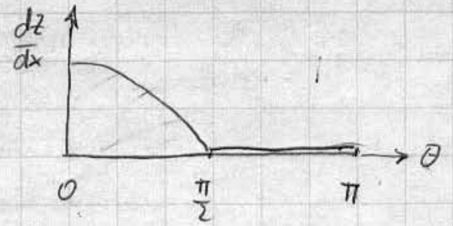
UE Fluids Quiz 1   Problem 1 Solution   S'09

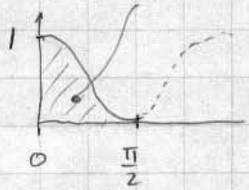
1) a) $$\frac{dz}{dx} = \begin{cases} 4\frac{h}{c}\left(1 - 2\frac{x}{c}\right) = 4\frac{h}{c}\cos\theta & \text{for } \theta < \frac{\pi}{2} \\ 0 & \text{for } \theta > \frac{\pi}{2} \end{cases}$$



Fourier Analysis:

$$\boxed{A_0} = \alpha - \frac{1}{\pi}\int_0^{\pi/2} 4\frac{h}{c}\cos\theta \, d\theta = \boxed{\alpha - \frac{4}{\pi}\frac{h}{c}} \quad 10 \qquad \text{since } \int_0^{\pi/2}\cos\theta\, d\theta = \sin\theta\Big|_0^{\pi/2} = 1$$

$$\boxed{A_1} = \frac{2}{\pi}\int_0^{\pi/2} 4\frac{h}{c}\cos^2\theta \, d\theta = \boxed{2\frac{h}{c}} \quad 10 \qquad \text{since } \int_0^{\pi/2}\cos^2\theta\, d\theta = \frac{\pi}{4}$$



Only A_0 and A_1 are needed for b), c).

b) $$C_\ell = \pi(2A_0 + A_1) = \pi\left(2\left(\alpha - \frac{4}{\pi}\frac{h}{c}\right) + 2\frac{h}{c}\right)$$

$$\boxed{C_\ell} = 2\pi\alpha - (8 - 2\pi)\frac{h}{c} = \boxed{2\pi\alpha - 1.717\frac{h}{c}} \quad 10$$

c) $$C_\ell = 2\pi(\alpha - \alpha_{L=0}) = 2\pi\alpha - (8-2\pi)\frac{h}{c}$$

$$\therefore \boxed{\alpha_{L=0}} = \boxed{\left(\frac{4}{\pi} - 1\right)\frac{h}{c} = 0.273\frac{h}{c}} \quad 10$$

UE Fluids Quiz 1 Problem 2 Solution S '09

a) Match $\sin n\theta$ coefficients: $\gamma = V_\infty(\sin\theta + 0.1\sin 3\theta) = 2V_\infty\left(A_0 \frac{1+\cos\theta}{\sin\theta} + \Sigma A_n \sin n\theta\right)$

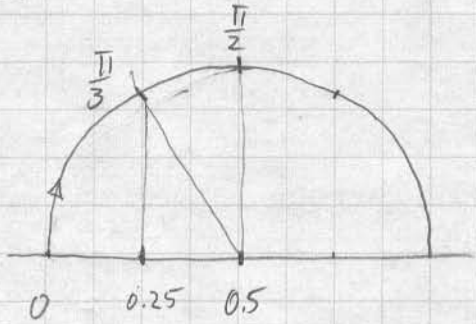
$\therefore \; V_\infty \sin\theta = 2V_\infty A_1 \sin\theta \rightarrow A_1 = 0.5$

$V_\infty \cdot 0.1\sin 3\theta = 2V_\infty A_3 \sin 3\theta \rightarrow A_3 = 0.05$

$$C_l = \pi(2\cancel{A_0} + A_1) = \pi A_1 = \boxed{0.5\pi}$$ 5

$$C_{m_{c/4}} = \frac{\pi}{4}(\cancel{A_2} - A_1) = -\frac{\pi}{4}A_1 = \boxed{-0.125\pi}$$ 5

b) $\Delta p = \rho V_\infty \gamma = \rho V_\infty^2(\sin\theta + 0.1\sin 3\theta)$



