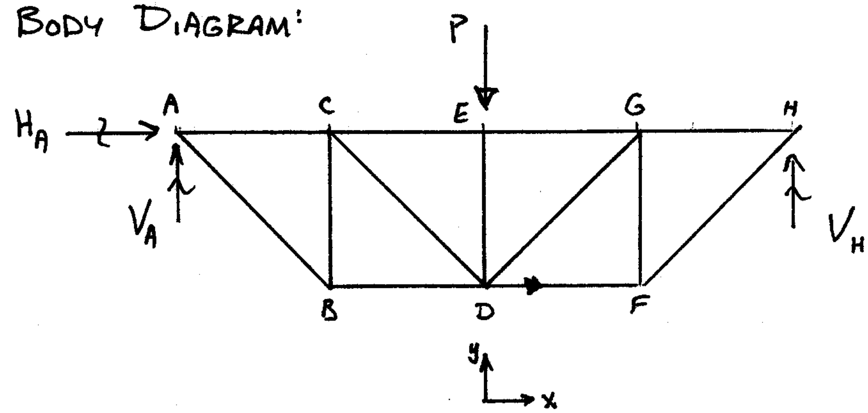
M~~X~~ 8

FREE BODY DIAGRAM:



APPLY EQUILIBRIUM TO FIND REACTIONS

$\xrightarrow{+} \sum F_x = 0 \qquad \boxed{H_A = 0\,N}$

$+\uparrow \sum F_y = 0 \qquad V_A + V_H - P = 0$

$+\circlearrowleft \sum M_A = 0 \qquad V_H(4L) - P(2L) = 0$

$$\boxed{\begin{array}{l} V_H = \frac{P}{2} \\ \Rightarrow V_A = \frac{P}{2} \end{array}}$$

TO DETERMINE THE DEFLECTION OF D, WE NEED TO EMPLOY COMPATIBILITY + CONSTITUTIVE LAWS.

OUR CONSTITUTIVE LAW FOR BAR DEFORMATION IS:

$$\delta_{ij} = \frac{F_{ij} L_{ij}}{AE}$$

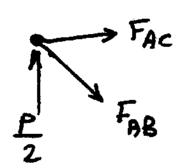
SO WE'LL NEED TO SOLVE FOR THE BAR FORCES IN ORDER TO DETERMINE THEIR EXTENSIONS, AND HENCE THE TRUSS DEFLECTION.

BECAUSE OF SYMMETRY, I ONLY NEED TO FIND HALF OF THE BAR FORCES. ALL OF THE PAIRS MIRRORED IN THE D-E AXIS WILL HAVE THE SAME BAR FORCE:

$$F_{AC} = F_{GH} \qquad F_{BD} = F_{DF}$$
$$F_{AB} = F_{FH} \qquad F_{CD} = F_{DG}$$
$$F_{BC} = F_{FC} \qquad F_{CE} = F_{EG}$$

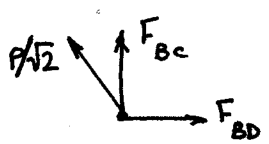
SOLVE FOR INDEPENDENT BAR FORCES:

MOJ @ A:



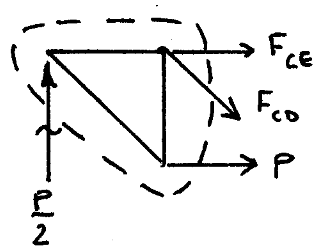
$$\Sigma F_y = \frac{P}{2} - F_{AB}\cos 45 = 0$$
$$F_{AB} = P/\sqrt{2}$$
$$\Sigma F_x = F_{AC} + F_{AB}\sin 45 = 0$$
$$F_{AC} = -\frac{P}{2}$$

MOJ @ B:



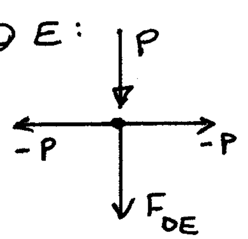
$$\Sigma F_y = 0 = F_{BC} + (P/\sqrt{2})\cos 45$$
$$F_{BC} = -P/2$$
$$\Sigma F_x = 0 = F_{BD} - (P/\sqrt{2})\sin 45$$
$$F_{BD} = P/2$$

MOS:



$$\Sigma F_y = 0 = \frac{P}{2} - F_{CD}\cos 45$$
$$F_{CD} = \frac{P}{\sqrt{2}}$$
$$\circlearrowleft M_D = 0 = -F_{CE}\cancel{\ell} - \frac{P}{\cancel{2}}(\cancel{2}\cancel{\ell}) = 0$$
$$F_{CE} = -P$$

MOJ @ E:

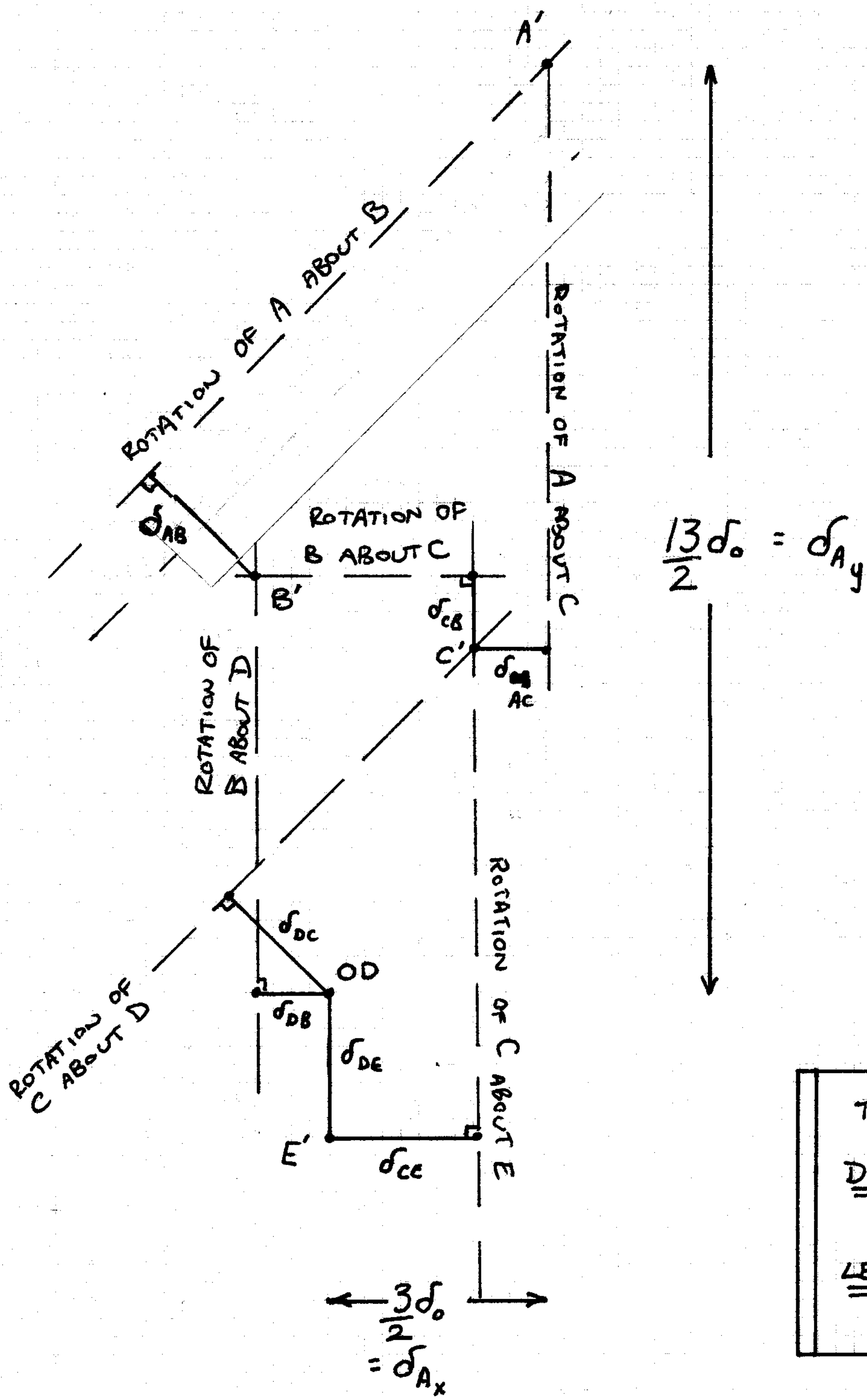


$$\Sigma F_y = 0$$
$$-P - F_{DE} = 0$$
$$F_{DE} = -P$$

BAR	FORCE $\left(\frac{F_{ij}}{P}\right)$	LENGTH $\left(\frac{L_{ij}}{L}\right)$	DEFORMATION $\delta_{ij}/\frac{PL}{AE}$
AB	$+1/\sqrt{2}$	$\sqrt{2}$	$+1$
AC	$-1/2$	1	$-1/2$
CB	$-1/2$	1	$-1/2$
CE	-1	1	-1
CD	$+\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$+1$
BD	$+\frac{1}{2}$	1	$+\frac{1}{2}$
ED	-1	1	-1
EG	-1	1	-1
DG	$+\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$+1$
DF	$+\frac{1}{2}$	1	$+\frac{1}{2}$
GF	$-1/2$	1	$-\frac{1}{2}$
GH	$-1/2$	1	$-\frac{1}{2}$
FH	$+1/\sqrt{2}$	$\sqrt{2}$	$+1$

NOW WE CAN GO AHEAD AND PLOT OUR TRUSS DEFLECTION DIAGRAM.

