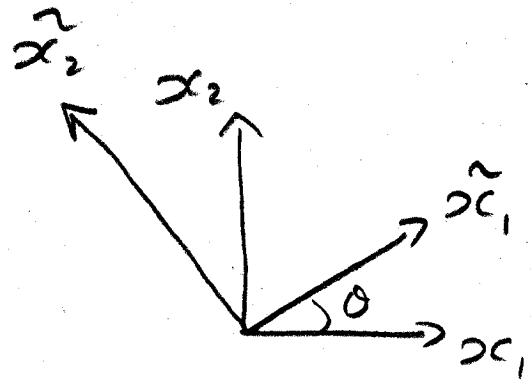
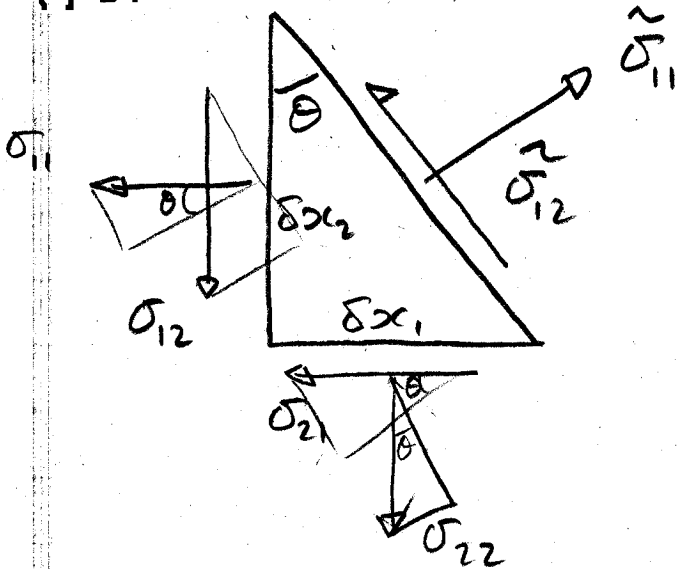


~~M12~~ M12



Consider thickness  $\delta x_3$

work in terms of  $\delta \tilde{x}_1$ ,  $\Rightarrow \delta x_2 = \delta \tilde{x}_1 \sin \theta$   
 $\delta x_2 = \delta \tilde{x}_1 \cos \theta$

$$\begin{aligned} \sum \vec{F}_i = 0: & \sigma_{11} \delta \tilde{x}_1 \delta x_3 - \sigma_{11} \delta \tilde{x}_1 \cos \theta \delta x_3 \cos \theta \\ & - \sigma_{12} \delta \tilde{x}_1 \cos \theta \delta x_3 \sin \theta - \sigma_{22} \delta \tilde{x}_1 \sin \theta \delta x_3 \sin \theta \\ & - \sigma_{21} \delta \tilde{x}_1 \sin \theta \delta x_3 \cos \theta = 0 \end{aligned}$$

$\delta x_3$ 's cancel,  $\sigma_{21} = \sigma_{12}$

$$\Rightarrow \tilde{\sigma}_{11} = \cos^2 \theta \sigma_{11} + \sin^2 \theta \sigma_{22} + 2 \cos \theta \sin \theta \sigma_{12}$$

and similarly for  $\tilde{\sigma}_{12}$

$$\frac{d\tilde{\sigma}_{11}}{d\theta} = -2\sigma_{11}\cos\theta\sin\theta + 2\sigma_{12}\sin\theta\cos\theta$$

$$+ (2\cos^2\theta - 2\sin^2\theta)\sigma_{12} = 0$$

Simplify  $\cos\theta\sin\theta = \frac{1}{2}(\sin 2\theta)$

$$2\cos^2\theta - 2\sin^2\theta = \cos 2\theta$$

$$\frac{1}{2}(\sigma_{11} + \cos 2\theta) - \frac{1}{2}(\sigma_{11} - \cos 2\theta)$$

