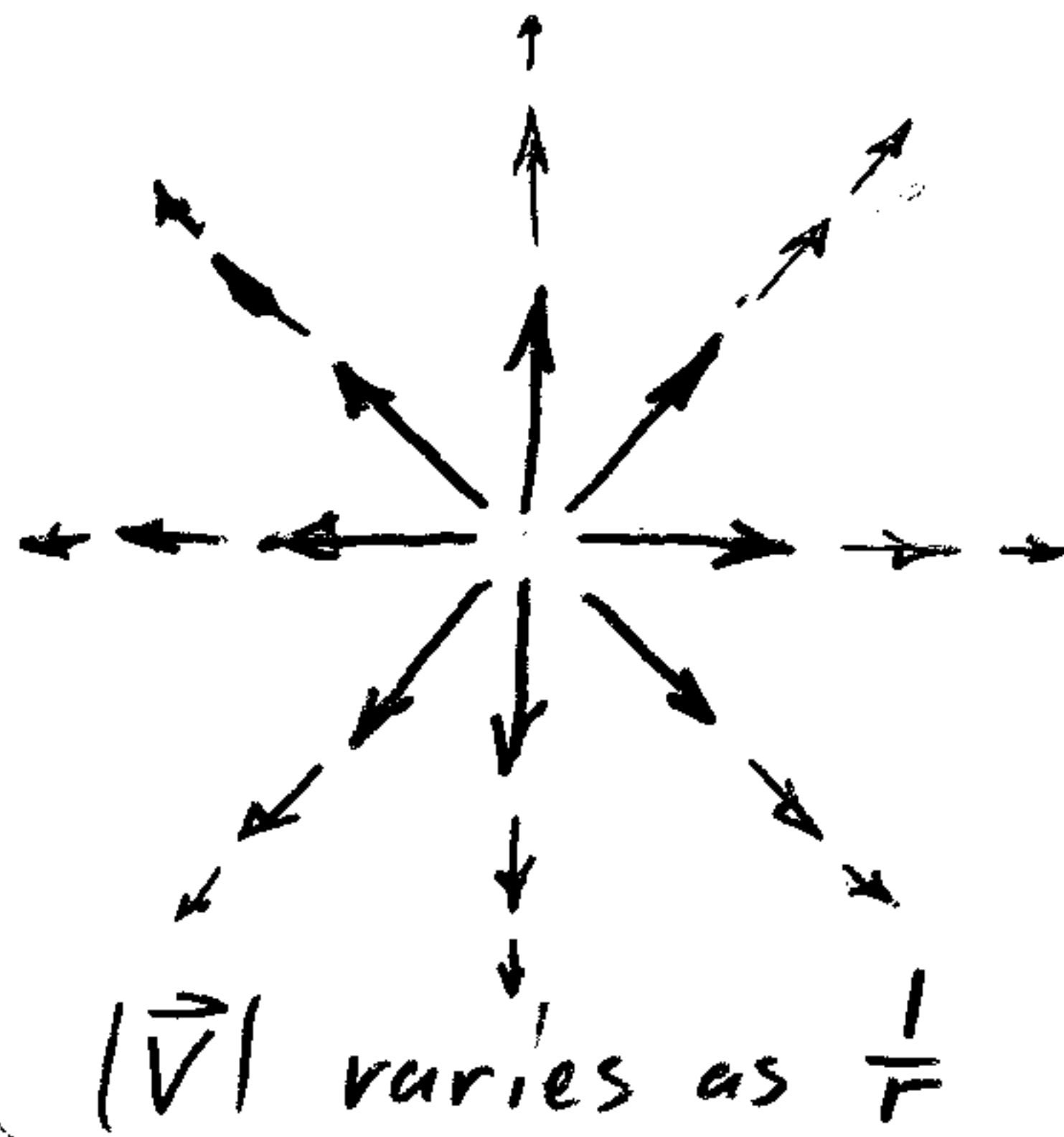


a)  $\psi = \arctan\left(\frac{y}{x}\right)$

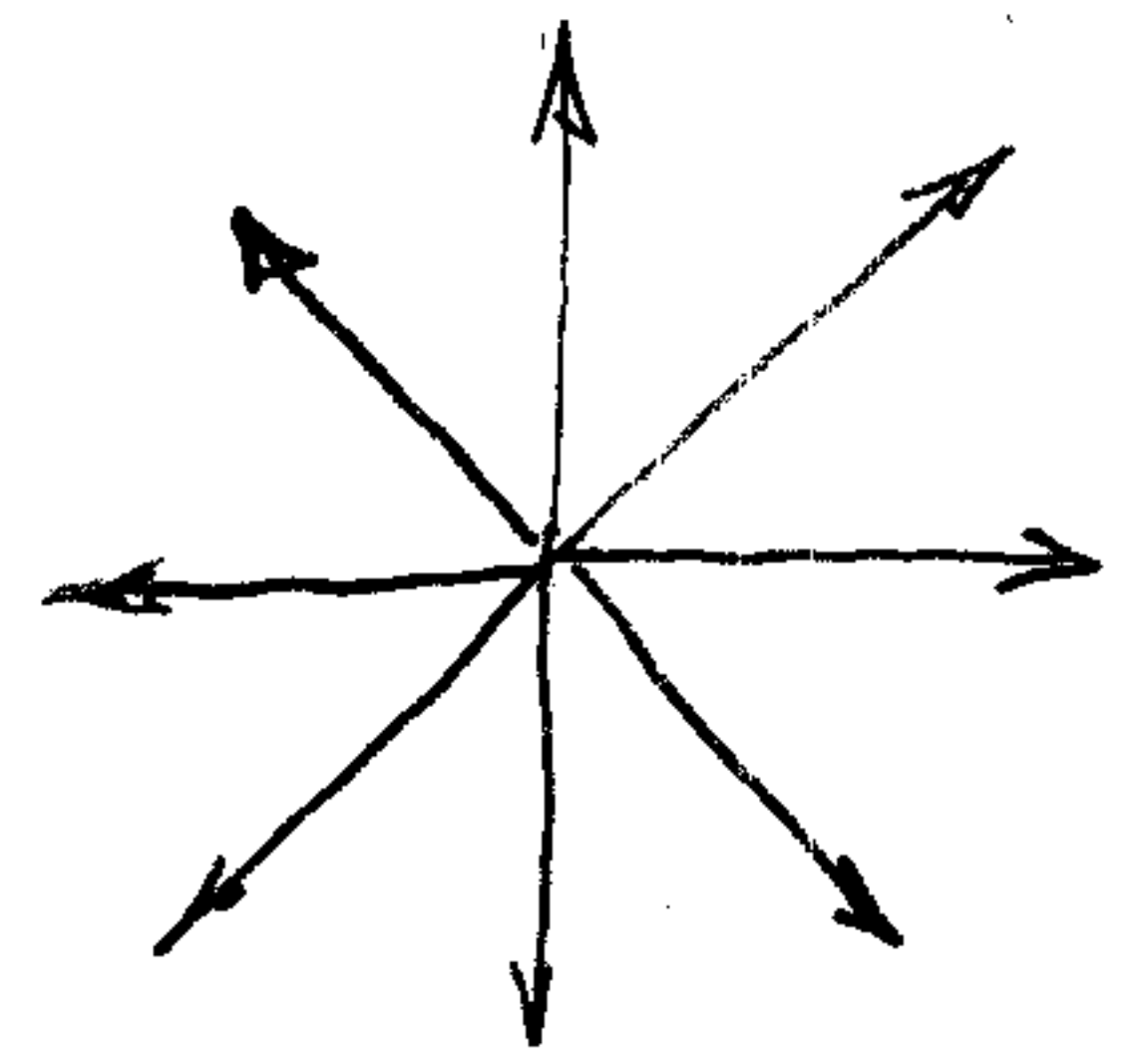
$u = \frac{\partial \psi}{\partial y} = \frac{x}{x^2+y^2}$