$\delta_o = \frac{PL}{AE}$



IF MY HINGE POINT A' ENDS UP DISPLACED FROM MY ORIGIN BY δ_{A_x} AND δ_{A_y}, THEN ~~MY~~ ORIGIN OD IS DISPLACED FROM A' BY $-\delta_{A_x}$ AND $-\delta_{A_y}$.

IF I NOW CONSIDER THE FIXED FRAME OA, WHERE A AND A' ARE THE SAME, I CAN FIND THE DEFLECTION OF D' IN THE FIXED FRAME, WHICH IS JUST ITS DISPLACEMENT FROM A', NAMELY

$$-\delta_{A_x}\hat{\imath} - \delta_{A_y}\hat{\jmath}.$$

THE JOINT D WILL TRANSLATE <u>DOWN</u> BY $\frac{13}{2}\frac{PL}{AE}$, AND <u>LEFT</u> BY $\frac{3}{2}\frac{PL}{AE}$

ESTIMATE OF TRUSS DEFLECTIONS

BARS IN EXPERIMENTAL TRUSS MADE OF STEEL
- HOLLOW WITH 22 mm OUTER DIAMETER AND
1.5 mm WALL THICKNESS (IGNORE END FITTINGS)

$$A \approx 2\pi r t \approx 10.5\times10^{-3} \times 2 \times \pi \times 1.5\times10^{-3}$$

$$\approx 100 \text{ mm}^2$$

$$L = 0.5\text{m}$$

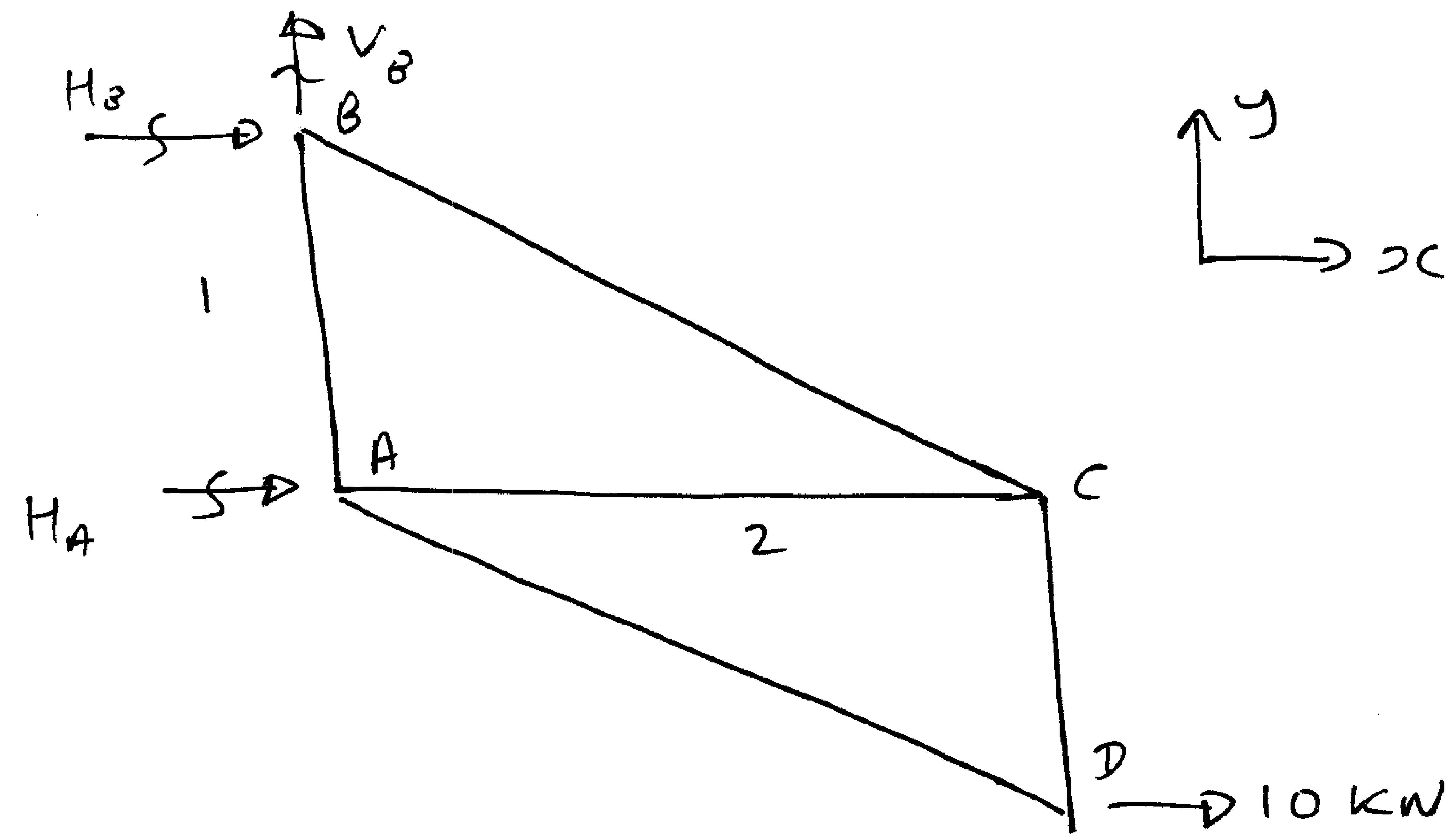
$$E = 210 \text{ GPa}$$

CENTER POINT DEFLECTION

$$\frac{\delta_D}{P} = \left(\frac{0.5}{100\times10^{-6} \times 210\times10^{9}}\right)\left(-\frac{13}{2}\hat{j} - \frac{3}{2}\hat{\imath}\right)$$

$$\boxed{\frac{\delta_D}{P} = -\frac{3.1\times10^{-7}}{2}\hat{j} - \frac{7.1\times10^{-8}}{2}\hat{\imath} \quad \frac{m}{N}}$$

M9



$$\sum \vec{F_x} = 0: \quad 10 + H_B + H_A = 0 \qquad (1)$$

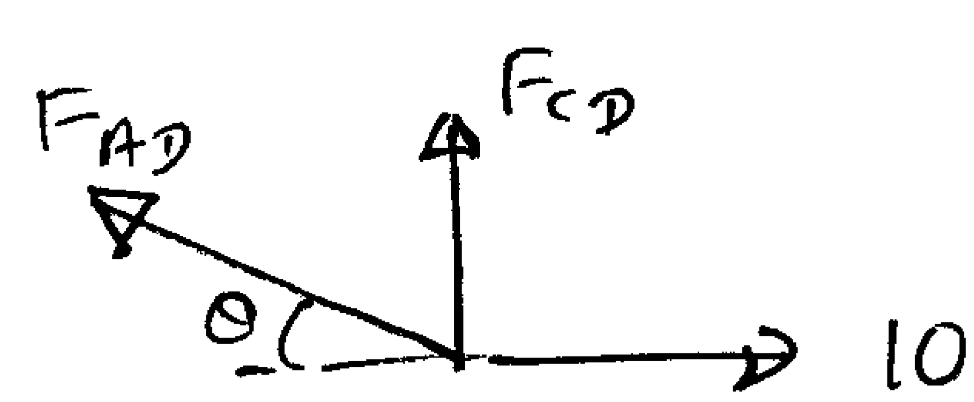
$$\sum F_y \uparrow = 0: \quad V_B + 0 = 0 \Rightarrow V_B = 0 \qquad (2)$$

$$\sum \circlearrowright M_B = 0: \quad H_A \cdot 1 + 10 \cdot 2 = 0 \Rightarrow H_A = -20\,kN \ (3) \Leftarrow$$

