UE Fluids — Quiz 3 Solution — S'09

Problem 1:

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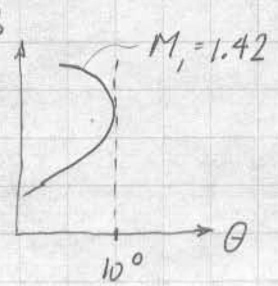
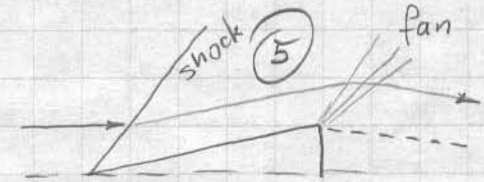
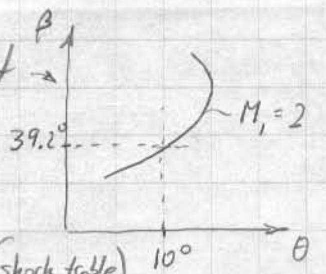
a) For $\theta = 10°$, minimum Mach for attached shock is $\boxed{M_\infty = 1.42}$



For $M_\infty > 1.42$ flow is:



b) For $M_1 = 2$, $\theta = 10° \rightarrow \beta = 39.2°$ from oblique shock chart



$M_{n_1} = M_1 \sin\beta = 1.26$

For $M_{n_1} = 1.26 \rightarrow M_{n_2} = 0.8071$, $\frac{p_2}{p_1} = 1.686$, $\frac{p_{o2}}{p_{o1}} = 0.9857$ (shock table)

$\boxed{p_a = p_\infty \cdot \left(\frac{p_2}{p_1}\right) = 1.686 \text{ atm}}$ 10 $\boxed{p_{oa} = p_{o\infty}\left(\frac{p_{o2}}{p_{o1}}\right) = p_\infty\left[1 + \frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} \cdot 0.9857 = 7.7126 \text{ atm}}$ 10

Alternative p_{oa} calc: $M_2 = \frac{M_{n2}}{\sin(\beta-\theta)} = 1.6544$, $p_{oa} = p_a\left[1 + \frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}} = 7.76$ atm close

c) $p_{ob} = p_{oa}$ since fan is isentropic, $\therefore \frac{p_{ob}}{p_b} = \frac{p_{oa}}{p_b} = \frac{7.7126}{0.6} = 12.8$

$$M_b = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_{ob}}{p_b}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = 2.32 \quad \text{(or use isentropic flow table)}$$

$M_a = 1.6544$ from part b)

$\nu_a = \nu(M_a) = 16.3°$, $\nu_b = \nu(M_b) = 34.6°$ (from Prandtl-Meyer table)

Fan turning angle: $\Delta\theta = \nu_b - \nu_a = 34.6° - 16.3° = 18.3°$

$\therefore \boxed{\theta_s = \Delta\theta - 10° = 8.3°}$ 15

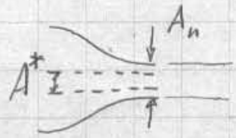
d) If θ_s was not constant (curved shear layer),

then M_b and hence p_b would vary along shear layer.

This contradicts p_b = constant statement 5

Problem 2.

a) Flow must be isentropic up to nozzle: $p_{o_n} = p_r$ 10



For $\frac{p_o}{p} = \frac{1.5}{1} = 1.5$, $\boxed{M_n = 0.78}$ subsonic nozzle flow

For $M = 0.78$, $\frac{A}{A^*} = 1.047$, $\boxed{\dot{m} = \rho^* a^* A^* = \frac{\gamma p_r}{\sqrt{(\gamma-1) h_r}} \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \cdot \left(\frac{A^*}{A}\right) \cdot A_n = 0.0335 \text{ kg/s}}$ 5

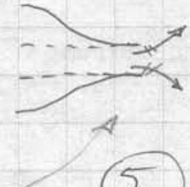
Alternatively, calculate $\rho_n = \frac{\rho_r}{(\rho_o/\rho)}$, $u_n = M_n a_n = M_n \sqrt{\frac{(\gamma-1) h_r}{(T_o/T)}}$, $\dot{m} = \rho_n u_n A_n$

where $\frac{\rho_o}{\rho} = 1.333$, $\frac{T_o}{T} = 1.122$ for $M = 0.78$ from isentropic table

b) Flow is still isentropic, but now it's choked, since $\frac{p_r}{p} = 4 > 1.893$ ($M=1$ case)

Choked flow: $A^* = A_n$, $\boxed{M_n = 1}$ 5

$\boxed{\dot{m} = \rho^* a^* A^* = \frac{\gamma p_r}{\sqrt{(\gamma-1) h_r}} \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{A^*}{A}\right)^{1} A_n = 0.0936 \text{ kg/s}}$ 5



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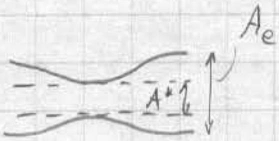
Exit flow is underexpanded supersonic exit flow

c) For matched flow, must have $p_e = p_B = 1\text{ atm}$, also we have $p_{oe} = p_r$

For $\frac{p_{oe}}{p_e} = \frac{4 \text{ atm}}{1 \text{ atm}} = 4 \rightarrow \boxed{M_e = 1.56}$, $\frac{A_e}{A^*} = 1.219$ 5



Since flow is choked $A_e^* = A_n = 0.0001 \text{ m}^2$

$\therefore \boxed{A_e = A_n \cdot \left(\frac{A_e}{A^*}\right) = 0.0001219 \text{ m}^2}$ 10

$\boxed{\dot{m} = \rho^* a^* A^* = 0.0936}$ kg/s, same as in b), since flow is still choked

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