$v = -\frac{\partial \psi}{\partial x} = \frac{y}{x^2+y^2}$

Streamlines:  $\arctan\left(\frac{y}{x}\right) = \theta = \text{const}$



$|\vec{V}|$  varies as  $\frac{1}{r}$



$\phi = x^2 + y^2$

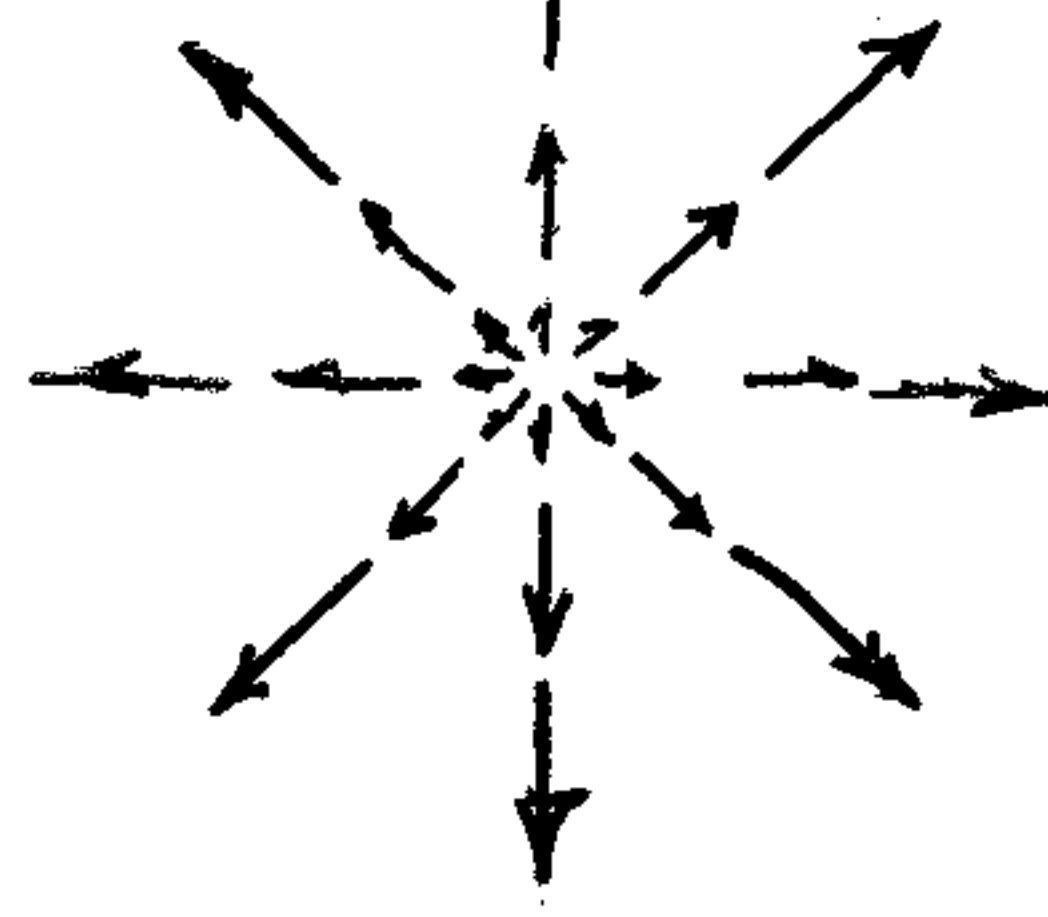
$u = \frac{\partial \phi}{\partial x} = 2x$