Substitute in (1) $\quad H_B = +10\ kN \Leftarrow$

<u>Bar Forces</u>

MoJ @ D



$\cos\theta = \frac{2}{\sqrt{5}}$

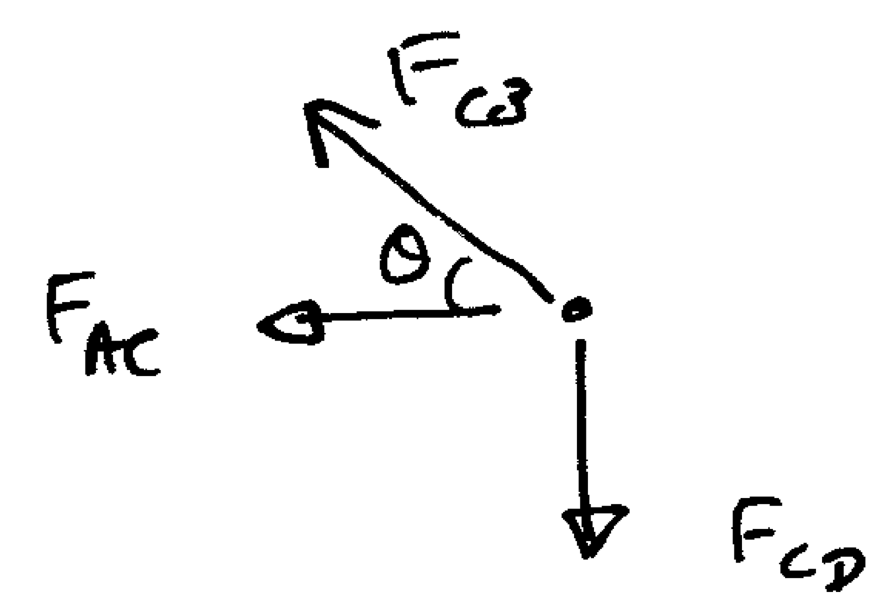
$\sin\theta = \frac{1}{\sqrt{5}}$

$$\sum \vec{F_x} = 0 \qquad -F_{AD}\cos\theta + 10 = 0 \Rightarrow F_{AD} = \frac{10 \cdot \sqrt{5}}{2} = +11.2\,kN$$

$$\sum F_y \uparrow = 0 \qquad F_{AD}\sin\theta + F_{CD} = 0$$

$$F_{CD} = -F_{AD}\sin\theta = -\frac{10 \cdot \cancel{\sqrt{5}}}{2} \cdot \frac{1}{\cancel{\sqrt{5}}} = -5\,kN \Leftarrow$$

MoJ @ C.



$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$

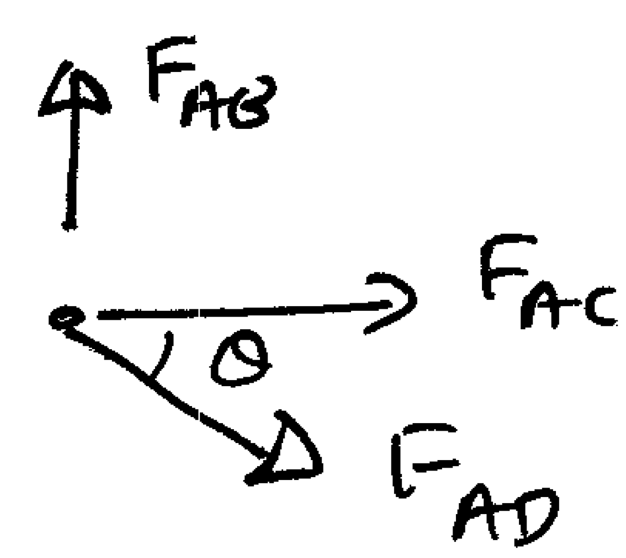
$\Sigma F_y \uparrow = 0 \qquad F_{CB}\sin\theta - F_{CD} = 0$

$F_{CB} = + \frac{F_{CD}}{\sin\theta} = \underline{-5}\sqrt{5}$ KN ⇐

$\Sigma \vec{F_x} = 0: \quad -F_{AC} \cancel{+} - F_{CD}\cos\theta = 0$

$F_{AC} = -F_{CD}\cos\theta = +5\cancel{\sqrt{5}} \cdot \frac{2}{\cancel{\sqrt{5}}} = +10$ KN ⇐

MoJ @ A



$\cos\theta = \frac{2}{\sqrt{5}}$

$\sin\theta = \frac{1}{\sqrt{5}}$

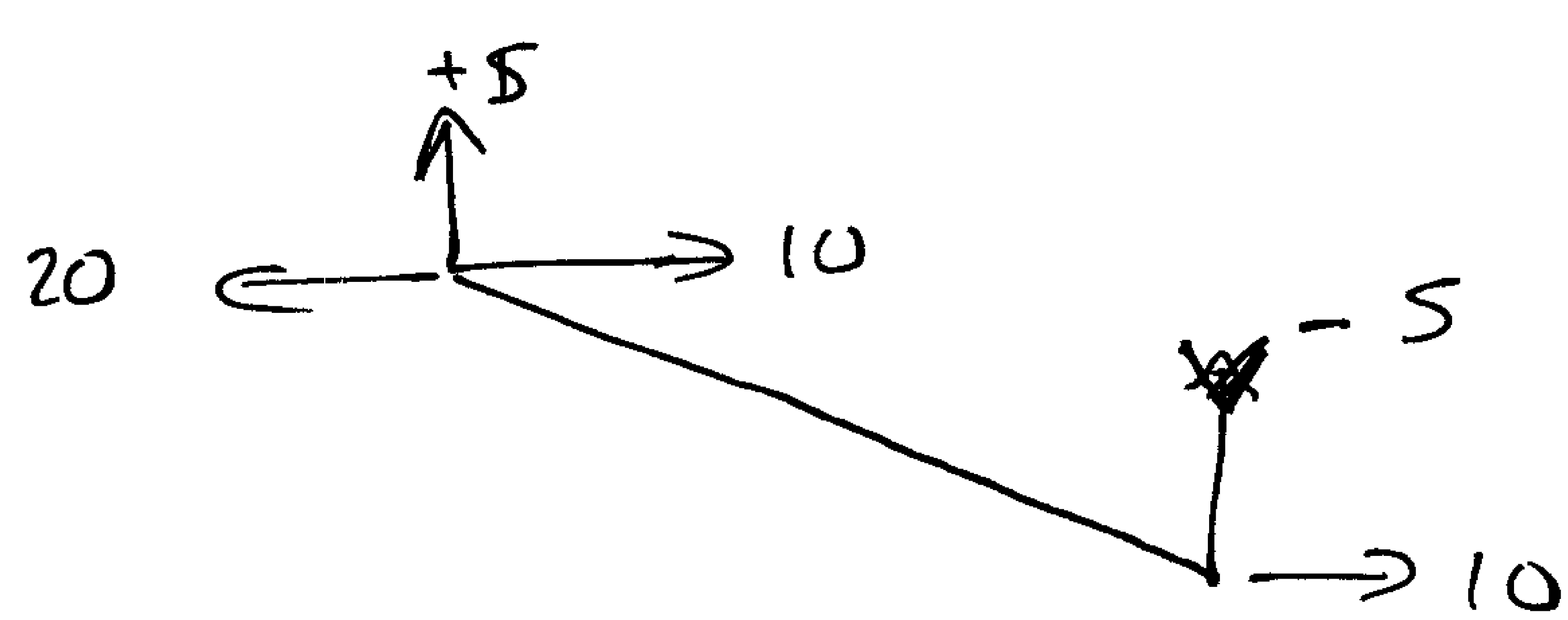
$\Sigma F_y \uparrow = 0 \qquad F_{AB} - F_{AD}\sin\theta = 0$