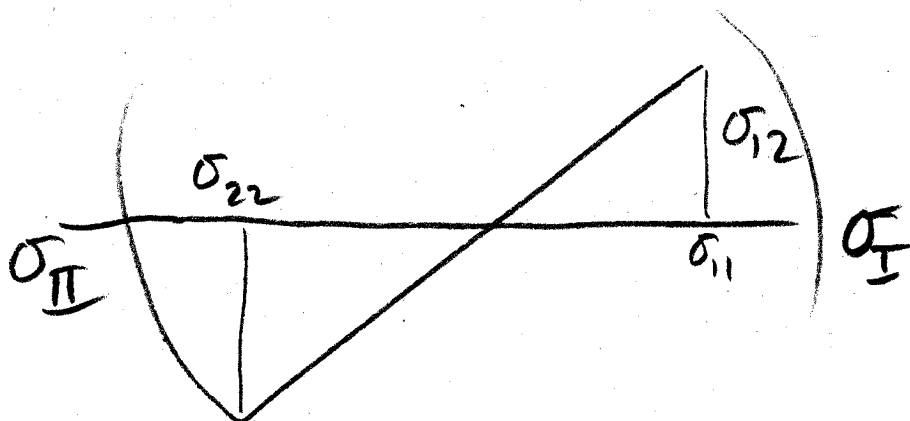
$$= \cos 2\theta$$

$$\Rightarrow \sigma_{22}\sin 2\theta - \sigma_{11}\sin 2\theta + 2\cos 2\theta\sigma_{12} = 0$$

$$(\sigma_{22} - \sigma_{11})\tan 2\theta + 2\sigma_{12} = 0$$

$$\tan 2\theta = \frac{2\sigma_{12}}{(\sigma_{22} - \sigma_{11})}$$

cf. Mohr's circle



$$\sum \vec{F}_{\vec{x}} = 0$$

$$\sigma_{12} \delta \tilde{x}_1 \delta x_3 + \tilde{\sigma}_{11} \delta x_1 \cos \theta \delta x_3 \sin \theta$$

$$- \sigma_{12} \delta \tilde{x}_1 \cos \theta \delta x_3 \cos \theta - \sigma_{22} \delta \tilde{x}_1 \sin \theta \delta x_3 \cos \theta$$

$$+ \sigma_{21} \delta \tilde{x}_1 \sin \theta \delta x_3 \sin \theta = 0$$

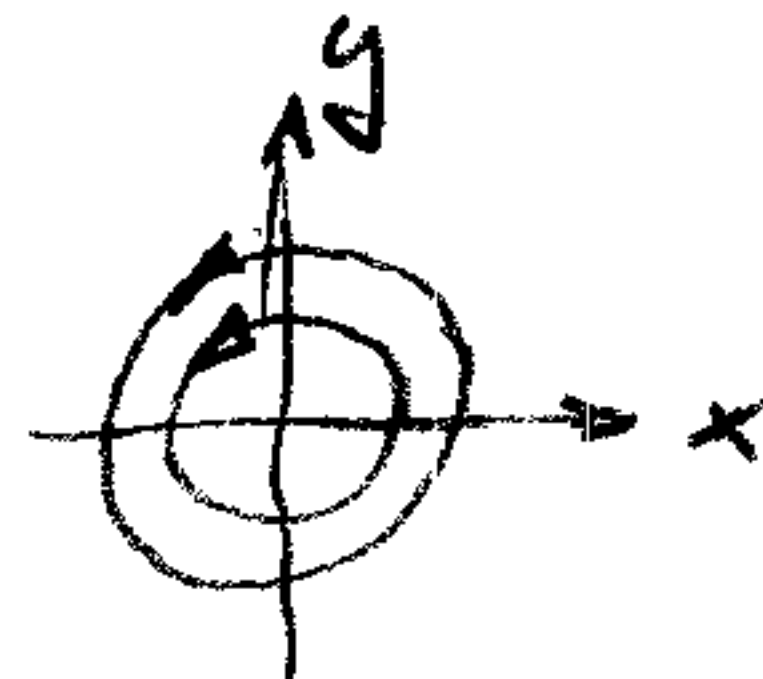
$$\Rightarrow \tilde{\sigma}_{12} = - \cos \theta \sin \theta \sigma_{11} + \cos \theta \sin \theta \sigma_{22}$$

$$+ (\cos^2 \theta - \sin^2 \theta) \sigma_{12} \Leftarrow$$

a)  $u = -y \quad v = x$

$\frac{dy}{dx} = \frac{v}{u} = \frac{-x}{-y} \rightarrow y dy = -x dx \rightarrow \frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$

$x^2 + y^2 = 2C$  circles of radius  $\sqrt{2C}$



b)  $\frac{Du}{Dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -y \cdot 0 + x \cdot (-1) = -x$