At $\frac{x}{c} = 0.25$, $\theta = \frac{\pi}{3} = 60°$

At $\frac{x}{c} = 0.50$, $\theta = \frac{\pi}{2} = 90°$

$\sin\frac{\pi}{3} = \sin 60° = \frac{\sqrt{3}}{2}$, $\sin 3\cdot\frac{\pi}{3} = \sin\pi = 0$

$\sin\frac{\pi}{2} = \sin 90° = 1$, $\sin 3\cdot\frac{\pi}{2} = -1$

At $x/c = 0.25$: $\boxed{\Delta p = \rho V_\infty^2\left(\frac{\sqrt{3}}{2} + 0.1\cdot 0\right) = 0.866}$ 5

At $x/c = 0.50$: $\boxed{\Delta p = \rho V_\infty^2(1 - 0.1\cdot 1) = 0.900}$ 5

UE Fluids Quiz 1 Problem 3 Solution S '09

a) Find A_n's corresponding to given Γ:

$$\Gamma = 2bV_\infty \sum A_n \sin n\theta = bV_\infty\left(0.06\sin\theta + 0.01\sin 2\theta\right) \qquad (*)$$

Matching $\sin n\theta$ terms:

$$2bV_\infty A_1 = bV_\infty\, 0.06 \rightarrow \boxed{A_1 = 0.03}$$

$$2bV_\infty A_2 = bV_\infty\, 0.01 \rightarrow \boxed{A_2 = 0.005}$$

(5)

Could also Fourier-analyze (*) for same result.

$$\boxed{C_L = \pi \frac{b^2}{S} A_1 = \pi \frac{4}{0.4} \cdot 0.03 = 0.942}$$

(5)

$$C_{D_i} = \frac{C_L^2}{\pi b^2/S}\left[1+\delta\right] \quad , \quad (1+\delta) = 1 + 2\left(\frac{A_2}{A_1}\right)^2 = 1.05556$$

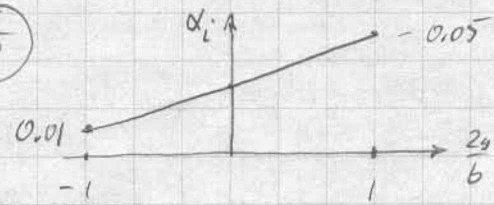
$$\boxed{C_{D_i} = 0.0298}$$

(5)

b) $$\alpha_i = \sum n A_n \frac{\sin n\theta}{\sin\theta} = A_1 + 2A_2 \frac{\sin 2\theta}{\sin\theta} = A_1 + 2A_2 \frac{2\sin\theta\cos\theta}{\sin\theta} = A_1 + A_2\, 4\cos\theta$$

$$\boxed{\alpha_i = A_1 + 4A_2 \frac{2y}{b} = 0.03 + 0.02\,\frac{2y}{b}}$$

(5)



c) $\Gamma(\theta) \overset{?}{=} \frac{1}{2} V_\infty\, c(\theta)\, c_\ell(\theta)$

Since $\Gamma(\theta)$ is known, must specify $c_\ell(\theta)$, V_∞ uniquely fix $c(\theta)$ (10)

d) $$M_{roll} = \int_{-b/2}^{b/2} L'\, y\, dy = \int_0^\pi \rho V_\infty \Gamma \cdot \frac{b}{2}\cos\theta \cdot \frac{b}{2}\sin\theta\, d\theta \quad ; \quad \cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$$

$$M_{roll} = \rho V_\infty^2 \frac{b^3}{4} \int_0^\pi \Big[0.06 \underbrace{\sin\theta}_{0} + 0.01 \underbrace{\sin 2\theta}_{\pi/2}\Big] \cdot \frac{1}{2}\sin 2\theta\, d\theta$$

$$\boxed{M_{roll} = \rho V_\infty^2 \frac{b^3}{16}\pi\, 0.01 = 0.005\,\pi}$$

(10)