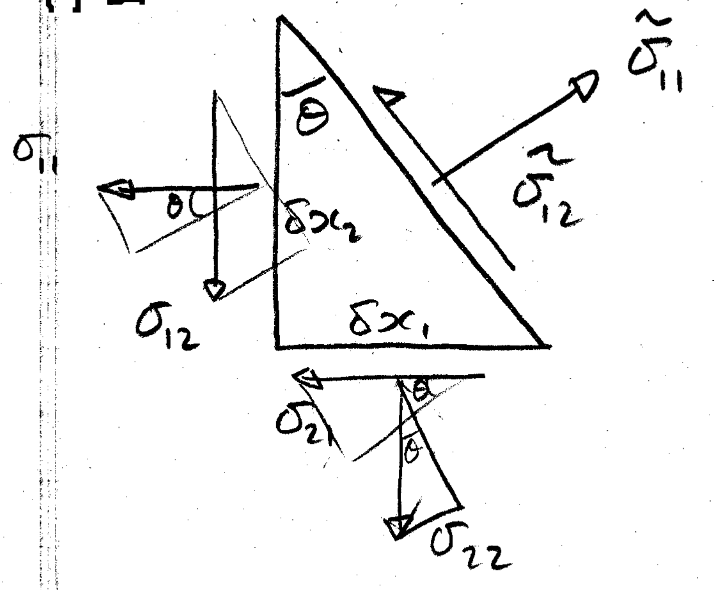
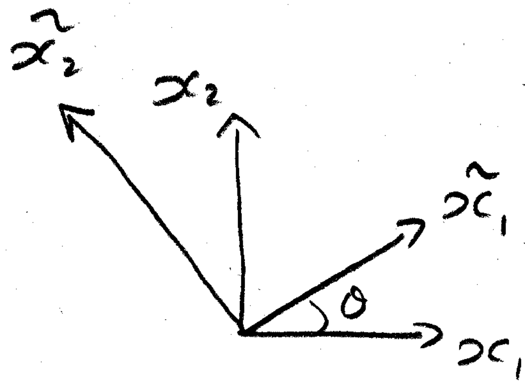
[redacted] M12





consider thickness δx_3

work in terms of $\delta\tilde{x}_1$ $\Rightarrow$ $\delta x_1 = \delta\tilde{x}_1 \sin\theta$

$\delta x_2 = \delta\tilde{x}_1 \cos\theta$

$$\Sigma F_{\tilde{1}} = 0: \quad \tilde{\sigma}_{11}\,\delta\tilde{x}_1\,\delta x_3 - \sigma_{11}\,\delta\tilde{x}_1 \cos\theta\,\delta x_3 . \cos\theta$$

$$- \sigma_{12}\,\delta\tilde{x}_1 \cos\theta . \delta x_3 . \sin\theta - \sigma_{22} . \delta\tilde{x}_1 \sin\theta . \delta x_3 \sin\theta$$

$$- \sigma_{21} . \delta\tilde{x}_1 \sin\theta . \delta x_3 \cos\theta = 0$$

δx_3's cancel, $\sigma_{21} = \sigma_{12}$

$$\Rightarrow \tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta . \sigma_{12}$$

and similarly for $\tilde{\sigma}_{12}$

$$\frac{d\tilde{\sigma}_{11}}{d\theta} = -2\sigma_{11}\cos\theta\sin\theta + 2\sigma_{22}\sin\theta\cos\theta$$

$$+ (2\cos^2\theta - 2\sin^2\theta)\,\sigma_{12} = 0$$

simplify $\cos\theta\sin\theta = \frac{1}{2}(\sin 2\theta)$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\frac{1}{2}(\cancel{1} + \cos 2\theta) - \frac{1}{2}(\cancel{1} - \cos 2\theta)$$