$\Sigma \vec{F_x} = 0$

$F_{AB} = F_{AD}\sin\theta$

$= \cancel{1/2} + 5\sqrt{5} \cdot \frac{1}{\sqrt{5}} = +5$ KN ⇐

check MOS



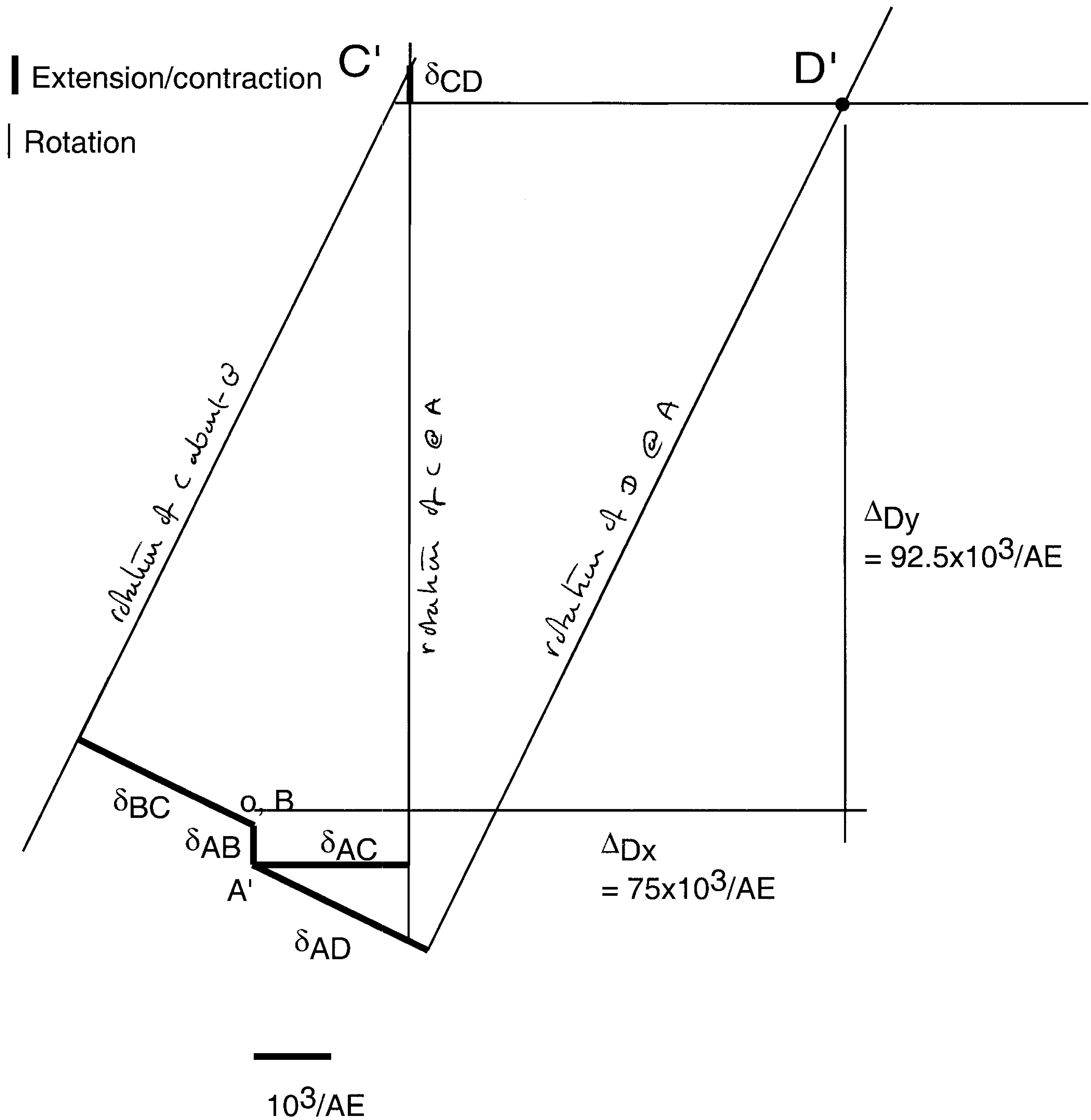
OK!

Bar	Force (kN)	F/P	Length	Length/I	$\delta / \frac{PL}{AE}$	$\frac{\delta}{AE} \times 10^3$
AB	+5	$+\frac{1}{2}$	1	1	$+ \cancel{5}\frac{1}{2}$	+5
AC	+10	+1	2	2	+ 2	+20
BC	$-5\sqrt{5}$	$-\sqrt{5}/2$	$\sqrt{5}$	$\sqrt{5}$	$-\frac{5}{2}$	−25
AD	$+5\sqrt{5}$	$+\sqrt{5}/2$	$\sqrt{5}$	$\sqrt{5}$	$+\frac{5}{2}$	+25
CD	−5	$-\frac{1}{2}$	1	1	$-\frac{1}{2}$	−5

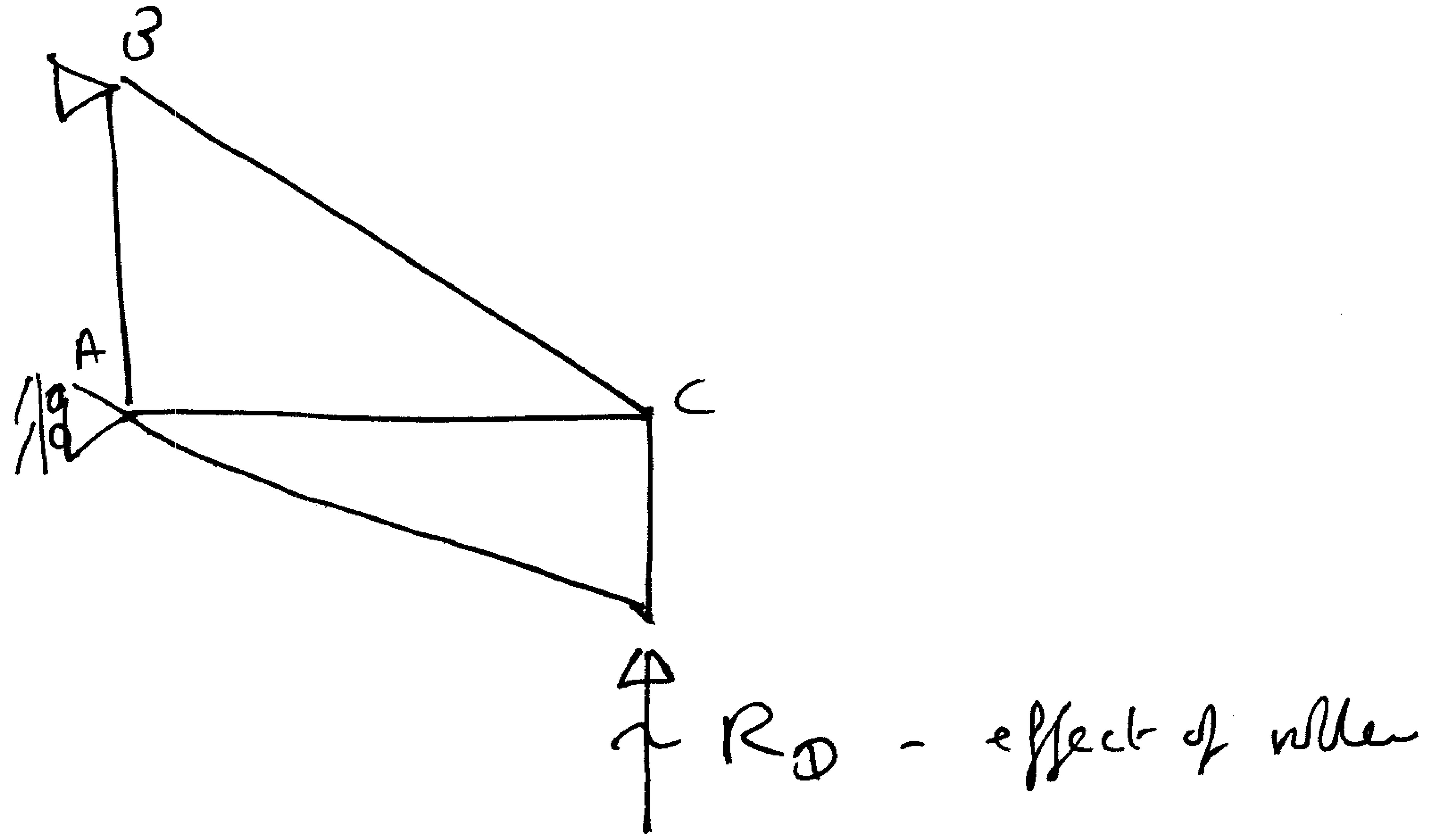
Draw displacement diagram (see attached)

$$\text{vertical deflection} = \frac{92.5 \times 10^3}{AE} = 2.64 \times 10^{-3}\ m$$
$$= 2.64\ mm \Leftarrow$$

