Note Title 5/5/2009

1 a.

$$Y(s) = \frac{10(s^2+75)}{(s+5)(s^2+10s+125)}$$

$$= \frac{10(s^2+75)}{(s+5)(s+5-j10)(s+5+j10)}$$

$$= \frac{K_1}{(s+5)} + \frac{K_2}{(s+5-j10} + \frac{K_3}{(s+5+j10)}$$

$$K_1 = \lim_{s\to -5} (s+5)\,Y(s)$$

$$= \lim_{s\to -5} \frac{10(s^2+75)}{s^2+10s+125}$$

$$= \frac{10(100)}{25-50+125}$$

$$K_1 = 10$$

$$K_2 = \lim_{s \to -5+j10} (s+5-j10)\, Y(s)$$

$$= \lim_{s \to -5+j10} \frac{10(s^2+75)}{(s+5)(s+5+j10)}$$

$$= \frac{\cancel{10}(\cancel{25} - j100 - \cancel{100} + \cancel{75})}{(j\cancel{10})(j20)}$$

$$K_2 = +j5$$

$$K_3 = -j5$$

$$\boxed{Y(s) = \frac{10}{(s+5)} + \frac{j5}{(s+5-j10)} - \frac{j5}{(s+5+j10)}}$$

1b.

Note $\quad k = j5 = 5e^{j90°}$

$$k^* = -j5 = 5e^{-j90°}$$

$$\boxed{y(t) = \left[10e^{-5t} + 10e^{-5t}\cos(10t + 90°)\right]u(t)}$$

2a $$Y(s) = \frac{5s^2 + 10s + 1}{s(s+1)^2}$$

$$= \frac{K_1}{s} + \frac{K_2}{(s+1)^2} + \frac{K_3}{(s+1)}$$

$$K_1 = \lim_{s \to 0} s\,Y(s)$$

$$= \lim_{s \to 0} \frac{5s^2 + 10s + 1}{(s+1)^2}$$

$$K_1 = 1$$

$$K_2 = \lim_{s \to -1} (s+1)^2\,Y(s)$$

$$= \lim_{s \to -1} \frac{5s^2 + 10s + 1}{s}$$

$$= \frac{5 - 10 + 1}{-1}$$

$$K_1 = 4$$

$$K_2 = \lim_{s \to -1} \frac{d}{ds} (s+1)^2 F(s)$$

$$= \lim_{s \to -1} \frac{d}{ds} \left(5s + 10 + \frac{1}{s} \right)$$

$$= \lim_{s \to -1} 5 - \frac{1}{s^2}$$

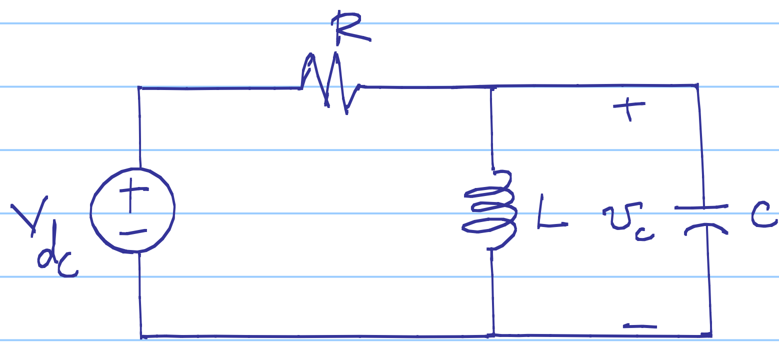
$$K_2 = 4$$

$$\boxed{Y(s) = \frac{1}{s} + \frac{4}{(s+1)^2} + \frac{4}{(s+1)}}$$

2b

$$\boxed{y(t) = \left[1 + 4te^{-t} + 4e^{-t}\right] u(t)}$$

3a



$$\frac{v_c(t) - V_{dc}}{R} + \frac{1}{L}\int_0^t v_c(x)\,dx + C\frac{d\,v_c(t)}{dt} = 0$$

$$\boxed{v_c(t) + \frac{R}{L}\int_0^t v_c(x)\,dx + RC\,\frac{d\,v_c(t)}{dt} = V_{dc}}$$

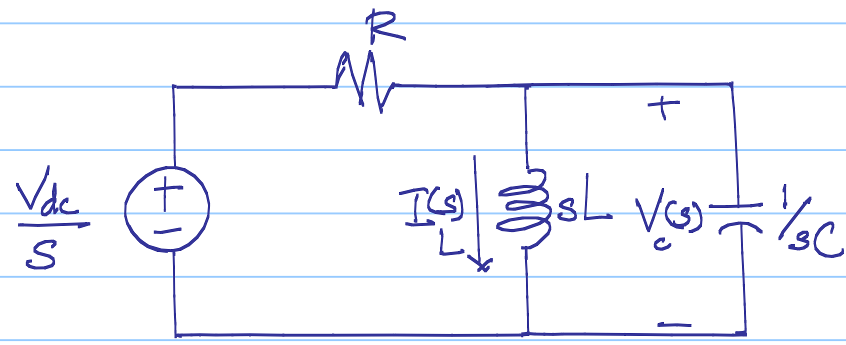
3b Taking the Laplace transform gives

$$V_c(s) + \frac{R}{L}\,\frac{V_c(s)}{s} + RC\left[sV_c(s) - \cancelto{0}{v_c(0)}\right] = \frac{V_{dc}}{s}$$