$v = \frac{\partial \phi}{\partial y} = 2y$

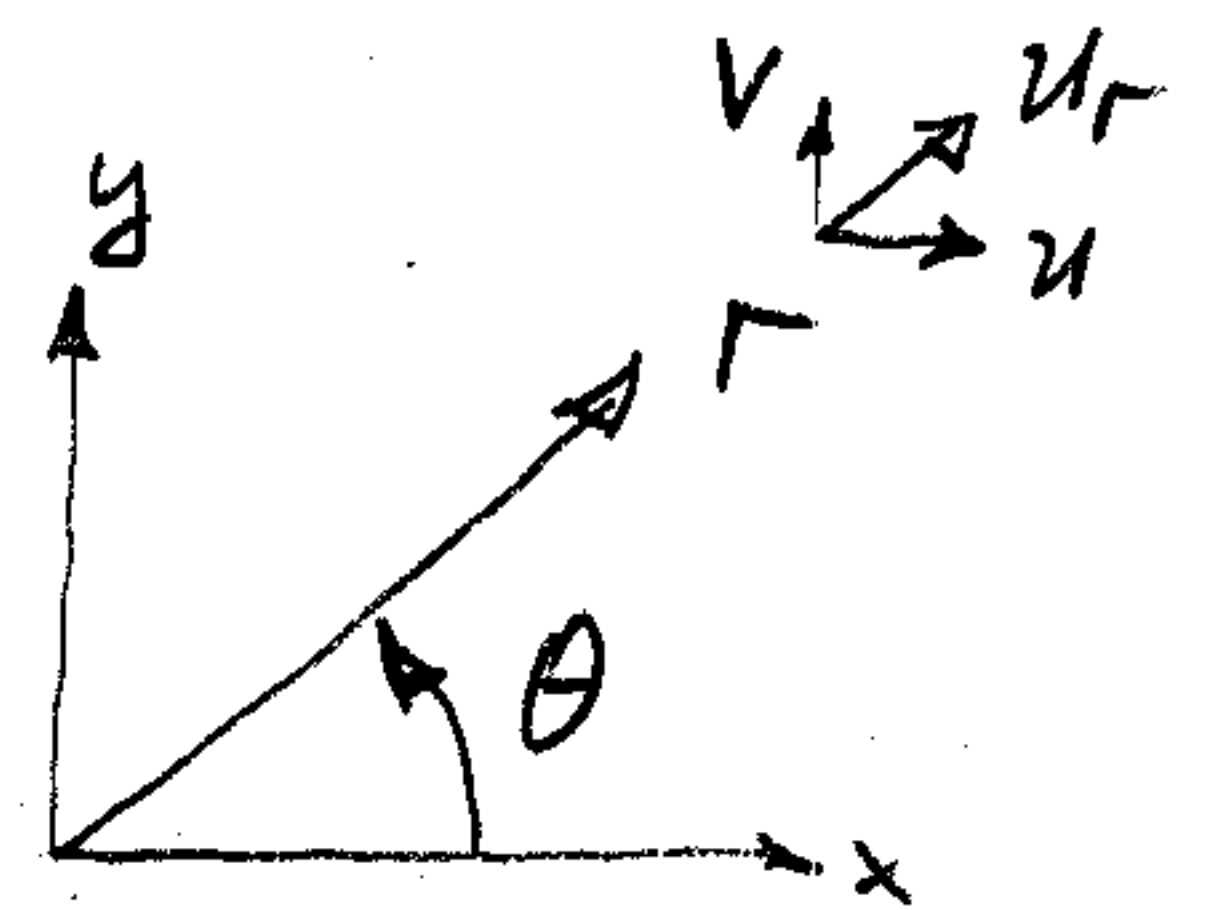
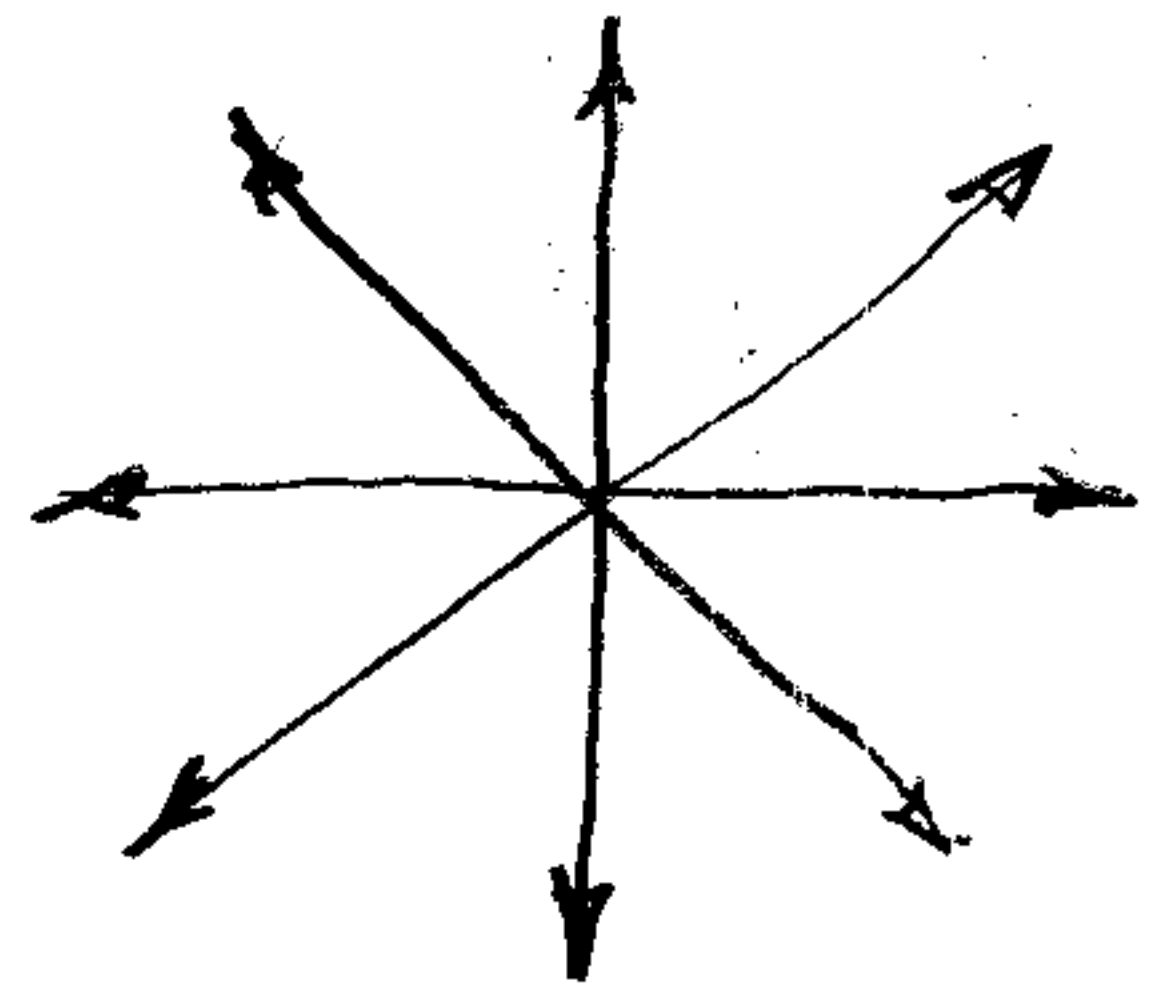
Streamlines:  $\frac{dy}{dx} = \frac{v}{u} = \frac{y}{x}$