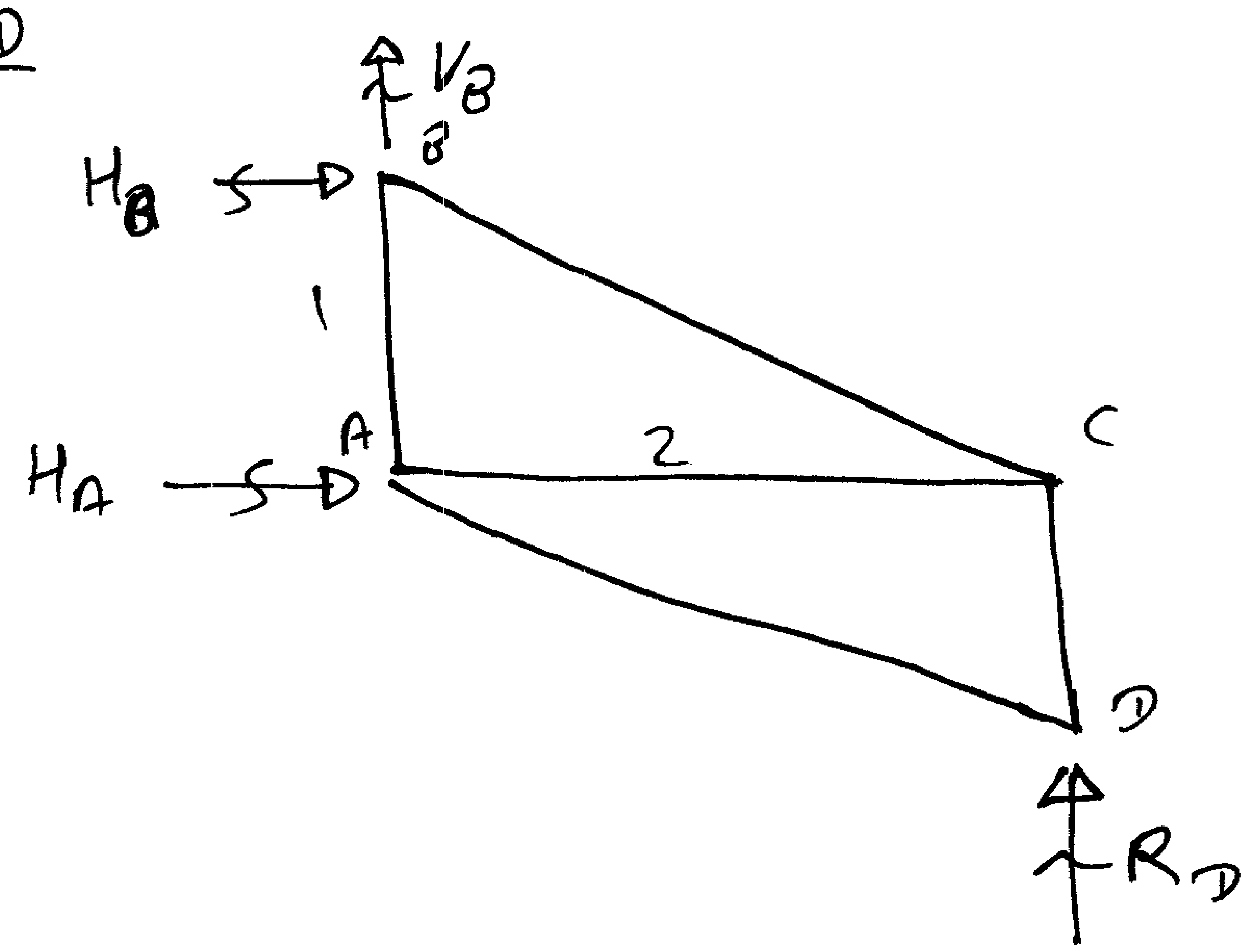
$$\text{Horizontal deflection} = \frac{75 \times 10^3}{AE} = 2.14 \times 10^{-3}\ m$$
$$= 2.14\ mm \Leftarrow$$


Extension/contraction
Rotation
C'
δ_{CD}
D'
rotation of C about B
rotation of C @ A
rotation of D @ A
Δ_{Dy}
$= 92.5 \times 10^3/AE$
δ_{BC}
o, B
δ_{AB}
δ_{AC}
A'
δ_{AD}
Δ_{Dx}
$= 75 \times 10^3/AE$
$10^3/AE$

M10 Use superposition (or set up as a set of unknowns)



FBD



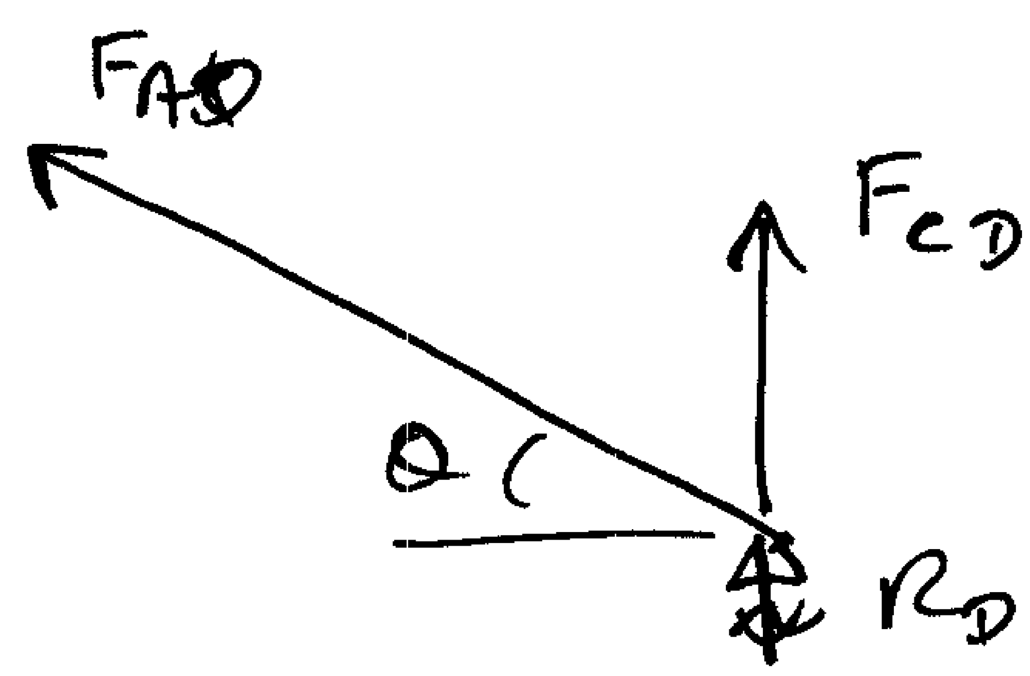
$\Sigma F_y \uparrow = 0: \quad V_B + R_D = 0$ ① $\quad V_B = -R_D$

$\Sigma \vec{F_x} = 0: \quad H_A + H_B = 0$ ②

$\Sigma M_B = 0: \quad H_A . 1 + R_D . 2 = 0 \qquad H_A = -2R_D$

$H_B = +2R_D.$

Bar forces: Method of joints

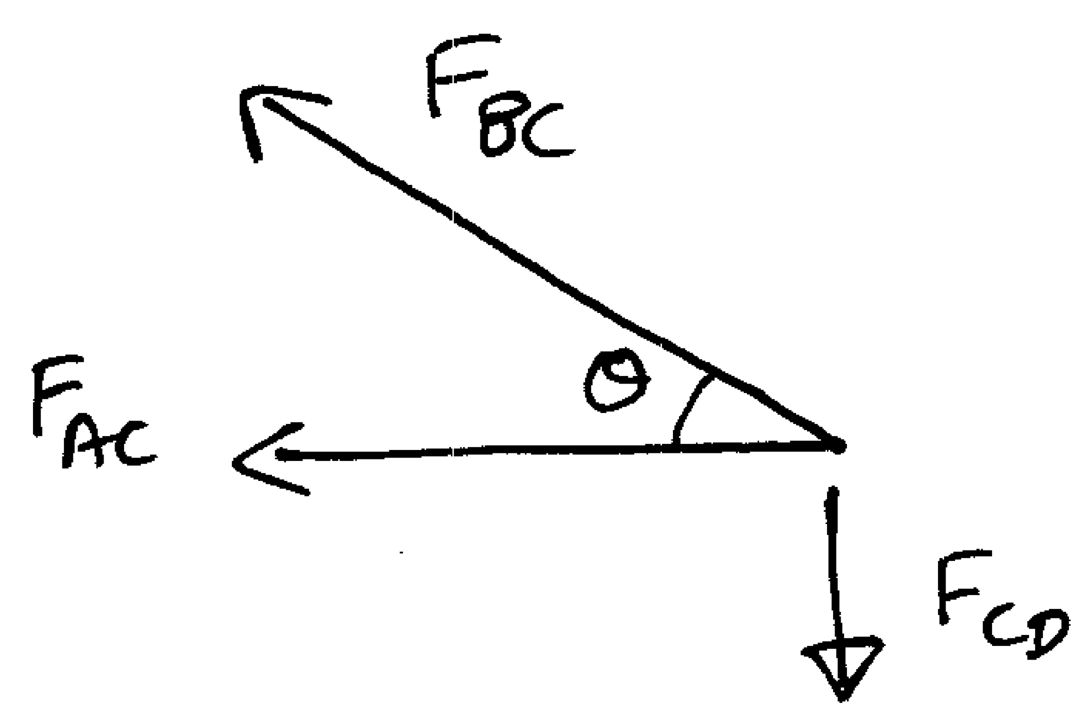


$$\cos\theta = \frac{2}{\sqrt{5}}$$
$$\sin\theta = \frac{1}{\sqrt{5}}$$

$\Sigma \vec{F_x} = 0$: $\quad F_{AD}\cos\theta = 0 \quad \Rightarrow F_{AD} = 0$

