**Problem T6.** (Thermodynamics)

Consider the following thermodynamic cycle. Assume all processes are quasi-static and involve an ideal gas.

Air undergoes a quasi-static thermodynamic cycle 1-2-3-4-1 as shown above. Process 1-2 is isothermal compression, 2-3 is constant volume heat addition, process 3-4 is adiabatic expansion, and process 4-1 is constant pressure cooling. The conditions at state 1 are $p_1 = 100\text{kPa}$, $T_1 = 300\text{K}$. The pressure ratio $(p_2/p_1)$ over process 1-2 is 10 and the peak temperature of the cycle is $1500\text{K}$. Assume that $c_p = 1.0035\text{kJ/kg-K}$ and $c_v = 0.7165\text{kJ/kg-K}$ are constants, and that $R = 0.287\text{kJ/kg-K}$.

a) For each leg of the cycle identify whether the heat added to the system, $Q$, and the work done by the system, $W$, are positive, negative or zero.

b) For each leg of the cycle calculate the work and heat transfer, the change in internal energy and the change in enthalpy.

c) What is the net work of the cycle?

d) What is the thermal efficiency of the cycle?

e) If you reversed the direction of the cycle and used it as a refrigerator, what is the maximum amount of heat you could you remove per Joule of power input?

(LO# 4, LO#6)
Problem T7 (Unified Thermodynamics)

MIT’s gas turbine power plant operates on a Brayton cycle. The current operating conditions are shown at [http://cogen.mit.edu/unified/](http://cogen.mit.edu/unified/). (The login name is “unified”, the password is a popular MIT acronym.) Assume the cycle is ideal and the gas behaves as an ideal gas with constant specific heats $c_p = 1.0035 \text{kJ/kg-K}$, and $c_v = 0.7165 \text{kJ/kg-K}$. The maximum temperature at entrance to the turbine is 1400K. From the web page, determine current atmospheric conditions and also the compressor discharge pressure (labeled COMP DEL PRESS). Use this to determine the pressure ratio for the cycle. (LO #4, LO#6)

a) Starting from the measured atmospheric conditions, calculate the pressure and temperature at each point for the ideal cycle.
b) Calculate the thermal efficiency and work per kilogram for the ideal cycle.
c) Assume the compressor discharge temperature is fixed. What is the difference in efficiency between a hot summer day and a cold winter day?
d) How does the efficiency you calculated in part (b) compare with the actual efficiency of the plant? [You can determine this from the data on the web page. If you get a full listing of variables, the fuel flow is listed under “total gas energy flow” and the net power is listed under “active load”. If you use the buttons next to the picture, you need to add up the main and primary fuel flows to get the total gas energy flow. Note that 1 BTU is 1055 Joules, and 1 KBTU/s is 10^3 BTU/s.] Why is the actual efficiency different from that you calculated in part (b)?
e) You modeled the compressor as quasi-static and adiabatic. Test how good this assumption is by comparing the temperature rise you calculate across the compressor with the measured temperature rise across the compressor.
Problem T8 (Unified Thermodynamics)

R=287 J/kg-K, \( c_p = 1003.5 \) J/kg-K, and \( c_v = 716.5 \) J/kg-K. (MO# 5)

Below is a schematic diagram of a rocket engine cycle called an expander cycle. The cold liquid propellants are pumped through the walls of the rocket nozzle to cool them. In doing so, they gain energy and turn into gases. Some of the energy in these propellant gases is then extracted in a turbine. The turbine is connected by a shaft to the liquid propellant pump. After passing through the turbine, the propellant gases flow to the combustion chamber where they are ultimately responsible for producing the heat at the walls of the rocket nozzle. So it is a little like pulling yourself up by your bootstraps. As you might expect, the design of a device like this depends on a careful analysis of the thermodynamics, fluid mechanics and structural/material behavior.