$\frac{dy}{y} = \frac{dx}{x}$

$y = Cx$



$|\vec{V}|$  varies as  $r$

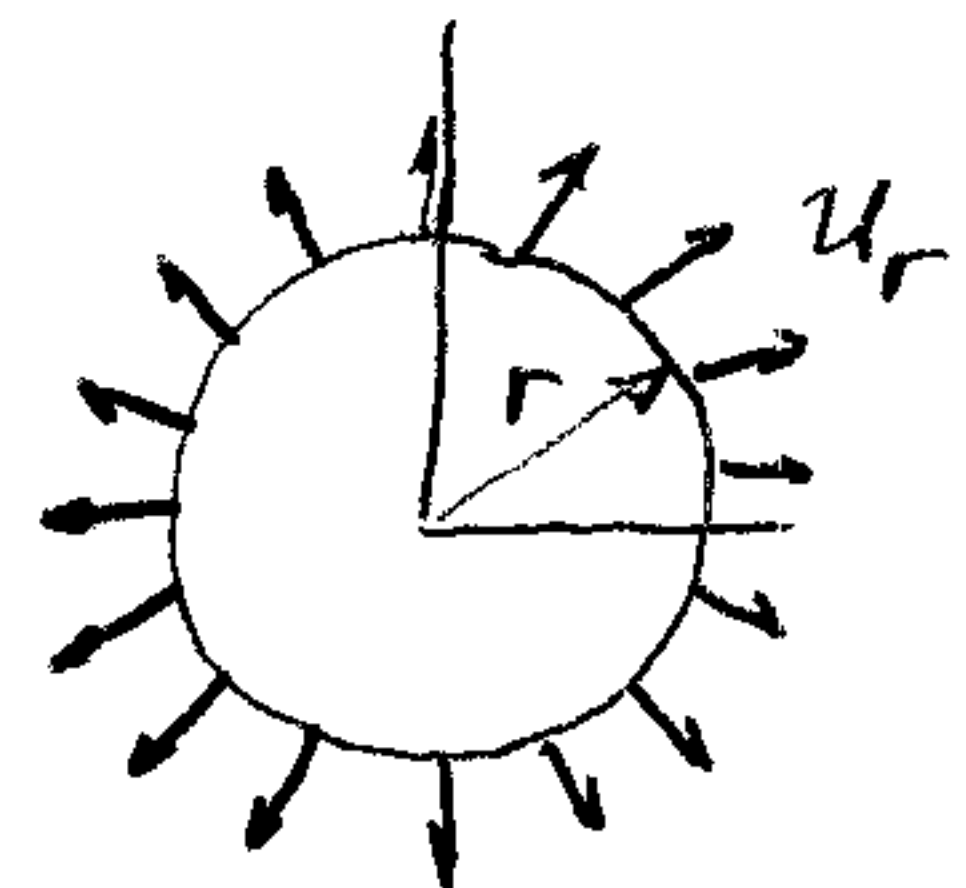


b) For  $\psi = \arctan\frac{y}{x}$ :  $u = \frac{x}{r^2} = \frac{\cos\theta}{r}$

$v = \frac{y}{r^2} = \frac{\sin\theta}{r}$

Radial velocity:  $u_r = u \cos\theta + v \sin\theta = \frac{1}{r}$

Volume flow rate:  $\dot{V} = 2\pi r u_r = 2\pi$  (constant)



For  $\phi = x^2 + y^2$ :  $u = 2x = 2r \cos\theta$

$v = 2y = 2r \sin\theta$

Radial velocity:  $u_r = u \cos\theta + v \sin\theta = 2r$

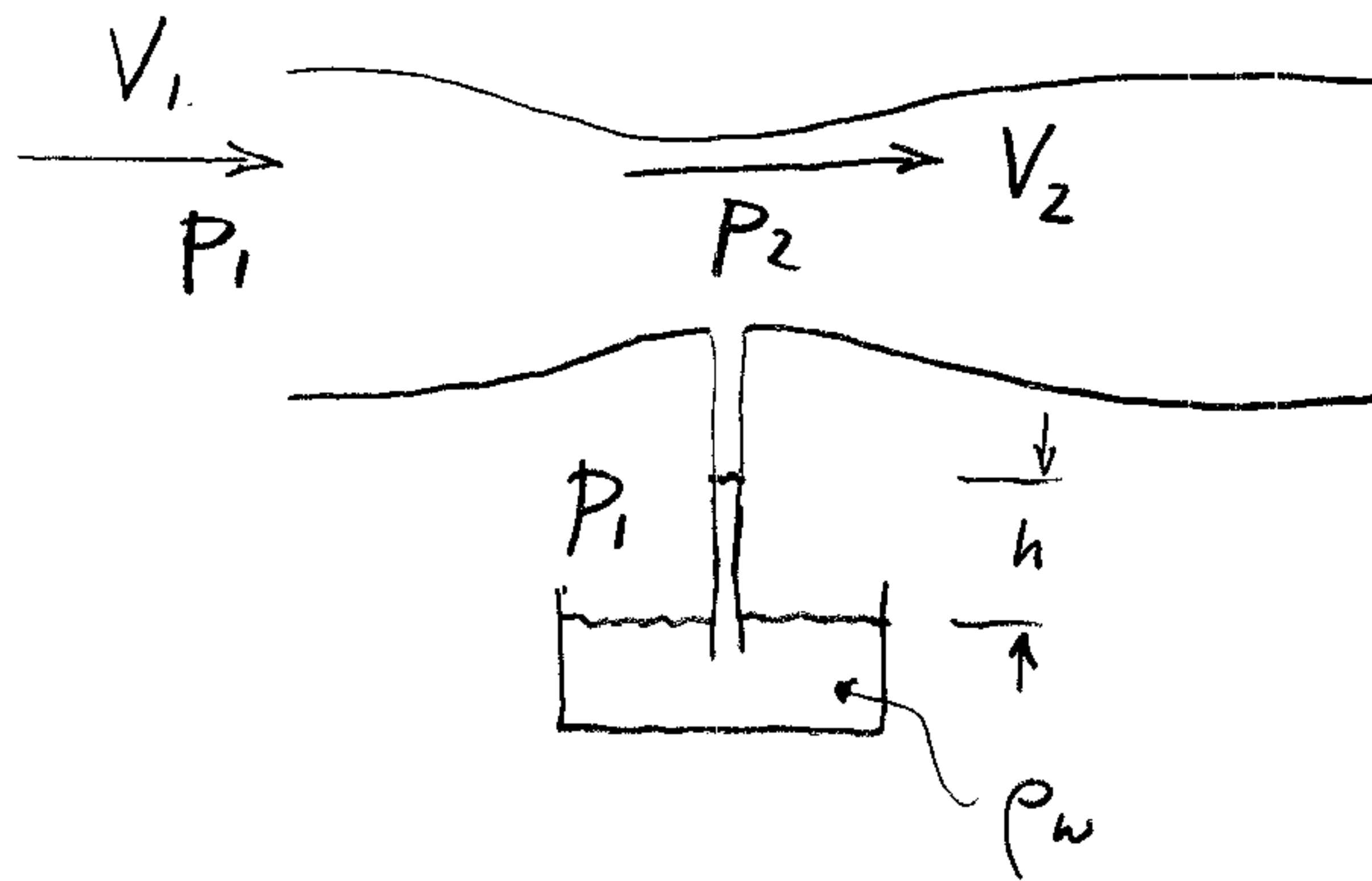
Volume flow rate:  $\dot{V} = 2\pi r u_r = 4\pi r^2$  (increases as  $r^2$ )