$\Sigma F_y \uparrow = 0 \quad F_{AD}\sin\theta + F_{CD} + R_D = 0$

$$F_{CD} = -R_D$$

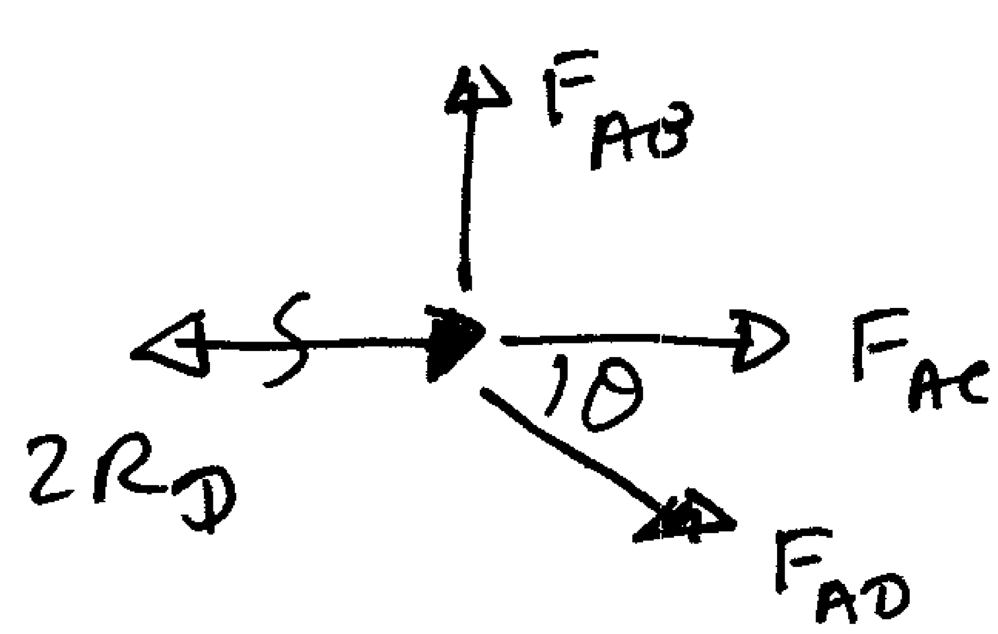


$\Sigma F_y \uparrow = 0$: $\quad F_{BC}\sin\theta - F_{CD} = 0$

$$\underline{F_{BC} = -R_D\sqrt{5}}$$

$\Sigma \vec{F_x} = 0$: $\quad -F_{AC} - F_{BC}\cos\theta = 0$

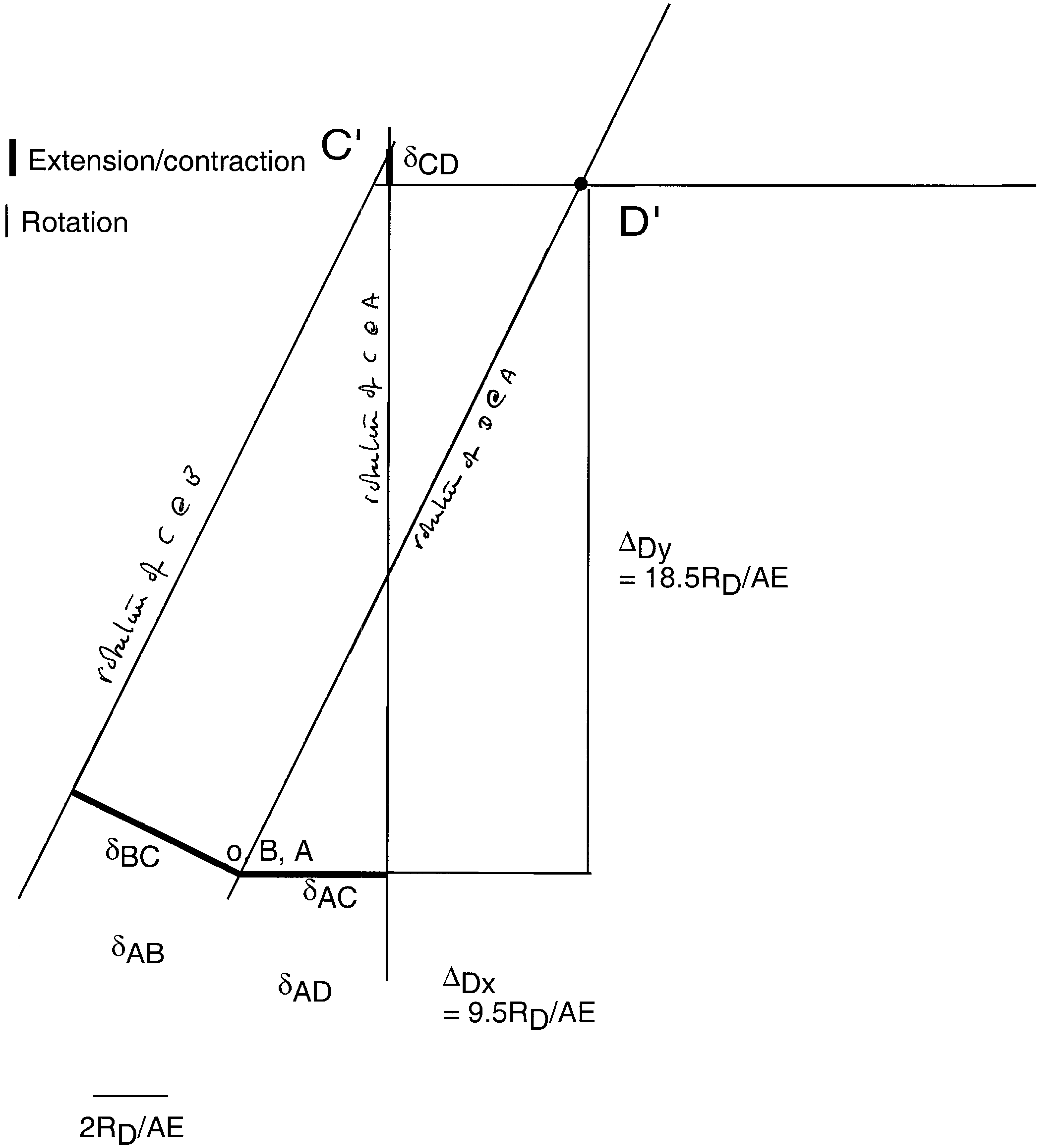
$$F_{AC} = R_D\cancel{\sqrt{5}}\cdot\frac{2}{\cancel{\sqrt{5}}} = 2R_D \Leftarrow$$



$\Sigma F_y \uparrow = 0 \quad F_{AB} - F_{AD}\sin\theta = 0$

$F_{AB} - 0 = 0$

$F_{AB} = 0$

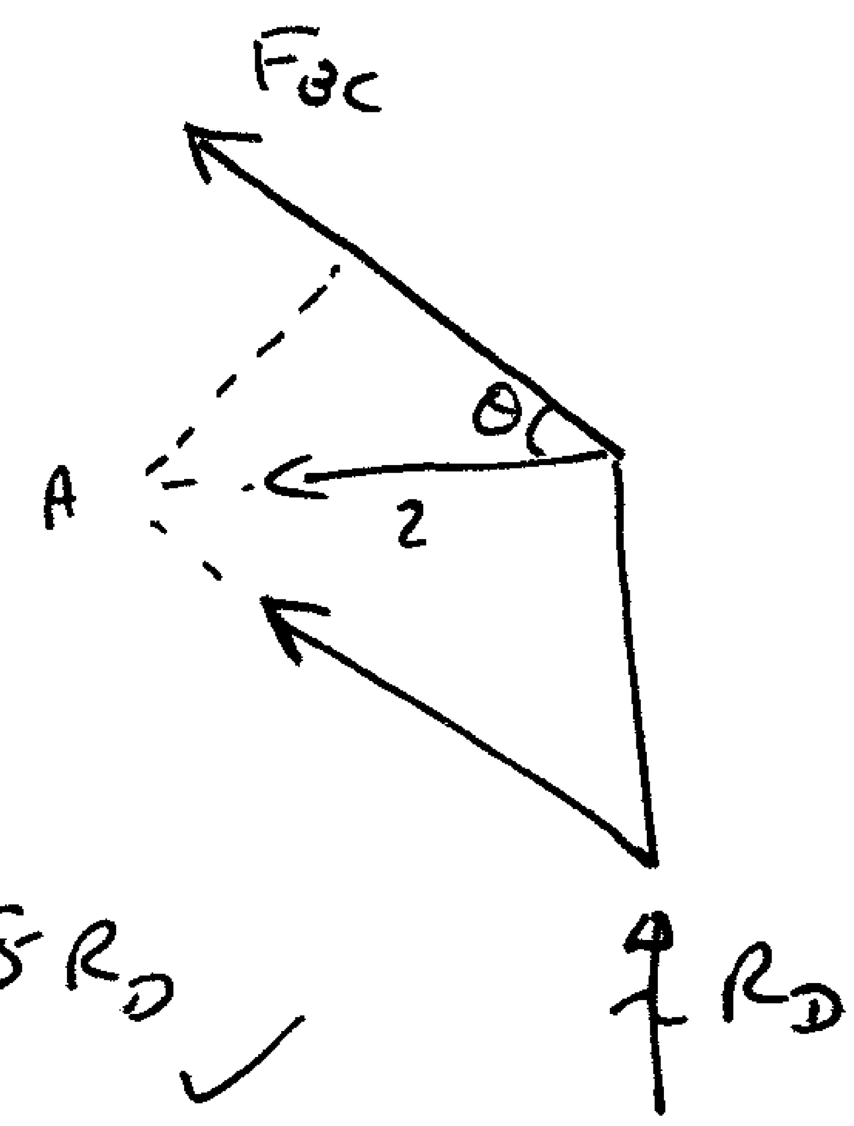


Extension/contraction
Rotation
C'
δCD
D'
rotation of C @ B
rotation of C @ A
rotation of D @ A
ΔDy
= 18.5RD/AE
δBC
o, B, A
δAC
δAB
δAD
ΔDx
= 9.5RD/AE
2RD/AE

check MoS

$\sin\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{2}{\sqrt{5}}$



$\Sigma M_A = 0$

$R_D . 2 + F_{BC} 2 \sin\theta = 0$

$F_{BC} = -\frac{\cancel{2} R_D \sqrt{5}}{\cancel{2}\ \cancel{1}} = -\sqrt{5} R_D$ ✓

check

Bar	Length	Force / R_D	δ / R_D/AE
AB	1	0	0
AC	2	2	4
BC	$\sqrt{5}$	$-\sqrt{5}$	-5
AD	$\sqrt{5}$	0	0
CD	1	-1	-1

