Flight Thrust, Power, and Energy Relations 5 Feb 09

Lab 1 Lecture Notes

Nomenclature

D	aircraft drag	T	propeller thrust
L	aircraft lift	T_c	thrust coefficient
W	total aircraft weight	P_{prop}	propulsive thrust power ($\equiv TV$)
W_e	empty aircraft weight	P_{shaft}	motor shaft power
W_p	payload weight	E_{shaft}	motor shaft energy expended
V	flight speed	R	propeller radius
d	flight distance	η_{prop}	overall propeller efficiency
S	reference area (wing area)	η_{v}	profile propeller efficiency (viscous loss)
b	wing span	η_{i}	Froude propeller efficiency (inviscid loss)
AR	wing aspect ratio	Re	chord Reynolds number
C_L	aircraft lift coefficient	c_ℓ	wing-airfoil profile lift coefficient
C_D	aircraft drag coefficient	c_d	wing-airfoil profile drag coefficient
CDA_0	drag area of non-wing components	ρ	air density

Thrust Power

Generation of thrust in flight requires the expenditure of power. For a propeller or a jet-engine fan, the shaft power and the thrust are related by the definition of propeller efficiency.

$$\frac{TV}{P_{\text{shaft}}} \equiv \frac{P_{\text{prop}}}{P_{\text{shaft}}} \equiv \eta_{\text{prop}} \tag{1}$$

The η_{prop} is the product of a viscous profile efficiency η_{v} which accounts for the viscous profile drag on the blades, and an inviscid Froude efficiency η_{i} which accounts for the kinetic energy lost in the accelerated propwash.

$$\eta_{\text{prop}} = \eta_{\text{v}}\, \eta_{\text{i}} \tag{2}$$

An upper limit and estimate of η_{i} is related to the dimensionless Thrust Coefficient T_c.

$$\eta_{\text{i}} \le \frac{2}{1+\sqrt{1+T_c}} \tag{3}$$

$$T_c \equiv \frac{T}{\frac{1}{2}\rho V^2\, \pi R^2} \tag{4}$$

Limiting cases are

$$T_c \gg 1\,,\ \eta_{\text{i}} \ll 1\ :\quad P_{\text{shaft}} \simeq \frac{T^{3/2}}{(2\pi\,\rho)^{1/2}}\,\frac{1}{R}\,\frac{1}{\eta_{\text{v}}} \quad , \qquad \text{(Heavy loading, low-speed takeoff)} \tag{5}$$

$$T_c \ll 1\,,\ \eta_{\text{i}} \simeq 1\ :\quad P_{\text{shaft}} \simeq T\,V\,\frac{1}{\eta_{\text{v}}} \quad , \qquad \text{(Light loading, high-speed cruise)} \tag{6}$$

Both T_c and η_i are seen to strongly depend on the thrust and flight speed, and also on the propeller radius. In contrast, η_{v} does not vary much and is often considered a constant.

Level-Flight Relations

In level flight we have $W = L$, which gives the velocity in terms of aircraft parameters.

$$W \;=\; L \;=\; \frac{1}{2}\rho V^2\, S\, C_L \tag{7}$$

$$V \;=\; \left(\frac{2\,W}{\rho\, S\, C_L}\right)^{1/2} \tag{8}$$

In steady level flight we also have $T = D$, in which case the thrust and the propulsive thrust power can then be given as follows.

$$T \;=\; D \;=\; \frac{1}{2}\rho V^2\, S\, C_D \;=\; W\,\frac{C_D}{C_L} \tag{9}$$

$$P_{\rm prop} \;=\; TV \;=\; DV \;=\; \frac{1}{2}\rho V^3\, S\, C_D \;=\; \left(\frac{2W^3}{\rho\, S}\right)^{1/2} \frac{C_D}{C_L^{3/2}} \tag{10}$$

In the level-flight case we can also express the thrust coefficient and hence the prop Froude efficiency in an alternative and somewhat more convenient manner.

$$T_c \;=\; \frac{T}{\frac{1}{2}\rho V^2\, \pi R^2} \;=\; \frac{D}{\frac{1}{2}\rho V^2\, \pi R^2} \;=\; \frac{S}{\pi R^2}\, C_D \tag{11}$$

Drag Breakdown

To obtain the required thrust or propulsive power via (9) or (10), we need the overall aircraft drag coefficient C_D, which is broken down into three basic components.

$$C_D \;=\; \frac{CDA_0}{S} \;+\; c_d(C_L, Re) \;+\; \frac{C_L^2}{\pi\, AR} \tag{12}$$

The first term gives the combined drag of all the non-wing components, such as the fuselage, tail, landing gear, etc. The second term is the wing profile drag, estimated from the wing's 2D airfoil $c_d(c_\ell, Re)$ data, and by assuming that the typical wing airfoil operates at $c_\ell \simeq C_L$. The last term is the induced drag coefficient C_{D_i}, which depends on C_L and the aspect ratio of the wing.

$$AR \;=\; \frac{b^2}{S} \tag{13}$$

Figure 1 shows the three C_D components versus C_L for a typical 1.5 m span light RC sport aircraft.