c)  $\phi = x^2 + y^2$  is not feasible to set up, since  $\nabla \cdot \vec{V} \neq 0$  for this flow, so it doesn't obey mass conservation in a low speed flow situation. Lack of mass conservation is further evidenced by  $\dot{V}$  increasing with  $r$ . Mass is being created "out of thin air" (pun intended)

To get required column height,  
we must have

$$P_1 - P_2 = \rho_w g h$$

$$\left. \begin{aligned} \rho_w &= 1000 \text{ kg/m}^3 \\ g &= 9.8 \text{ m/s}^2 \\ h &= 0.1 \text{ m} \end{aligned} \right\} \rightarrow P_1 - P_2 = 980 \text{ Pa}$$



Using Bernoulli:  $P_1 + \frac{1}{2} \rho V_1^2 = P_0 = P_2 + \frac{1}{2} \rho V_2^2$

$$\frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2 = P_1 - P_2 = 980 \text{ Pa}$$

Using Continuity:  $V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{1}{0.7}$

$$\frac{1}{2} \rho V_1^2 \left[ \frac{1}{0.7^2} - 1 \right] = P_1 - P_2$$

$$V_1 = \left( \frac{2(P_1 - P_2)}{\rho \left[ \frac{1}{0.7^2} - 1 \right]} \right)^{1/2} = \frac{2 \cdot 980 \text{ Pa}}{1.226 \text{ kg/m}^3 \left[ \frac{1}{0.7^2} - 1 \right]}$$

$$V_1 = 39 \text{ m/s} = 87.5 \text{ mph}$$

13-782 500 SHEETS, FILLER, 5 SQUARE  
 42-391 50 SHEETS EYE EASE, 5 SQUARE  
 42-392 100 SHEETS EYE EASE, 5 SQUARE  
 42-393 200 SHEETS EYE EASE, 5 SQUARE  
 42-394 100 RECYCLED WHITE, 5 SQUARE  
 42-395 200 RECYCLED WHITE, 5 SQUARE  
 Made in U.S.A.



M13

Need  $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n = 0$

$f_n = 0$

a) ①  $\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = 0$

Since  $\frac{\partial \sigma_{11}}{\partial x_1} = 0$ ,  $\sigma_{21} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = 0$

②  $\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} = 0$

Since  $\sigma_{12} = \sigma_{22} = \sigma_{32} = 0$  no additional information

③  $\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3}$

Since  $\sigma_{23} = \sigma_{33} = 0 \Rightarrow \frac{\partial \sigma_{13}}{\partial x_1} = 0$

$\frac{\partial \sigma_{31}}{\partial x_3} = \frac{\partial \sigma_{13}}{\partial x_1} = 0$

+ Since  $\sigma_{13} = 0 @ \pm h$

$\therefore \sigma_{13} = 0$  everywhere in  $x_3$

$\sigma_{13} = \text{constant in } x_1 \Leftarrow$

from ①

$$6) \quad \sigma_{11} = C \left( \frac{M}{I} \right) x_3 x_1$$

$$\frac{\partial \sigma_{11}}{\partial x_1} = \left( \frac{CM}{I} \right) x_3$$

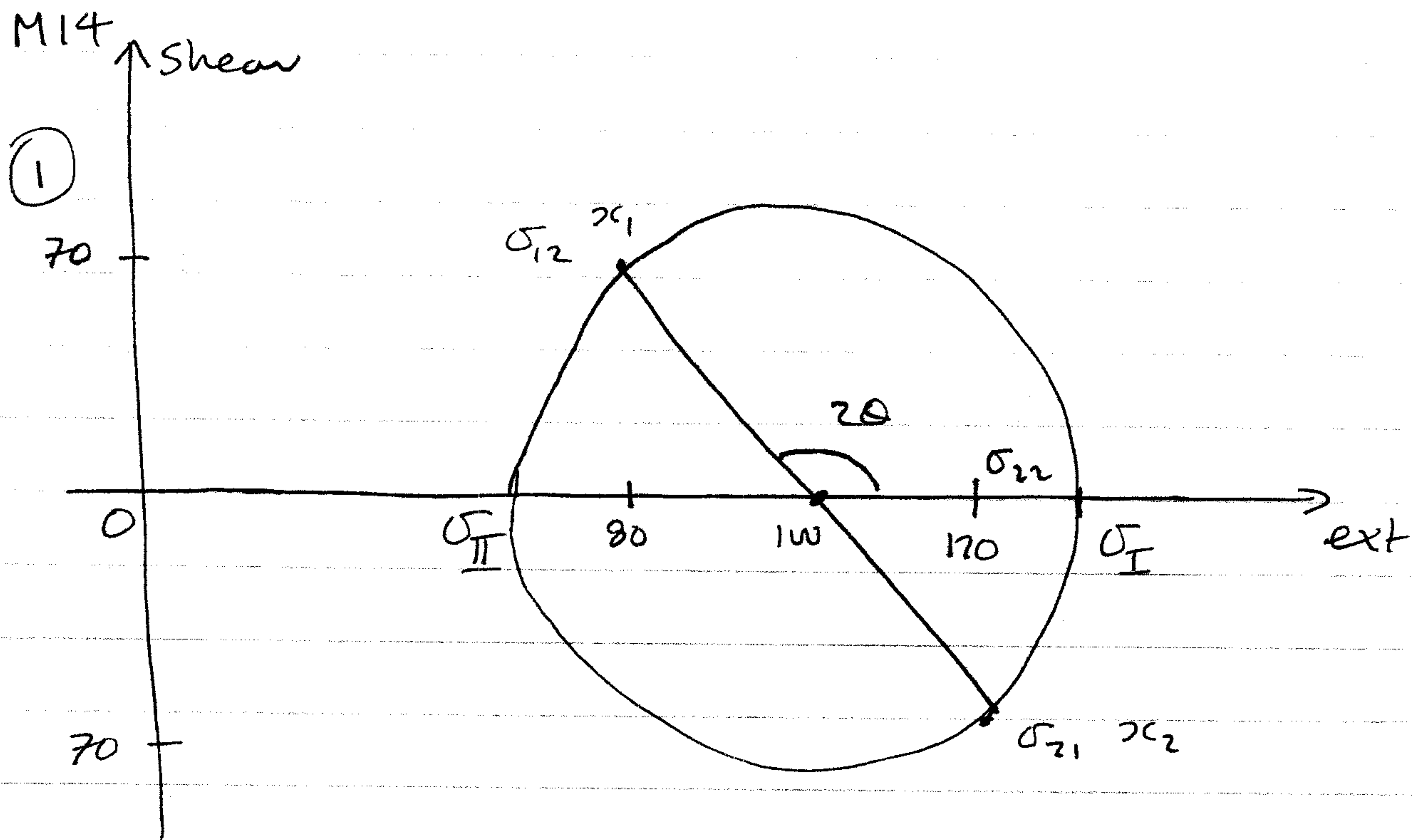
$$\Rightarrow \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{31}}{\partial x_3} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = - \left( \frac{CM}{I} \right) x_3$$

