Improved Performance Estimates for Optimization 7 Apr 09

Lab 10 Lecture Notes

Nomenclature

W	total weight ($= W_{\rm wing} + W_{\rm fuse} + W_{\rm pay}$)	b	wing span
S	reference area (wing area)	c	average wing chord
AR	wing aspect ratio	$T_{\max}$	maximum thrust
V	flight speed	C_L	lift coefficient
c_r	root wing chord	C_D	total drag coefficient
c_t	tip wing chord	c_d	wing profile drag coefficient
λ	taper ratio ($= c_t/c_r$)	CDA_0	drag area of non-wing components
E	Young's modulus or wing material	δ	tip deflection
ρ	air density	τ	airfoil thickness/chord ratio
g	gravity acceleration	ε	airfoil camber/chord ratio

Assumed Design Variables

The primary design variables here are assumed to be $\{AR, S\}$. These can easily be converted to alternative sets such as $\{b, c\}$. Other design variables such as τ, λ, etc., should also be considered. These will be denoted by "$\ldots$" in the argument lists below.

Basic Relations

The following dependent auxilliary functions and constraint functions are assumed to be known from suitable analyses:

$$c(AR, S) = \sqrt{S/AR} \tag{1}$$

$$b(AR, S) = \sqrt{S\ AR} \tag{2}$$

$$W_{\rm wing}(AR, S, \ldots) = 0.6\, \tau\, c^2\, b\, \rho_{\rm foam}\, g \tag{3}$$

Maximum-Payload Case

The maximum payload and wing bending tip deflection/span ratio is then computed from the following relations:

$$C_D(AR, S, \ldots) = \frac{CDA_0}{S} + c_{d_{\rm slow}}(C_L; Re, \tau) + \frac{C_L^2}{\pi\, AR} \tag{4}$$

$$W(AR, S, \ldots) = T_{\max} \frac{C_L}{C_D} \tag{5}$$

$$W_{\rm pay}(AR, S, \ldots) = W - W_{\rm wing} - W_{\rm fuse} \tag{6}$$

$$\frac{\delta}{b}(AR, S, \ldots) = 0.018\, \frac{W_{\rm fuse} + W_{\rm pay}}{E\tau(\tau^2 + \varepsilon^2)} (1+\lambda)^3 (1+2\lambda) \frac{b^2}{c^4} \tag{7}$$

The flight speed and corresponding Reynolds number can then also be determined from the level flight lift=weight relation.

$$V(AR, S, \ldots) = \left[\frac{2W}{\rho\, C_L\, S}\right]^{1/2} \tag{8}$$

$$Re(AR, S, \ldots) = \frac{V\, c}{\nu} \tag{9}$$

Maximum-Speed Case

For the maximum-speed case without payload, the same basic relations as above are used. But now, the total weight is computed differently, with $W_{\rm pay}$ now omitted. Also, the $C_{L_{\rm min}}$ corresponding to $V_{\rm max}$ is now computed rather than being an independent design variable.

$$W(AR, S, \ldots) = W_{\rm wing} + W_{\rm fuse} \tag{10}$$

$$C_{L_{\rm min}}(AR, S, \ldots) = \frac{2W}{\rho V_{\rm max}^2 S} \tag{11}$$

$$Re_{\rm max}(AR, S, \ldots) = \frac{V_{\rm max}\, c}{\nu} \tag{12}$$

$$C_D(AR, S, \ldots) = \frac{CDA_0}{S} + c_{d_{\rm fast}}(C_{L_{\rm min}}; Re_{\rm max}, \tau) + \frac{C_{L_{\rm min}}^2}{\pi AR} \tag{13}$$

$$V_{\rm max}(AR, S, \ldots) = \left[\frac{2\,T_{\rm max}}{\rho\, C_D\, S}\right]^{1/2} \tag{14}$$

Note also that the same C_D function is used here as for the maximum-payload case, but the resulting C_D value will be different because of the different C_L and Re values, and possibly a different $c_{d_{\rm fast}}$ function as well.

Thrust versus Flight Speed

Appendix A shows some propeller thrust measurements versus voltage and flight speed V. For the maximum available voltage from the battery, the thrust is closely approximated by the following linear function of velocity.

$$T_{\rm max}(V) \simeq T_0 + T_1 V \tag{15}$$

The maximum thrust therefore depends on the design variables through $V_{(AR,S,\ldots)}$, as given by relation (8).

$$T_{\rm max}(AR, S, \ldots) = T_{\rm max}(V_{(AR,S,\ldots)}) \tag{16}$$

Profile Drag

Appendix B gives an approximate relation for the wing profile drag c_d versus $c_\ell \simeq C_L$, Reynolds number, and airfoil thickness ratio.

$$c_d(c_\ell, Re, \tau) \simeq \left[c_{d_0} + c_{d_2}(c_\ell - c_{\ell_0})^2\right]\left(1 + k_\tau \tau^3\right)\left(\frac{Re}{Re_{\rm ref}}\right)^a \tag{17}$$

This therefore depends on the design variables through $Re_{(AR,S,\ldots)}$, as given by relation (9).

$$c_d(AR, S, \ldots) = c_d(C_L, Re_{(AR,S,\ldots)}, \tau) \tag{18}$$

Note that all the airfoils have a camber ratio of roughly $\varepsilon \simeq 0.03$, which gives a good tradeoff between high-C_L and low-C_L performance. So there's little reason to consider the camber as another design variable in this case.

$W_{\rm pay}$ Function Calculation

All the above equations are too complicated to be explicitly solved for $W_{\rm pay}(AR, S, \ldots)$. However, it is possible to solve them by a reasonably simple iterative procedure. We note that the influence of the design variables on $T_{\max}$ and c_d is relatively small, so these auxilliary functions can be lagged in an iteration loop.

Given some design variable values $\{AR, S, \ldots\}$, the iteration to compute W_{pay} proceeds as follows.

0) Assume some reasonable $T_{\max}$ and $c_{d_{\text{slow}}}$ values.

1) Compute C_D from (4),

2) Compute W from (5), and then W_{pay} from (6).

3) Compute V from (8) and Re from (9).

4) Compute $T_{\max}$ from (15)

The iteration can be repeated by starting again at 1) with the new $T_{\max}$ and c_d values. Convergence is obtained once all the values cease to change significantly. The tip deflection ratio δ/b can be computed from (7) after the above iteration is finished.

$V_{\max}$ Function Calculation

A similar type of iteration can be used to compute $V_{\max}$ for the no-payload maximum-speed flight case. The no-payload weight $W = W_{\text{fuse}} + W_{\text{pay}}$ is now used.

0) Assume some reasonable initial $T_{\max}$, $c_{d_{\text{fast}}}$, and $C_{L_{\min}}$ values. The latter will be small at high speed, and can be initially assumed zero.

1) Compute C_D from (13) and $V_{\max}$ from (14),

2) Compute $C_{L_{\min}}$ from (11), and $Re_{\max}$ from (12).

3) Compute $T_{\max}$ from (15)

Faster-converging iteration methods such as Newton's Method can also be used, but at much greater complexity.

Mission Score Function Calculation

The overall Mission Score is computed simply as a weighted sum of the W_{pay} and $V_{\max}$ which result from the above iterations.

$$\text{Mission Score}(AR, S, \ldots) \;=\; W_{\text{pay}} \;+\; k\, V_{\max} \tag{19}$$