Draw displacement diagram

D displaces upward $\Delta D_y = \frac{18.5 R_D}{AE}$

Since D is on a roller $\delta^{(M9)} + \delta^{(M10)} = 0$

$\frac{92.5 \times 10^3}{\cancel{AE}} + \frac{18.5 R_D}{\cancel{AE}} = 0 \qquad R_D = -\frac{92.5 \times 10^3}{18.5} = -5\text{ kN} \Leftarrow$

Horizontal deflection:

$$\Delta D_x^{M9} + \Delta D_x^{M10} = \frac{75 \times 10^3}{AE} + \frac{(9.5 \times -5) \times 10^3}{AE} = 786 \times 10^{-6}$$

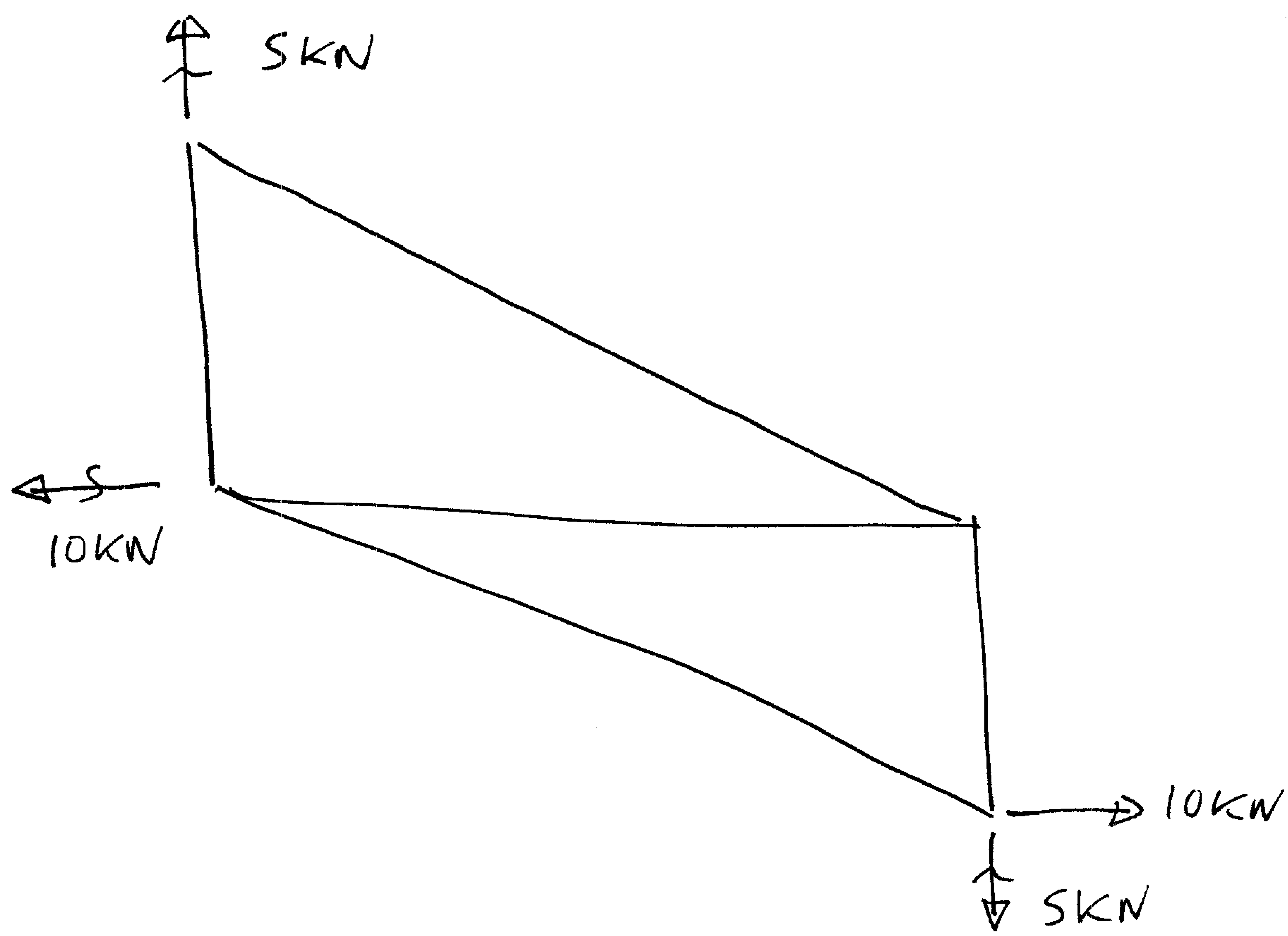
$$= 0.79\ \text{mm}$$

Reactions

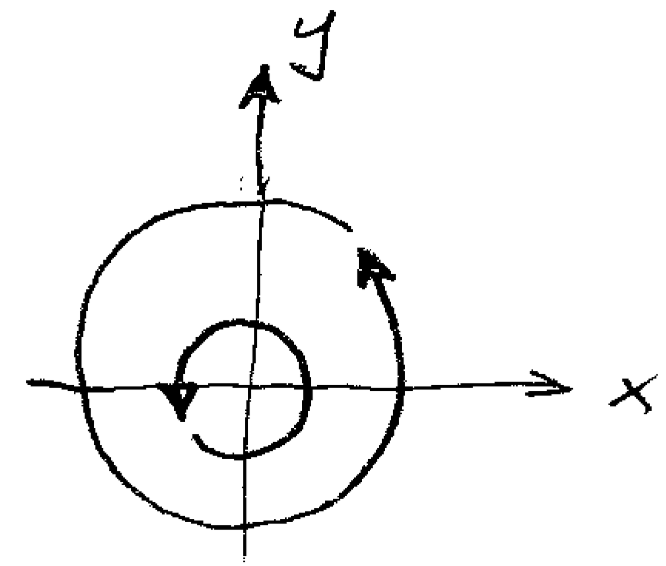
$$H_B = H_B^{M9} + H_B^{M10} = +10 + 2(-5) = 0$$

$$H_A = H_A^{M9} + H_A^{M10} = -20 + (-2(-5)) = -10\ \text{kN}$$

$$V_B = V_B^{M9} + V_B^{M10} = 0 - (-5) = +5\ \text{kN}$$



$$\frac{dy}{dx} = \frac{v}{u} = \frac{x}{-y}$$



$$y\,dy = -x\,dx$$

$$\tfrac{1}{2}y^2 = -\tfrac{1}{2}x^2 + C$$

$$x^2 + y^2 = 2C \qquad \text{circles of radius } \sqrt{2C}$$

For steady flow, with $\rho = \text{const}$, must have $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\left.\begin{aligned} u &= \frac{-y}{x^2+y^2} & \frac{\partial u}{\partial x} &= \frac{y \cdot 2x}{(x^2+y^2)^2} \\ v &= \frac{x}{x^2+y^2} & \frac{\partial v}{\partial y} &= \frac{-x \cdot 2y}{(x^2+y^2)^2} \end{aligned}\right\} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \overset{\checkmark}{=} 0$$