$$\sigma_{31} = - \left( \frac{CM}{I} \right) \frac{x_3^2}{2} + D$$

but  $\sigma_{31} = 0$  for  $\pm h$ .

$$\therefore D = \frac{CM}{I} \frac{h^2}{2} \Rightarrow \sigma_{31} = \frac{CM}{2I} (h^2 - x_3^2) \Leftarrow$$

from ③  $\frac{\partial \sigma_{13}}{\partial x_1} = 0$  i.e. Shear stress constant along length.



$$\text{Radius} = \sqrt{20^2 + 70^2} = 72.8 \text{ MPa} = \text{Max Shear.}$$

2) Max

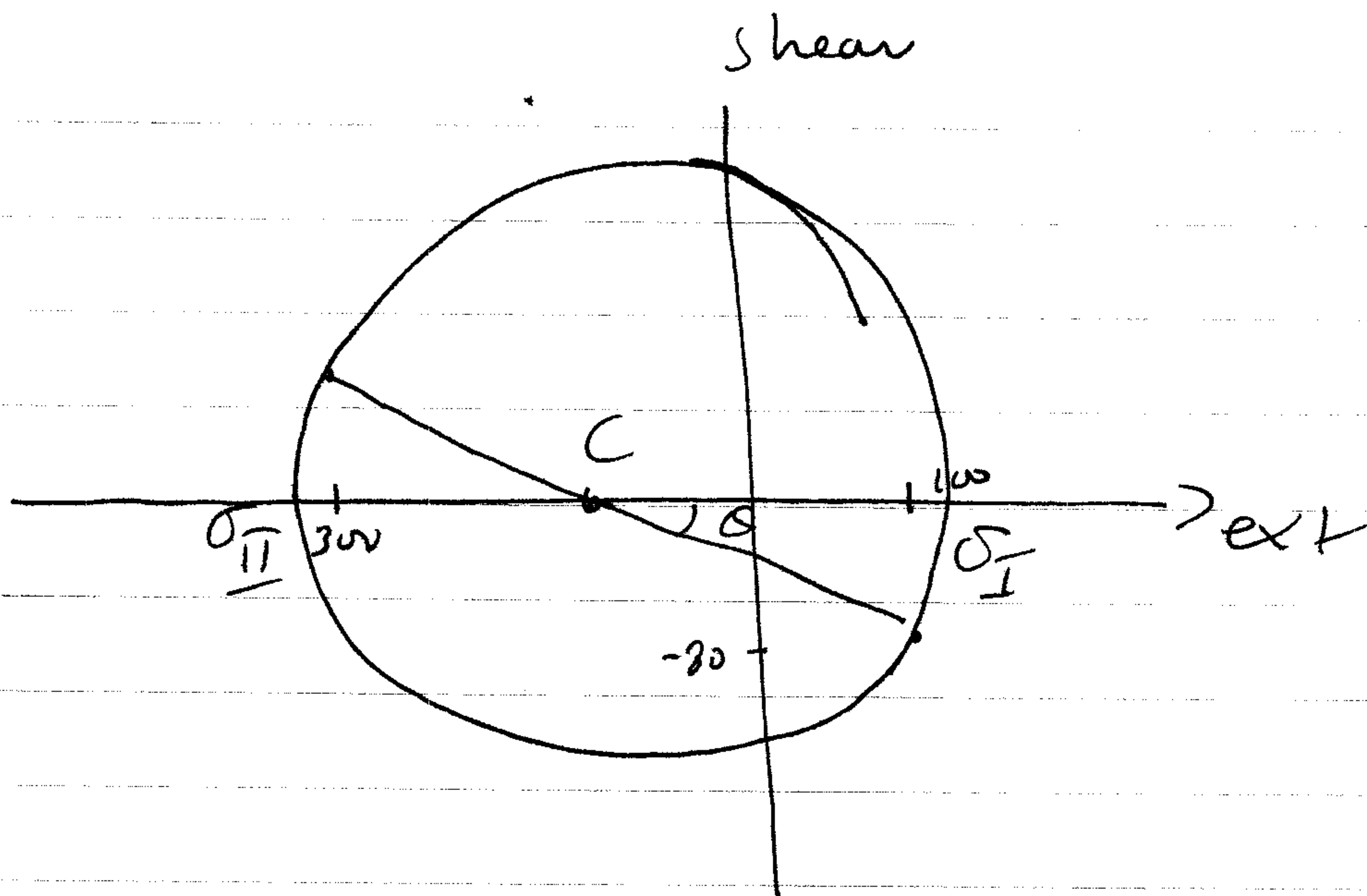
$$\text{Max Principal Stress} = 100 + 72.8 = 172.8 \text{ MPa} \leftarrow$$

$$\text{Min Principal Stress} = 100 - 72.8 = 27.2 \text{ MPa} \leftarrow$$

③ From  $x_1$   $d_{-}^n$

$$2\theta = \frac{1}{2} \left( 180 - \tan^{-1} \left( \frac{70}{20} \right) \right) = 53^\circ \text{ clockwise from } x_1$$

b)  
1)