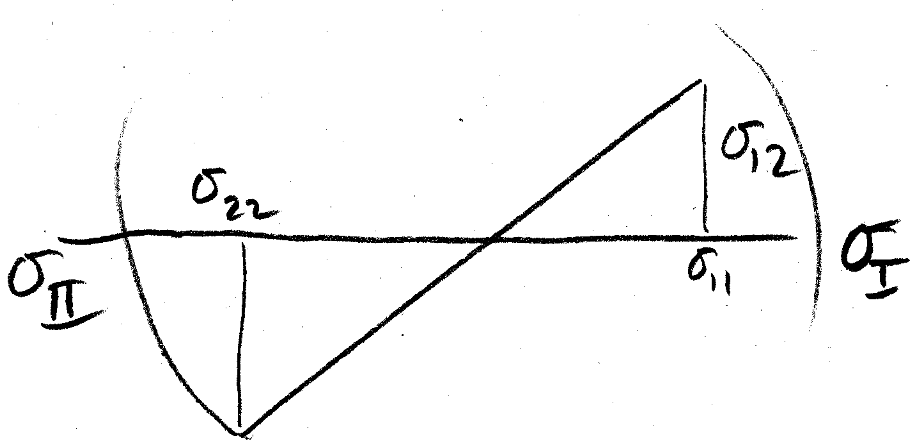
$$= \cos 2\theta$$

$$\Rightarrow \sigma_{22}\sin 2\theta - \sigma_{11}\sin 2\theta + 2\cos 2\theta\,\sigma_{12} = 0$$

$$(\sigma_{22} - \sigma_{11})\tan 2\theta + 2\sigma_{12} = 0$$

$$\tan 2\theta = \frac{2\sigma_{12}}{(\sigma_{22} - \sigma_{11})}$$

cf. Mohr's circle



$$\sum F_{\tilde{x}} = 0$$

$$\sigma_{12}\, \delta\hat{x}_1\, \delta x_3 + \tilde{\sigma}_{11}\, \delta x_1 \cos\theta\, \delta x_3 \sin\theta$$

$$- \sigma_{12}\, \delta\hat{x}_1 \cos\theta\, \delta x_3 \cos\theta - \sigma_{22}\, \delta\tilde{x}_1 \sin\theta . \delta x_3 \cos\theta$$

$$+ \sigma_{21}\, \delta\hat{x}_1 \sin\theta\, \delta x_3 \sin\theta = 0$$

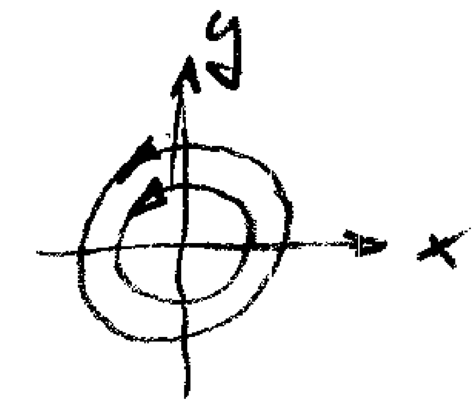
$$\Rightarrow \tilde{\sigma}_{12} = - \cos\theta \sin\theta\, \sigma_{11} + \cos\theta \sin\theta\, \sigma_{22}$$

$$+ (\cos^2\theta - \sin^2\theta)\, \sigma_{12} \Leftarrow$$

a) $u = -y \qquad v = x$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{-x}{y} \rightarrow y\,dy = -x\,dx \rightarrow \frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$$

$$x^2 + y^2 = 2C \qquad \text{circles of radius } \sqrt{2C}$$



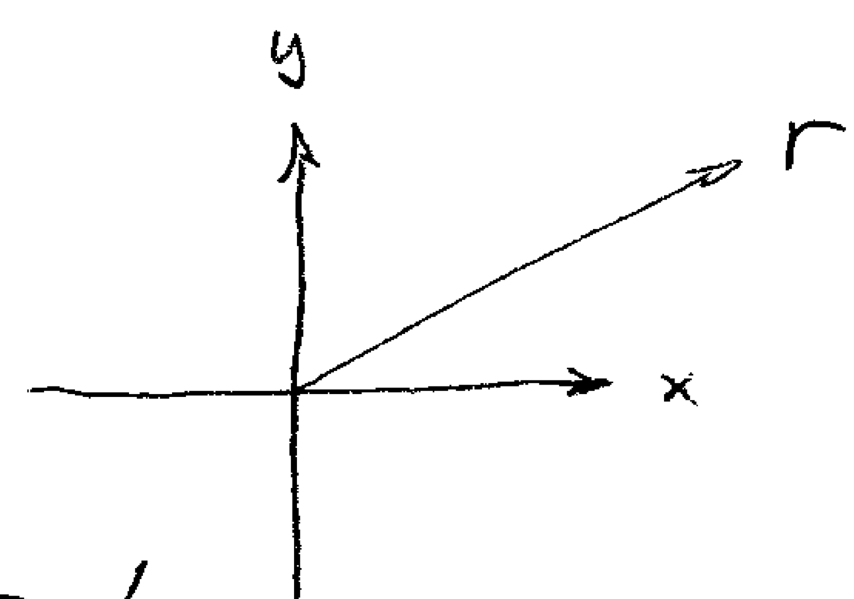
b) $$\frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -y \cdot 0 + x \cdot (-1) = -x$$