To simplify the problem, we will consider only one of the propellants and assume that it is always in gaseous form and behaves as an ideal gas with constant specific heats. Assume R=260 J/kg-K, \( c_p = 2800 \) J/kg-K, and \( c_v = 2540 \) J/kg-K. (LO #4)

a) The cold propellant enters the pump at \( p = 1 \) MPa, \( T = 200 \) K and \( c = 50 \) m/s. The flow rate is 100 kg/s. The flow leaves the pump at \( p = 5 \) MPa and \( c = 50 \) m/s. Assuming the pump behaves adiabatically and quasi-statically, what are the conditions at the pump exit? How much shaft work and flow work were done by propellant during the pumping process?

b) Assume that 1300 kJ/kg are absorbed by the gas as it passes along the walls of the rocket nozzle. At entrance to the turbine, it is now flowing at \( c=100 \) m/s and remains at \( p = 5 \) MPa. What are the thermodynamic conditions at this point of the engine. How much shaft work and flow work were done by the propellant during the heating process?

c) Assuming the turbine behaves quasi-statically and adiabatically, what are the conditions at the exit of the turbine assuming the velocity is \( c = 120 \) m/s? How much shaft work and flow work were done by the propellant in the turbine?
Problem T9 (Unified Thermodynamics)

There is a group of people at MIT who are working to develop micro-heat engines (gas turbines, rockets, etc.). These small (<1 cm³) engines are being manufactured using microfabrication technology similar to that developed to make microelectronic devices. Microengines have a very high ratio of surface area-to-volume (area scales with the characteristic length squared, volume scales with the characteristic length cubed). Therefore, it is no longer a good approximation to neglect heat transfer for many of the flow processes as we often do for larger devices.

Consider the following micro-rocket engine (it is about the thickness of a credit card and produces about 15N of thrust). It burns methane and oxygen and operates on an expander cycle like that described in homework T8. The combustion chamber pressure is 125 atm and the combustion chamber temperature is 3000 K. The velocities in the combustion chamber are small. Assume the flow behaves as an ideal gas with $c_p = 1.5 \text{ kJ/kg-K}$, $\gamma = 1.2$.

a) If the flow is adiabatic, what is the exit velocity when the rocket exhausts into standard atmospheric conditions? What are the static temperature and Mach number for this exit velocity?

b) What would the exit velocity, static temperature and Mach number be if the flow were fully-expanded to $p_e = 0$ in space?

c) Assume that about 20% of the total enthalpy of the flow in the combustion chamber is lost due to heat transfer to the walls of the combustion chamber. Now what is the exit velocity at standard atmospheric conditions and in space? (Assume that the chamber pressure is the same whether or not there is heat transfer.)

(LO# 4)
**16.01/16.02 Unified Engineering I, II**  
**Fall 2003**

**Problem Set #3**

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Name: ____________________  

Due Date: 9/23/03  

Problems: T6, T7, T8, T9, C5, C6, C7

Announcements:
Problem C5. (Unified Computers and programming)

1. Convert the following base 10 numbers into 8-bit 2’s complement notation
   0, -1, -12

2. Perform each of the following additions assuming that the bit strings represent values in 2’s complement notation. Identify the cases in which the answer is incorrect because of overflow.
   
   \[
   \begin{array}{c}
   \text{1111} \\
   + \text{1111}
   \end{array}
   \quad \begin{array}{c}
   \text{01111} \\
   + \text{10001}
   \end{array}
   \quad \begin{array}{c}
   \text{01110} \\
   + \text{10010}
   \end{array}
   \]

   \textbf{Hint}: The bit strings are only 5 bits long so your answer should also be 5 bits long.

3. Write an algorithm to convert a negative decimal number into a binary number in 2’s complement form. Assume that the number ranges from +127 to -128

   \textbf{Hint}: You already know how to convert a positive decimal number into binary notation. Think about determining sign and inverting bit positions.

Problem C6. (Unified Computers and programming)

1. What is the excess-16 representation of 12? How many bits do you need to represent a number in excess-16 format?

2. Convert 29/8 into binary 8-bit floating-point representation.

3. Sketch the basic von Neumann architecture and describe each component in a few lines.

4. Write an assembly language program (using the language described in the machine language handout) to add two positive numbers. Assume that the numbers are present in memory locations 0xFE and 0xFF. Turn in both a hard copy and an electronic copy of your code.
Problem C7. (Unified Computers and programming)

1. Write an algorithm to implement the subtraction operation for two positive integers in assembly language.

   **Hint:** You have to convert one of the numbers into two’s complement notation. How do you do that using only the operations available to you in assembly language?

2. Implement your algorithm in the assembly language described in the machine language handout. Test your implementation using the SimpleSim simulator. Turn in both a hard copy and an electronic copy of your code.