2) Center @  $\frac{100 + (-300)}{2} = -100 \text{ MPa}$

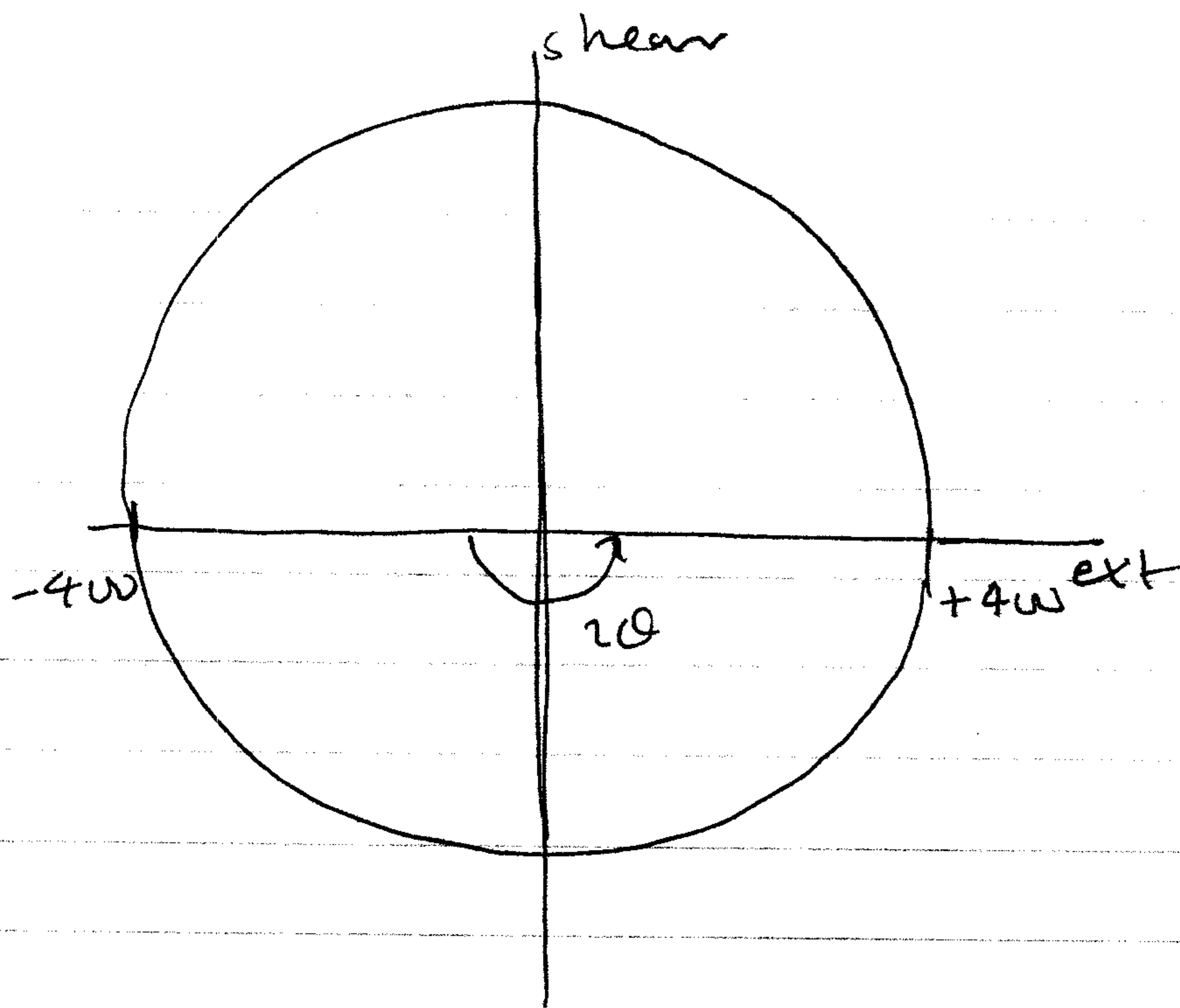
Radius =  $\sqrt{200^2 + 80^2} = 215.4 \text{ MPa} = \text{Max Shear}$

$$\sigma_I = -100 + 215.4 = 115.4 \text{ MPa} \leftarrow$$

$$\sigma_{II} = -100 - 215.4 = -315.4 \text{ MPa} \leftarrow$$

3)  $\tan \theta = \frac{1}{2} \tan^{-1} \left( \frac{80}{200} \right) = 10.9^\circ$  counterclockwise

c)



center at origin

Max shear = 400 MPa

$$\sigma_I = 400 \text{ MPa}$$

$$\sigma_{II} = -400 \text{ MPa}$$

Max  $\sigma_I$  in  $x_2$  direction  $\therefore 90^\circ$  CCW from  $x_1$ .

M15) i) Consider element of initial sides 1 unit in length

deformed element has sides  $1 + \epsilon_1, 1 + \epsilon_2, 1 + \epsilon_3$

$$\therefore \text{Volume (deformed)} = (1 + \epsilon_1)(1 + \epsilon_2)(1 + \epsilon_3)$$

neglecting high order terms ( $\epsilon_1 \epsilon_2, \epsilon_1 \epsilon_3, \epsilon_2 \epsilon_3, \epsilon_1 \epsilon_2 \epsilon_3$ )

$$\text{deformed volume} = 1 + \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$\therefore \frac{V_{\text{def}} - V_{\text{undef}}}{V_{\text{undef}}} = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad \Leftarrow$$

ii)



$$\text{ii a) } \varepsilon_{11} = \frac{\partial U_1}{\partial x_1} = (x_1 + 0.5x_2) \times 10^{-3}$$

$$\varepsilon_{22} = \frac{\partial U_2}{\partial x_2} = (0.5x_2 - x_1) \times 10^{-3}$$

$$\varepsilon_{33} = \frac{\partial U_3}{\partial x_3} = 0$$

$$\begin{aligned} \varepsilon_{12} &= \frac{1}{2} \left( \frac{\partial U_1}{\partial x_2} + \frac{\partial U_2}{\partial x_1} \right) = \frac{1}{2} (-x_2 + 0.5x_1 + 0.5x_1 - x_2) \\ &= \frac{1}{2} (x_1 - 2x_2) \times 10^{-3} \end{aligned}$$

$$\varepsilon_{23} = \frac{1}{2} \left( \frac{\partial U_2}{\partial x_3} + \frac{\partial U_3}{\partial x_2} \right) = 0$$

$$\varepsilon_{13} = \frac{1}{2} \left( \frac{\partial U_3}{\partial x_1} + \frac{\partial U_1}{\partial x_3} \right) = 0$$