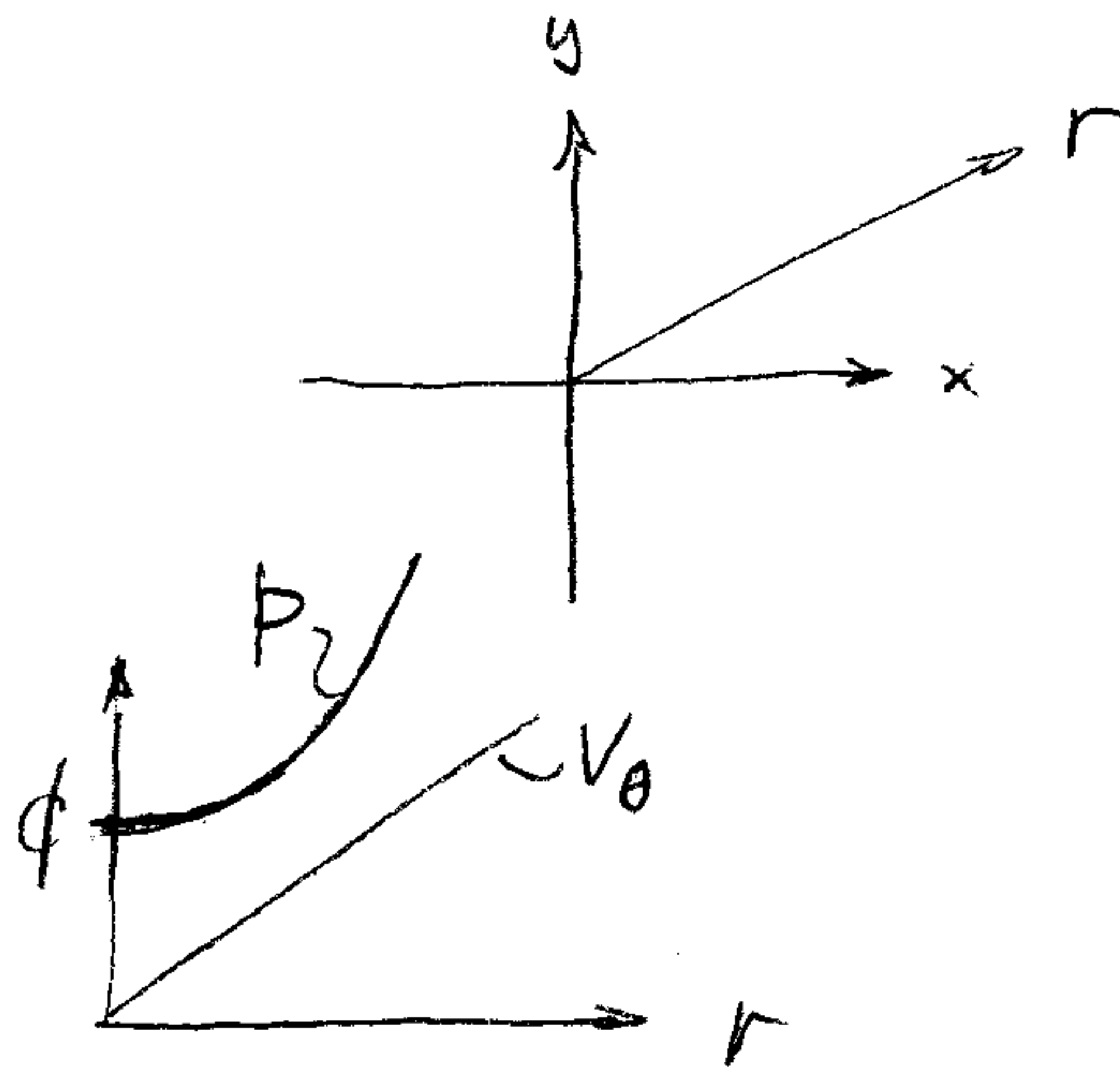
$\frac{Dv}{Dt} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -y \cdot 1 + x \cdot 0 = -y$

using momentum eq'n:  $\frac{\partial p}{\partial x} = -\rho \frac{Du}{Dt} = \rho x$

$\frac{\partial p}{\partial y} = -\rho \frac{Dv}{Dt} = \rho y$

$\nabla p = \frac{\partial p}{\partial x} \hat{i} + \frac{\partial p}{\partial y} \hat{j} = \rho x \hat{i} + \rho y \hat{j}$

c)  $\left. \begin{aligned} \frac{\partial p}{\partial x} &= \rho x \\ \frac{\partial p}{\partial y} &= \rho y \end{aligned} \right\} p = \frac{1}{2} \rho (x^2 + y^2) + C$

