Design Variable Concepts

19 Mar 09

Lab 7 Lecture Notes — Addendum

Alternative Objectives

Frequently, an engineering design problem will involve competing objectives. For example, an alternative objective to minimize flight power (or maximize payload), is to maximize the maximum-attainable flight speed. The level-flight thrust relations still apply to this flight condition, but now the thrust is a known quantity, and equal to the maximum available thrust from the powerplant.

$$T_{\max} = \frac{1}{2}\rho V_{\max}^2 S \left[\frac{CDA_0}{S} + c_d(C_{L_{\min}}; Re_{\max}) + \frac{C_{L_{\min}}^2}{\pi e AR} \right] \qquad (1)$$

$$\text{where} \quad C_{L_{\min}} = \frac{2W/S}{\rho V_{\max}^2}$$

This can be solved for $V_{\max}$, or equivalently $C_{L_{\min}}$, by numerical means if necessary. If one can assume that at $V_{\max}$ the induced drag is negligible, and c_d is some constant, then we have

$$V_{\max}(AR, S) \simeq \left[\frac{2\,T_{\max}}{\rho\,(CDA_0 + S\,c_d)} \right]^{1/2} \qquad (2)$$

which is a suitable alternative objective function.

The figure shows the isolines of $V_{\max}$ versus the design variables, calculated using (1). The slight upturn in the isolines for decreasing AR is the Re effect in c_d. The sharp downturn near $AR = 0$ is due to the induced drag term. Isolines computed using the approximation (2) would be level.

Now the stiffness-constrained optimum results in an unreasonably small wing, which means that other more practical constraints are likely to come into play.

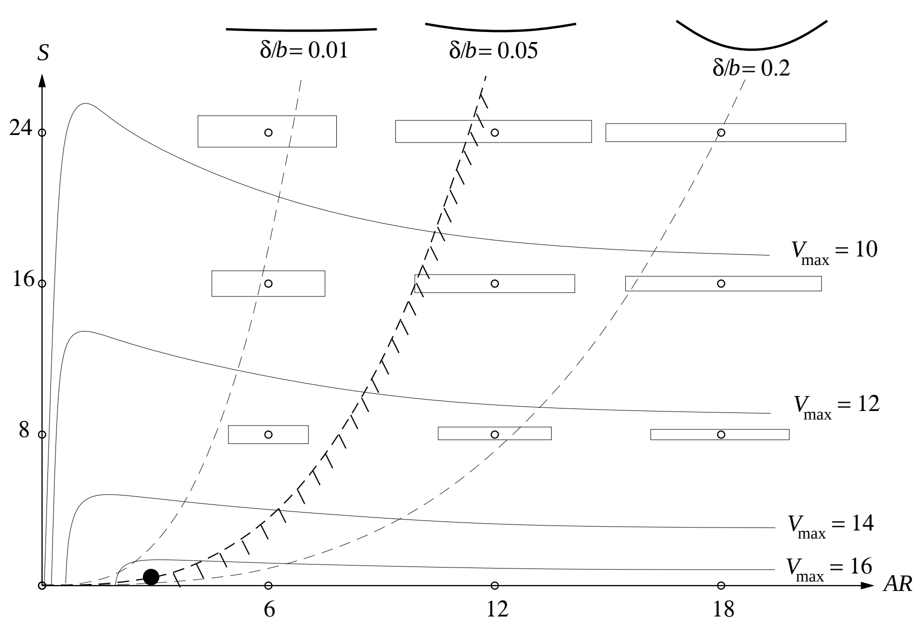

Figure 1: Objective function contours (isolines) in design space of a rectangular wing. Dot shows the stiffness-constrained optimum point.

Iterative Calculation of $V_{\max}$

Although equation (1) cannot be explicitly solved for $V_{\max}$, it is possible to solve it by a reasonably simple iterative procedure.

We first use equation (2) to give a first approximation $\tilde{V}_{\max}$.

$$\tilde{V}_{\max}(AR, S) = \left[\frac{2\,T_{\max}}{\rho\,(CDA_0 + S\,c_d)} \right]^{1/2} \tag{3}$$

This is then used to compute an estimated $\tilde{C}_L$ and $\tilde{Re}$.

$$\tilde{C}_L(AR, S) = \frac{2W/S}{\rho\,\tilde{V}_{\max}^2} \tag{4}$$

$$\tilde{Re}(AR, S) = \frac{\tilde{V}_{\max} c}{\nu} \tag{5}$$

These then allow an improved estimation of C_{D_i}, or c_d, or both.

$$\tilde{C}_{D_i}(AR, S) = \frac{\tilde{C}_L^2}{\pi\, e\, AR} \tag{6}$$

$$\tilde{c}_d(AR, S) = c_d(\tilde{C}_L, \tilde{Re}) \tag{7}$$

Note that this new C_{D_i} is much better than the first estimate, which merely assumed $C_{D_i} = 0$. The new c_d may or may not be better than the initial estimate of assuming c_d was some constant.

In any case, the new $\tilde{C}_{D_i}$ and/or new $\tilde{c}_d$ are then used in equation (1) to get an improved $V_{\max}$ estimate.

$$V_{\max}(AR, S) = \left[\frac{2\,T_{\max}}{\rho\,(CDA_0 + S\,\tilde{c}_d + S\,\tilde{C}_{D_i})} \right]^{1/2} \tag{8}$$

The iteration can be repeated by starting again at (4), although the additional accuracy gain is likely to be minimal.

The $V_{\max}$ calculated by this procedure will fully include the C_{D_i} contribution to the total drag, and will also include Reynolds number effects if the $c_d(C_L, Re)$ function is used. The resulting $V_{\max}$ isolines should resemble those in Figure 1.