16.01/16.02 Unified Engineering I, II
Fall 2003

Problem Set 12

Name: ____________________

Due Date: 11/25/03

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Announcements:
F12. For the two flows given by …

\[ \psi(x, y) = \arctan(y/x) \quad \phi(x, y) = x^2 + y^2 \]

a) Determine the velocity fields, and sketch the streamlines.
b) Determine the volume flow rate through a circle of radius \( r \).
c) Which of these flows is not feasible to set up in a lab? Explain.

F13. A venturi has a minimum throat area of 0.7 times the inlet/outlet areas. The water tank is open to ambient atmospheric pressure. Determine the sea-level wind speed \( V \) which is needed at the inlet to raise the water column 10 cm.

\[ V \quad \text{(water column 10 cm)} \]

F14. \( \phi_1(x, y) \) and \( \phi_2(x, y) \) are known to be physically-possible flows (i.e. satisfy mass conservation), and their corresponding pressure fields \( p_1(x, y) \) and \( p_2(x, y) \) are known via the Bernoulli equation.

a) A third flow is now defined by \( \phi_3(x, y) = \phi_1 + \phi_2 \). Explain how you would obtain its corresponding pressure field \( p_3 \).
b) Yet another flow \( \phi_4 = \partial \phi_1 / \partial x \) is defined. Is this a physically-possible flow?
Problem M13 (Materials and Structures)

a) The state of axial stress through the thickness of a beam in pure bending (i.e. loaded only by a moment) is given by:

\[
\sigma_{11} = \frac{M}{I} x_3 \quad \text{for } -h < x_3 < +h
\]

M is the bending moment (which is constant in \(x_1\)), and I is the second moment of area of the cross-section of the beam (which is also constant). If \(\sigma_{12} = \sigma_{32} = \sigma_{22} = \sigma_{33} = 0\) what can you say about the variation of the shear stress \(\sigma_{13}\) with \(x_1, x_2\) and \(x_3\)?

b) The bending moment M now varies as a function of \(x_1\) according to \(M = cx_1\). The axial stress is still give by \(\sigma_{11} = \frac{M}{I} x_3\). Again, \(\sigma_{12} = \sigma_{32} = \sigma_{22} = \sigma_{33} = 0\). How does \(\sigma_{13}\) vary with \(x_1, x_2\) and \(x_3\)? Note \(\sigma_{13} = 0\) for \(-h = x_3\) and \(x_3 = +h\) (the top and bottom surfaces of the beam are free surfaces and do not have any stress acting on them).
Problem MS14

For each of the states of plane stress shown acting on the differential elements drawn below do the following:
1) Draw a Mohr's circle describing the stress state.
2) Determine the principal stresses and the maximum shear stress
3) Calculate the angle from the $x_1$ direction as shown to the more tensile principal stress direction. Note whether the angle is clockwise or counterclockwise.
Problem M15 (Materials and Structures)

i) By considering the change in volume of an infinitessimal element undergoing small elongational strains show that the volumetric strain \( \frac{\Delta V}{V} = \epsilon_1 + \epsilon_2 + \epsilon_3 \)

ii) A continuous body experiences a displacement field, \( u_n \) that is described by:

\[
\begin{align*}
\dot{u}_1 &= [0.5(\dot{x}_1^2 \dot{x}_2^2) + 0.5\dot{x}_1 \dot{x}_2] \times 10^{-3} \text{ mm} \\
\dot{u}_1 &= [0.25(\dot{x}_1^2 \dot{x}_2^2) - \dot{x}_1 \dot{x}_2] \times 10^{-3} \text{ mm} \\
\dot{u}_3 &= 0.
\end{align*}
\]

Determine:

a) The 6 components of the strain tensor as a function of position (i.e. in terms of \( x_1, x_2, x_3 \))

b) The rigid body rotation about \( x_3 \) as a function of position (i.e. in terms of \( x_1, x_2, x_3 \)).

c) The principal strains and the principal strain directions at \( x_1 = 5 \text{ mm} \) and \( x_2 = 7 \text{ mm} \).

d) The volumetric strain at \( x_1 = 5 \text{ mm} \) and \( x_2 = 7 \text{ mm} \).
Problem M16

The purpose of this question is to demonstrate the equivalence of the two methods at our disposal for transforming strain (and stress) and for calculating the principal values and directions.

Given a state of plane strain: $e_{11} = -0.000200$, $e_{22} = +0.000400$, $e_{12} = -0.000200$, do the following:

a) Draw a Mohr's circle for the strain state. Note you may find it convenient to work in terms of "micro-strain" (strain/10$^{-6}$).

b) From the Mohr's circle determine the principal strains $e_I$ and $e_{II}$ and principal directions, $\tilde{x}_1$ and $\tilde{x}_2$. You should specify the directions as counterclockwise angles with respect to the original $x_1$ and $x_2$ coordinate system.

c) Determine the direction cosines for the transformation between $x_1, x_2$ and $\tilde{x}_1, \tilde{x}_2$.

d) Using the appropriate tensor operation show that the original strain tensor ($\underline{e}_{nm}$) transforms to the principal strain tensor ($\underline{\tilde{e}}_{pq}$).