$$V_c(s)\left[s + \frac{R}{L} + RC\,s^2\right] = V_{dc}$$

$$\boxed{V_c(s) = \frac{V_{dc}/RC}{\left[s^2 + (1/RC)\,s + 1/LC\right]}}$$

3C



$$V_c(s) = \frac{Z_{eq}}{R + Z_{eq}} \frac{V_{dc}}{s}$$

where $$\frac{1}{Z_{eq}} = \frac{1}{sL} + \frac{1}{1/sC}$$

$$= \frac{1 + s^2LC}{sL}$$

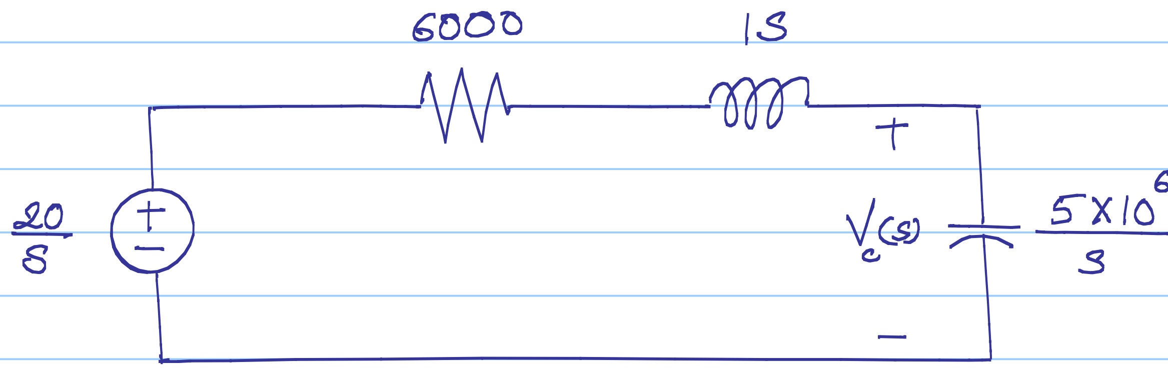
$$V_c(s) = \frac{1}{R\,(1+s^2LC)/_{sL} + 1} \frac{V_{dc}}{s}$$

$$I_L(s) = \frac{V_c(s)}{sL}$$

$$= \frac{1}{(R + s^2 RLC + sL)} \frac{V_{dc}}{s}$$

$$\boxed{I_L(s) = \frac{1}{s} \frac{V_{dc}/(RLC)}{\left[s^2 + (1/RC)s + (1/LC)\right]}}$$

4



$$V_c(s) = \frac{5\times10^6/s}{s + 6000 + 5\times10^6/s}\ \frac{20}{s}$$

$$= \frac{100 \times 10^6}{s\,(s^2 + 6000\,s + 5\times10^6)}$$

$$= \frac{100\times10^6}{s\,(s + 1\times10^3)(s + 5\times10^3)}$$

$$V_c(s) = \frac{K_1}{s} + \frac{K_2}{(s+1\times10^3)} + \frac{K_3}{(s+5\times10^3)}$$

$$K_1 = \lim_{s \to 0} s\, V_c(s)$$

$$= \lim_{s \to 0} \frac{100 \times 10^6}{(s + 1\times 10^3)(s + 5\times 10^3)}$$

$$K_1 = 20$$

$$K_2 = \lim_{s \to -1\times 10^3} (s + 1\times 10^3)\, V_c(s)$$

$$= \lim_{s \to -1\times 10^3} \frac{100 \times 10^6}{s(s + 5\times 10^3)}$$

$$= \frac{100 \times \cancel{10^6}}{-\cancel{10^3}(4 \times \cancel{10^3})}$$

$$K_2 = -25$$

$$K_3 = \lim_{s \to -5\times10^3} (s+5\times10^3)\, V_c(s)$$

$$= \lim_{s \to -5\times10^3} \frac{100\times10^6}{s(s+1\times10^3)}$$

$$= \frac{100\times\cancel{10}^6}{-5\times\cancel{10}^3(-4\times\cancel{10}^3)}$$

$$K_3 = 5$$

Therefore

$$V_c(s) = \frac{20}{s} + \frac{-25}{(s+1\times10^3)} + \frac{5}{(s+5\times10^3)}$$

$$\boxed{v_c(t) = \left[20 - 25\, e^{-10^3 t} + 5\, e^{-5\times10^3 t}\right] u(t)}$$