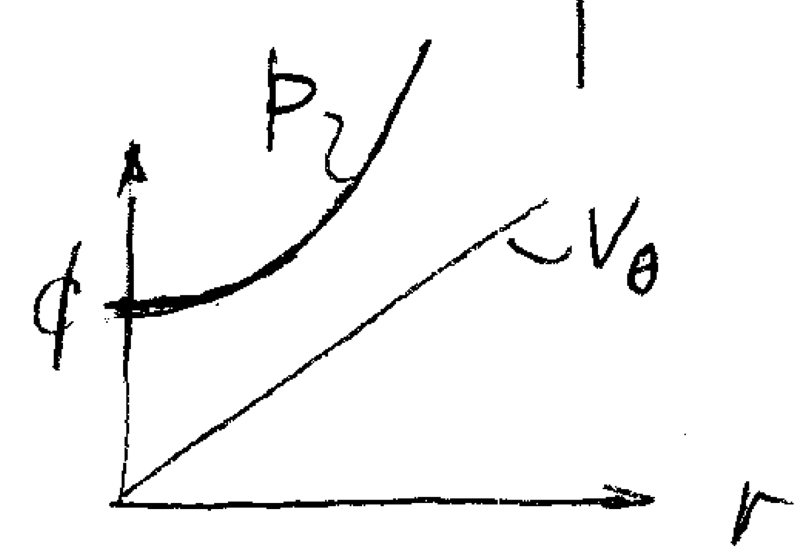
$$\frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -y \cdot 1 + x \cdot 0 = -y$$

using momentum eq'n: $$\frac{\partial p}{\partial x} = -\rho\frac{Du}{Dt} = \rho x$$
$$\frac{\partial p}{\partial y} = -\rho\frac{Dv}{Dt} = \rho y$$

$$\nabla p = \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} = \rho x\,\hat{i} + \rho y\,\hat{j}$$

c) $$\left.\begin{aligned}\frac{\partial p}{\partial x} &= \rho x \\ \frac{\partial p}{\partial y} &= \rho y\end{aligned}\right\} \quad p = \frac{1}{2}\rho(x^2 + y^2) + \cancel{C}$$





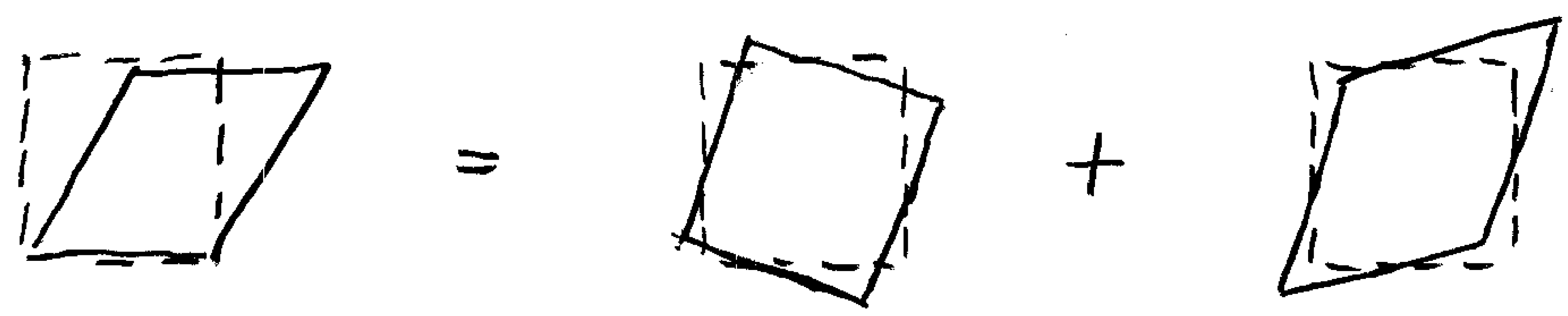
a) $u = Cy \qquad v = 0$

$$\xi_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -C \qquad \circlearrowleft +\xi_z \qquad \circlearrowright -\xi_z$$

$$\text{or, } \omega_z = \frac{1}{2}\xi_z = -\frac{1}{2}C$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}C$$

b) <u>Simple shearing</u> motion, which is a 50-50 combination of rotation and shear



$-\omega_z \qquad \varepsilon_{xy}$

$$\frac{\partial u}{\partial y} = -\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$C = -\left(-\frac{1}{2}C\right) + \frac{1}{2}C$$