UE Fluids Quiz 2 Solution S '09

1) a) Steady adiabatic flow has $h_{o_1} = h_{o_2}$ or $\boxed{\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}\left[1+\frac{\gamma-1}{2}M_1^2\right] = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2}\left[1+\frac{\gamma-1}{2}M_2^2\right]}$ (10)

b) Isentropic flow has $\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma}$ (10)

2) a) For static quantities across the shock $()_1 = ()_i$, $()_2 = ()_f$ (10)

Given: $\frac{p_2}{p_1} = \frac{p_f}{p_i} = 1.8$, from shock table $\rightarrow \frac{\rho_2}{\rho_1} = \frac{\rho_f}{\rho_i} = 1.516$, so $\boxed{\rho_f = 1.516\ kg/m^3}$

b) From shock table for $\frac{p_2}{p_1} = 1.8$, $M_1 = 1.30 = \frac{V_1}{a_1}$, and $a_1 = \sqrt{\frac{\gamma p_1}{\rho_1}} = \sqrt{\frac{1.4\cdot 10^5}{1}} = 374\ m/s$

In shock frame $V_1 = M_1 \cdot a_1 = 1.3 \cdot 374\ m/s = \boxed{486.4\ m/s = V_s}$ (10)

tube frame: V_s ←
shock frame: V_1 → $V_1 = V_s$

c) From shock table for $\frac{p_2}{p_1} = 1.8$, $M_2 = 0.786$, and $a_2 = \sqrt{\frac{\gamma p_2}{\rho_2}} = 407.7\ m/s$

$V_2 = M_2 \cdot a_2 = 0.786 \cdot 407.7\ m/s = 320.46\ m/s$, $\boxed{V_f = V_s - V_2 = 165.9\ m/s}$ (10)

V_f, V_s ← ; $V_2 = V_s - V_f$ → V_2

d) $\boxed{p_{o_f} = p_f\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}} = 2.706\times 10^5\ Pa}$ (10)

<u>or</u> using shock table for $\frac{p_2}{p_1} = 1.8$: $\frac{p_{o_2}}{p_1} = 2.714$, $p_{o_f} = p_{o_2} = p_i \cdot \frac{p_{o_2}}{p_1} = 2.71\times 10^5\ Pa$ (same)

e) In tube frame, $M_f = \frac{V_f}{a_2} = \frac{165.9\ m/s}{407.7\ m/s} = 0.407$, $\boxed{p_{o_f} = p_f\left(1+\frac{\gamma-1}{2}M_f^2\right)^{\frac{\gamma}{\gamma-1}} = 2.017\times 10^5\ Pa}$ (10)

3) a) Given: $\frac{p_{o_2}}{p_1} = \frac{p_{stag}}{p_\infty} = \frac{3\ atm}{1\ atm} = 3.0$, from shock table $\rightarrow \boxed{M_1 = M_\infty = 1.38}$ (10)

b) Given $\frac{p_{o_2}}{p_1} = \frac{3\ atm}{2\ atm} = 1.5$ not on shock table (subsonic flow, no shock).

Using isentropic table for $\frac{p_o}{p} = 1.5 \rightarrow \boxed{M_1 = M_\infty = 0.78}$ (10)

c) For shock $M_1 = 1.38$, $\frac{p_{o_2}}{p_{o_1}} = 0.963$, $\frac{p_2}{p_1} = 2.055$

$\therefore p_{o_1} = \frac{p_{o_2}}{p_{o_2}/p_{o_1}} = \frac{3\ atm}{0.968} = 3.10\ atm$ (10)

$p_2 = p_1 \cdot \frac{p_2}{p_1} = 1\ atm \cdot 2.055 = 2.055\ atm$