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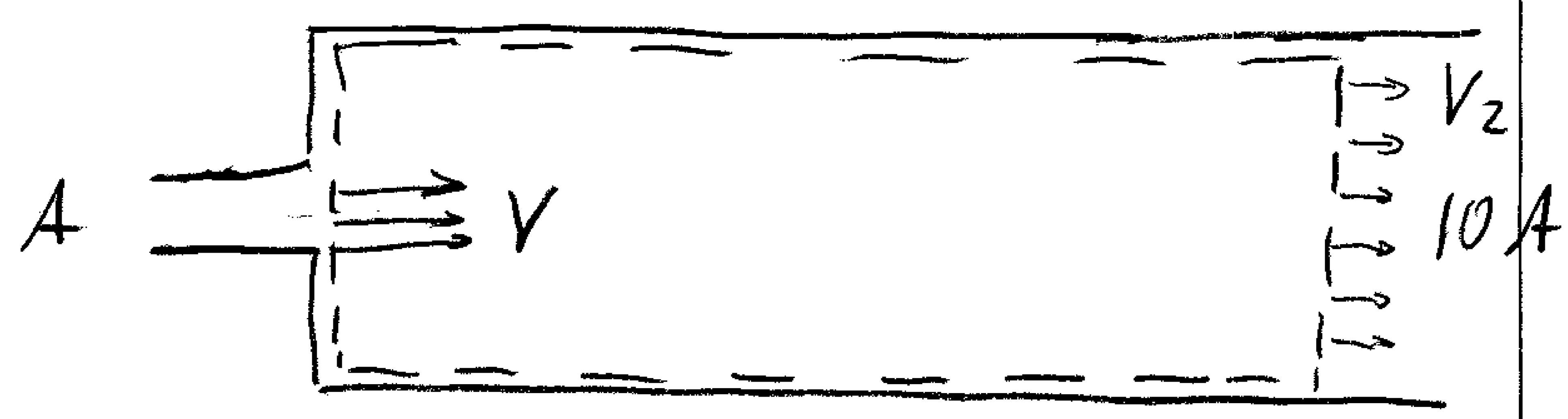
Control Volume:



a) mass conservation

$$\oiint \rho \vec{V}\cdot\hat{n}\, dA = -\rho V A + \rho V_2 (10A) = 0 \rightarrow \boxed{V_2 = \frac{1}{10} V}$$

b) momentum conservation

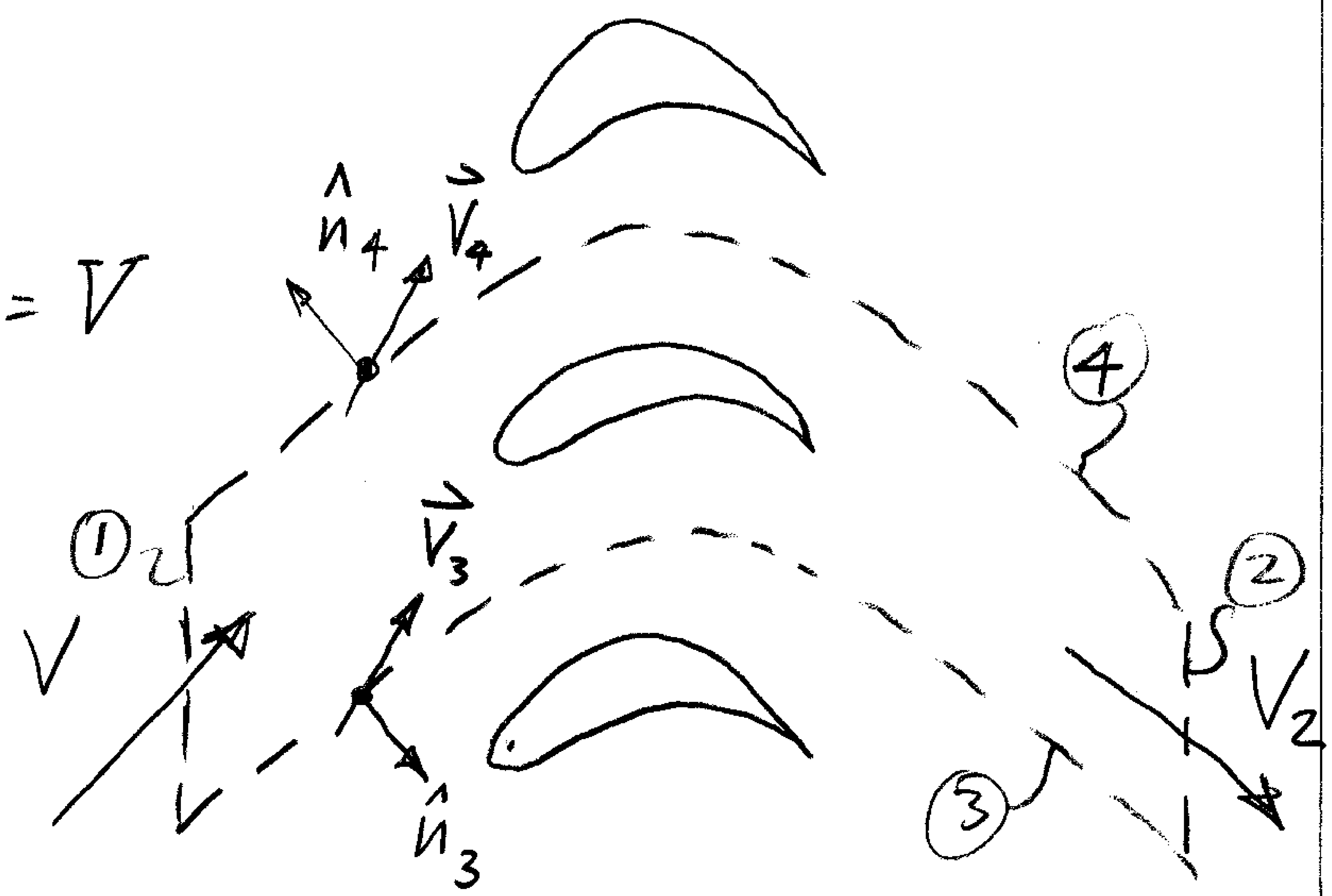
$$0 = \oiint \left[p\hat{n} + \rho(\vec{V}\cdot\hat{n})\vec{V} \right] dA = -p_1 \cdot 10A - \rho V^2 A + p_2 \cdot 10A + \rho V_2^2 10A$$

$$0 = (p_2 - p_1) 10A + \rho\left(-V^2 + \frac{1}{10}V^2\right) A$$

$$\boxed{p_2 - p_1 = \frac{9}{100}\rho V^2 A}$$

Control Volume:

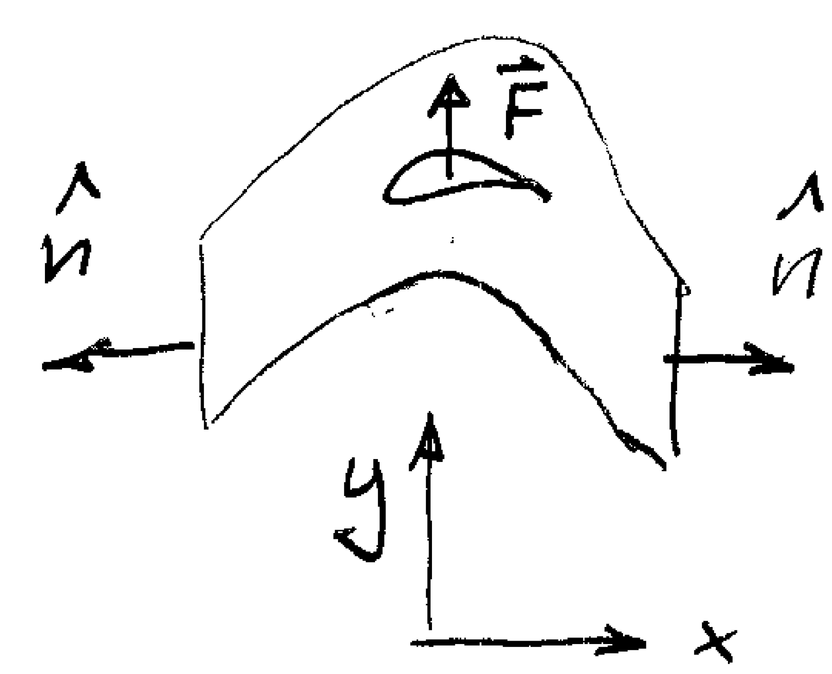
By mass conservation, $V_2 = V$



Because flow is periodic, $\vec{V}_3 = \vec{V}_4$, $p_3 = p_4$
And since $\hat{n}_4 = -\hat{n}_3$, then sides 3 and 4 will cancel in momentum integral.

$$\oint_{①} \left(p\hat{n} + \rho \vec{V}\cdot\hat{n}\,\vec{V}\right) dA + \oint_{②} \left(p\hat{n} + \rho \vec{V}\cdot\hat{n}\,\vec{V}\right) dA = -\vec{F}$$

By symmetry, $\oint_{①} p\hat{n}\, dA + \oint_{②} p\hat{n}\, dA = 0$



$$\oint_{①} \left(\rho \vec{V}\cdot\hat{n}\,\vec{V}\right) dA + \oint \rho \vec{V}\cdot\hat{n}\,\vec{V}\, dA = -\vec{F}$$

$$-\rho V \tfrac{\sqrt{2}}{2} \cdot V \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix} h + \rho V \tfrac{\sqrt{2}}{2} \cdot V \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix} = -\vec{F}$$

$$\boxed{\vec{F} = \begin{bmatrix} 0 \\ \rho V^2 h \end{bmatrix} \quad \text{Vertical Force}}$$