of 350 MPa, and a value of fracture toughness of 30.0 MPa(m)$^{1/2}$. The particular steel has a modulus of 200 GPa, a Poisson's ratio of 0.31, a value of the tensile yield stress of 350 MPa, a value of the tensile ultimate strength of 500 MPa, and a value of fracture toughness of 50.0 MPa(m)$^{1/2}$.



The objective of the design is to minimize weight. The density of the aluminum is 2.7 Mg/m^3, that of the steel is 8.0 Mg/m^3.

(a) Using the von Mises criterion and the tensile ultimate strength as the failure criterion, the necessary thickness for the aluminum is determined to be 10 mm for the critical loading. Determine which of the two materials better fulfills the design objective using this failure criterion. Explain carefully using equations as needed.

→ The primary expression of the von Mises criterion is:

$$(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2 = 2\sigma_{ref}^2$$

& using ultimate as specified here

- This is plane stress, so $\sigma_{III} = 0$
- Stress state orientation and thus principal stresses will not change with material, but only to related to same stress factor, A:

PROBLEM #3M (continued)

$\sigma_{11} = 3A \qquad \sigma_{22} = -A \qquad \sigma_{12} = -\frac{3}{2}A$

This will give: $\sigma_I = C_1 A \qquad \sigma_{II} = C_2 A$

Thus, the critical value of A will depend on the σ_{ult} of the material:

$$C_3 A = \sigma_{ult}$$

$$\Rightarrow \frac{A_{aluminum}}{A_{steel}} = \frac{350\ MPa}{500\ MPa} = 0.7$$

→ The stress is related to thickness via:

$$\sigma = \frac{P}{wt} \Rightarrow \sigma \propto 1/t$$

The critical loading and other geometry do not change with material. Thus:

$$\frac{A_{aluminum}}{A_{steel}} = 0.7 = \frac{1/t_{aluminum}}{1/t_{steel}}$$

→ weight/mass will be proportional to the density times the thickness (with all other geometry being unchanged)

$$\frac{m_{aluminum}}{m_{steel}} = \frac{\rho_{al}}{\rho_{steel}}\frac{t_{al}}{t_{steel}} = \frac{2.7\ Mg/m^3}{8.0\ Mg/m^3}\frac{1}{0.7}$$

$$= 0.5$$

$\Rightarrow$ $\boxed{\text{Aluminum minimizes weight/mass}}$

PROBLEM #3M (continued)

(b) Using the damage tolerance approach for Mode I considerations and using only the stress normal to a critical crack size of 16.0 mm as the failure criterion, the necessary thickness for the aluminum is determined to be 8.0 mm for the critical loading. Determine which of the two materials better fulfills the design objective using this failure criterion. Explain carefully using equations as needed.

→ The Griffith equation governing this is

$$\lambda \sigma \sqrt{\pi a} = K$$

→ Looking to use critical Mode I value of the fracture toughness, K_{IC},

→ Use largest normal stress perpendicular to potential crack ⇒ largest principal stress

→ as in (a), largest principal stress will be ratioed to stress factor A : $C_\phi A$

$a_{cr} = 8.0$ mm

Ratioing:
$$\frac{\lambda C_\phi A_{aluminum} \sqrt{\pi (8.0\,mm)}}{\lambda C_\phi A_{steel} \sqrt{\pi (8.0\,mm)}} = \frac{K_{IC\,aluminum}}{K_{IC\,steel}}$$

$$\Rightarrow \frac{A_{aluminum}}{A_{steel}} = \frac{30.0\,MPa(m)^{1/2}}{50.0\,MPa(m)^{1/2}} = 0.6$$

→ again, the stress is related to thickness and thus:

$$\frac{A_{aluminum}}{A_{steel}} = 0.6 = \frac{1/t_{aluminum}}{1/t_{steel}}$$

 Name SOLUTIONS

PROBLEM #3M (continued)

→ Again, weight/mass is proportional to the density times the thickness

$$\frac{M_{aluminum}}{M_{steel}} = \frac{\rho_{al}}{\rho_{steel}} \frac{t_{al}}{t_{steel}} = \frac{2.7\, Mg/m^3}{8.0\, Mg/m^3} \frac{1}{0.6}$$

$$= 0.55$$

⇒ $\boxed{\text{Aluminum minimizes weight/mass}}$