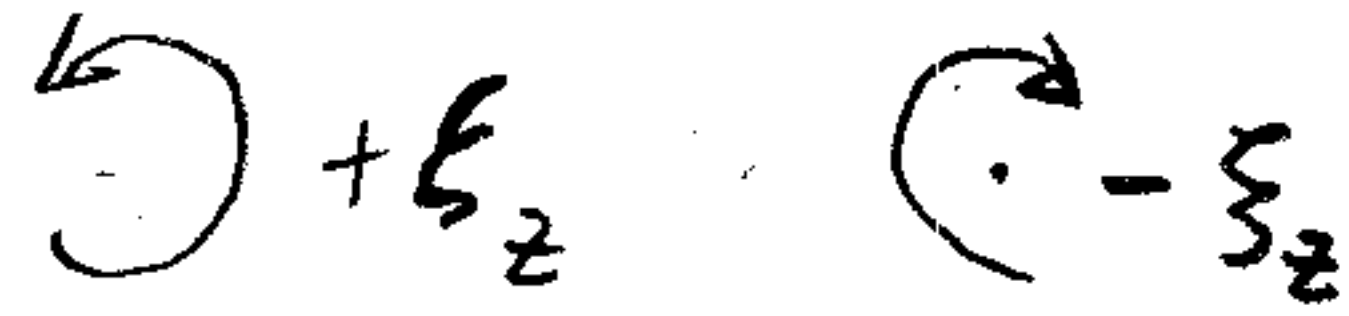


19-782  
42-361  
42-362  
42-369  
42-392  
42-399  
500 SHEETS, FILLER, 5 SQUARE  
50 SHEETS, EYE-EASE, 5 SQUARE  
100 SHEETS, EYE-EASE, 5 SQUARE  
200 SHEETS, EYE-EASE, 5 SQUARE  
100 RECYCLED WHITE, 5 SQUARE  
200 RECYCLED WHITE, 5 SQUARE  
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a)  $u = Cy \quad v = 0$

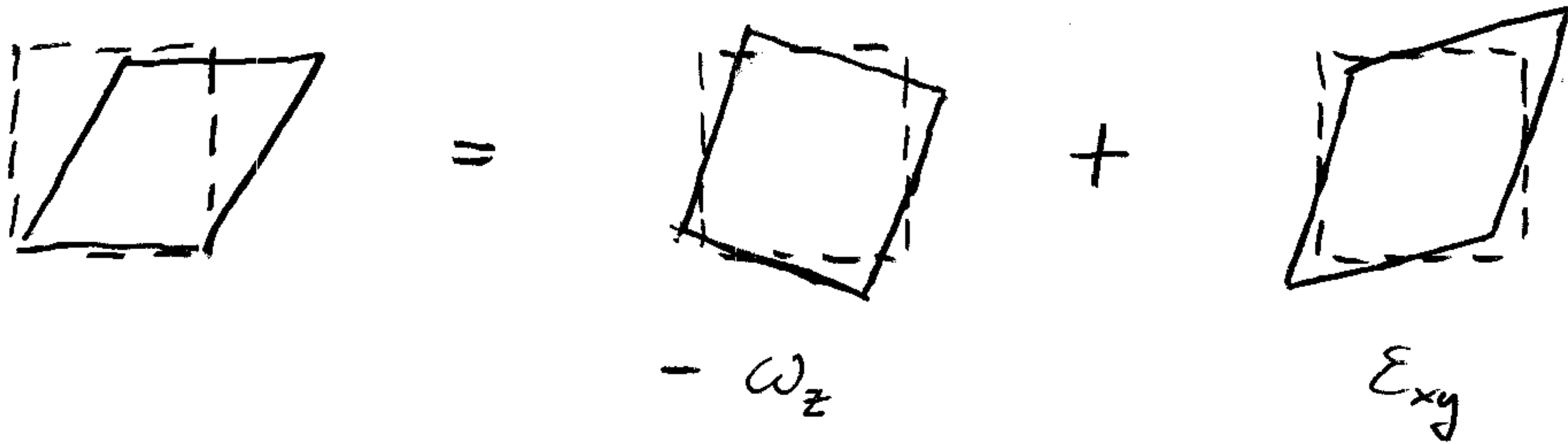
$$\xi_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -C$$



or,  $\omega_z = \frac{1}{2} \xi_z = -\frac{1}{2} C$

$$\epsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{1}{2} C$$

b) Simple shearing motion, which is a 50-50 combination of rotation and shear



$$\frac{\partial u}{\partial y} = -\frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$C = - \left( -\frac{1}{2} C \right) + \frac{1}{2} C$$

13-782 500 SHEETS FILLER 5 SQUARE  
 42-381 50 SHEETS EYE-EASE 5 SQUARE  
 42-382 100 SHEETS EYE-EASE 5 SQUARE  
 42-389 200 SHEETS EYE-EASE 5 SQUARE  
 42-392 100 RECYCLED WHITE 5 SQUARE  
 42-399 200 RECYCLED WHITE 5 SQUARE  
 Made in U.S.A.