For a typical operating point at $C_L = 1.0$ (low speed) and $C_L = 0.3$ (high speed), indicated by the symbols in Figure 1, the three components contribute roughly the following percentages to the total drag:

C_L	CDA_0/S	c_d	$C_L^2/\pi AR$	C_D
1.0	0.0167	0.0335	0.0406	0.0909
0.3	0.0167	0.0220	0.0037	0.0424
1.0	18 %	37 %	45 %	100 %
0.3	39 %	52 %	9 %	100 %

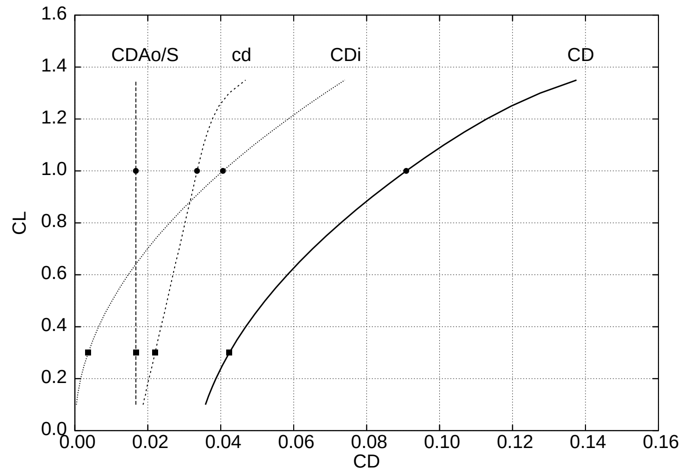

Figure 1: Drag polar and drag polar components for electric sport aircraft. $A\!\!R = 9.0$

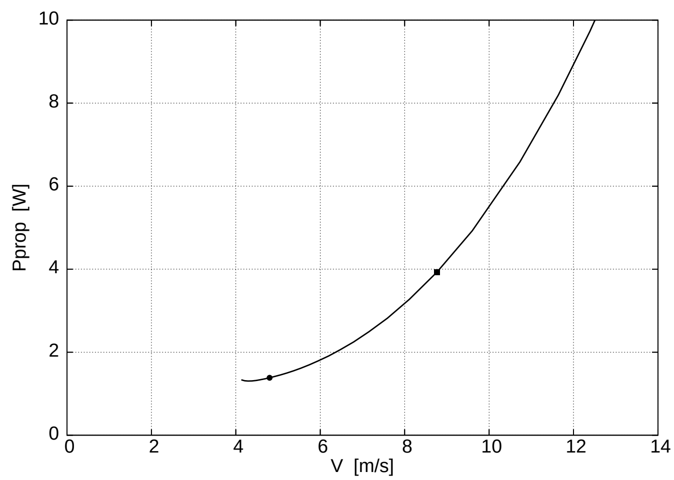

Figure 2: Propulsive thrust power $P_{\text{prop}} = DV = TV$ for electric sport aircraft.

The corresponding propulsive power is shown in Figure 2.

Minimum Flight Energy for Given Payload

It is of great interest to reduce the flight energy (and hence the fuel consumption) of an aircraft as much as possible, while still carrying the required payload. Assuming that the total aircraft weight does not change appreciably during flight, the time t and shaft energy E_{shaft} required to fly a distance d is

$$t = \frac{d}{V} \tag{14}$$

$$E_{\text{shaft}} = P_{\text{shaft}}\, t = \frac{T\,d}{\eta_{\text{prop}}} \tag{15}$$

Combining the relations and assumptions above, we have the following summary relations for a sustained level flight of distance d.

$$E_{\text{shaft}} \;=\; \frac{1}{\eta_{\text{v}}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{S}{\pi R^2}\,C_D}\right)(W_e+W_p)\;d\;\frac{C_D}{C_L} \tag{16}$$

$$P_{\text{shaft}} \;=\; \frac{1}{\eta_{\text{v}}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{S}{\pi R^2}\,C_D}\right)\left(\frac{2(W_e+W_p)^3}{\rho\, S}\right)^{1/2}\frac{C_D}{C_L^{3/2}} \tag{17}$$

where
$$C_D \;=\; \frac{CDA_0}{S} \;+\; c_d(C_L, Re) \;+\; \frac{C_L^2}{\pi}\frac{S}{b^2} \tag{18}$$

The auxilliary C_D expression (18) is the same as (12), restated here for convenience.

Parameter Coupling and Design Optimization

Any change in the aircraft variables which permits a reduction of E_{shaft} as given by equation (16), with a fixed W_p, will give a reduction in energy or fuel consumption. However, it's essential to realize that most of the variables and parameters in equations (16) and (18) are coupled in an actual design application, so the effect of changing one will have multiple side effects, with the net effect being nonobvious.

One example which might appear if one attempts to reduce E_{shaft} by increasing the wing area S. One immediately-apparent benefit of a larger S is:

- Pro: Reduction of the first CDA_0 term in (18)

There are also obvious drawbacks:

- Con: Increase in the first propeller loss term in (16)
- Con: Increase in the last induced drag term in (18)

Furthermore, there will likely be additional drawbacks which are not explicitly apparent:

- Con: Increase in the empty weight W_e because of more wing material, etc.

Other Pros and Cons may be present in addition to those listed above, depending on the situation.

Much of the activity which occurs during aircraft design and sizing consists of identifying and quantifying such couplings. Knowing the couplings then allows suitable tradeoffs to be performed, in order to find the best set of design parameters to maximize the design objective. Once a good or optimum design has been reached, all its competing tradeoffs are in balance, so that there are no more "easy" design changes which can be made without adversely affecting something else.