6) Rigid body rotation

$$\Theta = \frac{1}{2} \left( \frac{\partial U_1}{\partial x_2} - \frac{\partial U_2}{\partial x_1} \right)$$

$$= \frac{1}{2} \left( -x_2 + 0.5x_1 - 0.5x_1 - x_2 \right) \times 10^{-3}$$

$$= \frac{1}{2} (-2x_2) = -x_2 \times 10^{-3}$$

c) at  $x_1 = 5\text{mm}$ ,  $x_2 = 7\text{mm}$ .

$$\epsilon_{11} = \left( \frac{5+7}{2} \right) \times 10^{-3} = 8.5 \times 10^{-3}$$

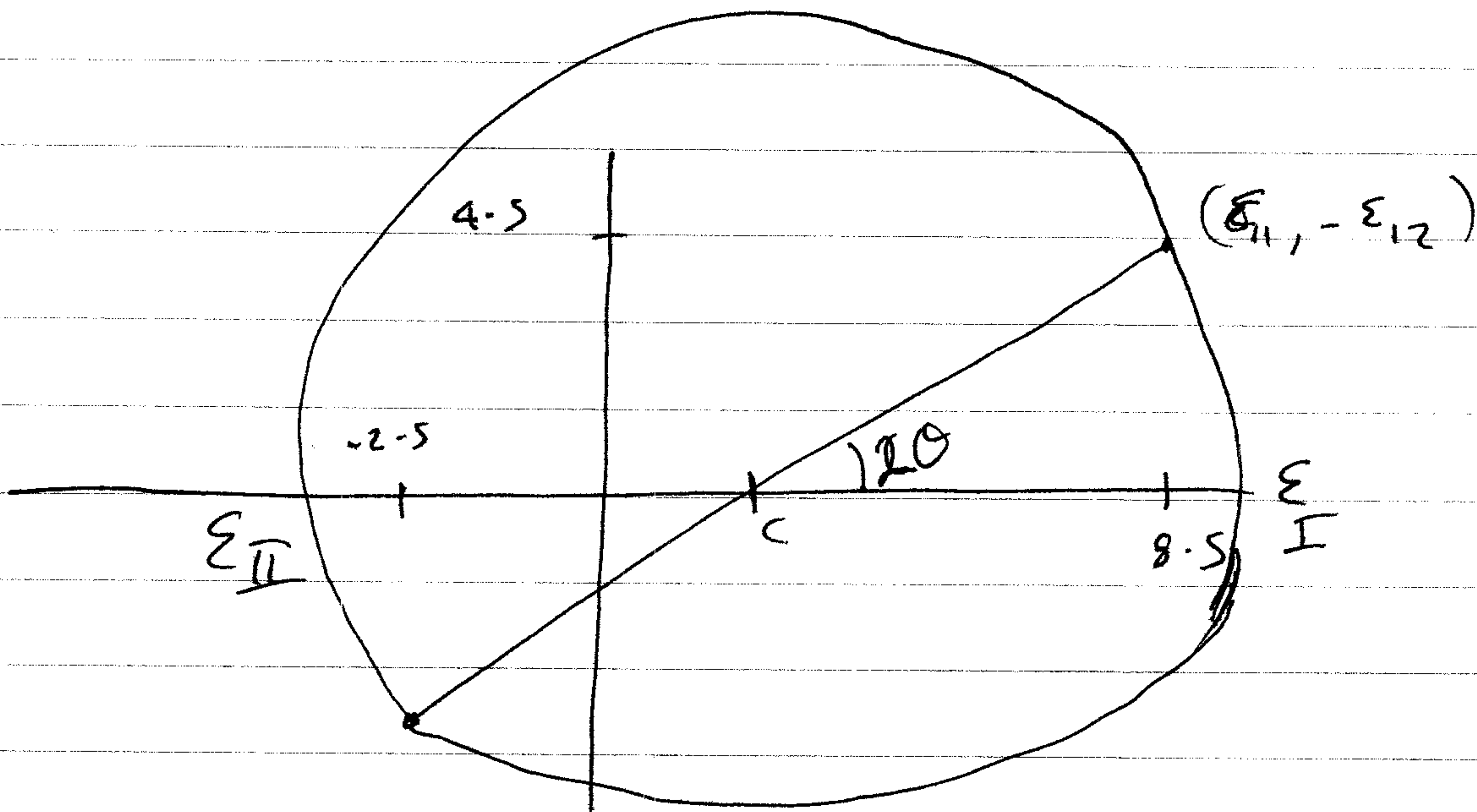
$$\epsilon_{22} = \left( \frac{7-5}{2} \right) \times 10^{-3} = -2.5 \times 10^{-3}$$

$$\epsilon_{12} = \frac{1}{2} (5-14) \times 10^{-3} = -4.5 \times 10^{-3}$$

$$\epsilon_{33} = 0$$

d) Volumetric strain =  $(8.5 + (-2.5) + 0) \times 10^{-3} = 1.5 \times 10^{-3}$

c)



Center @  $\frac{1}{2} [8.5 + (-2.5)] \times 10^{-3} = +3 \times 10^{-3}$

Radius =  $10^3 \sqrt{(4.5)^2 + (5.5)^2} = 7.11 \times 10^{-3}$

$\epsilon_I = 3 \times 10^{-3} + 7.11 \times 10^{-3} = 10.11 \times 10^{-3}$

$\epsilon_{II} = 3 \times 10^{-3} - 7.11 \times 10^{-3} = -4.11 \times 10^{-3}$

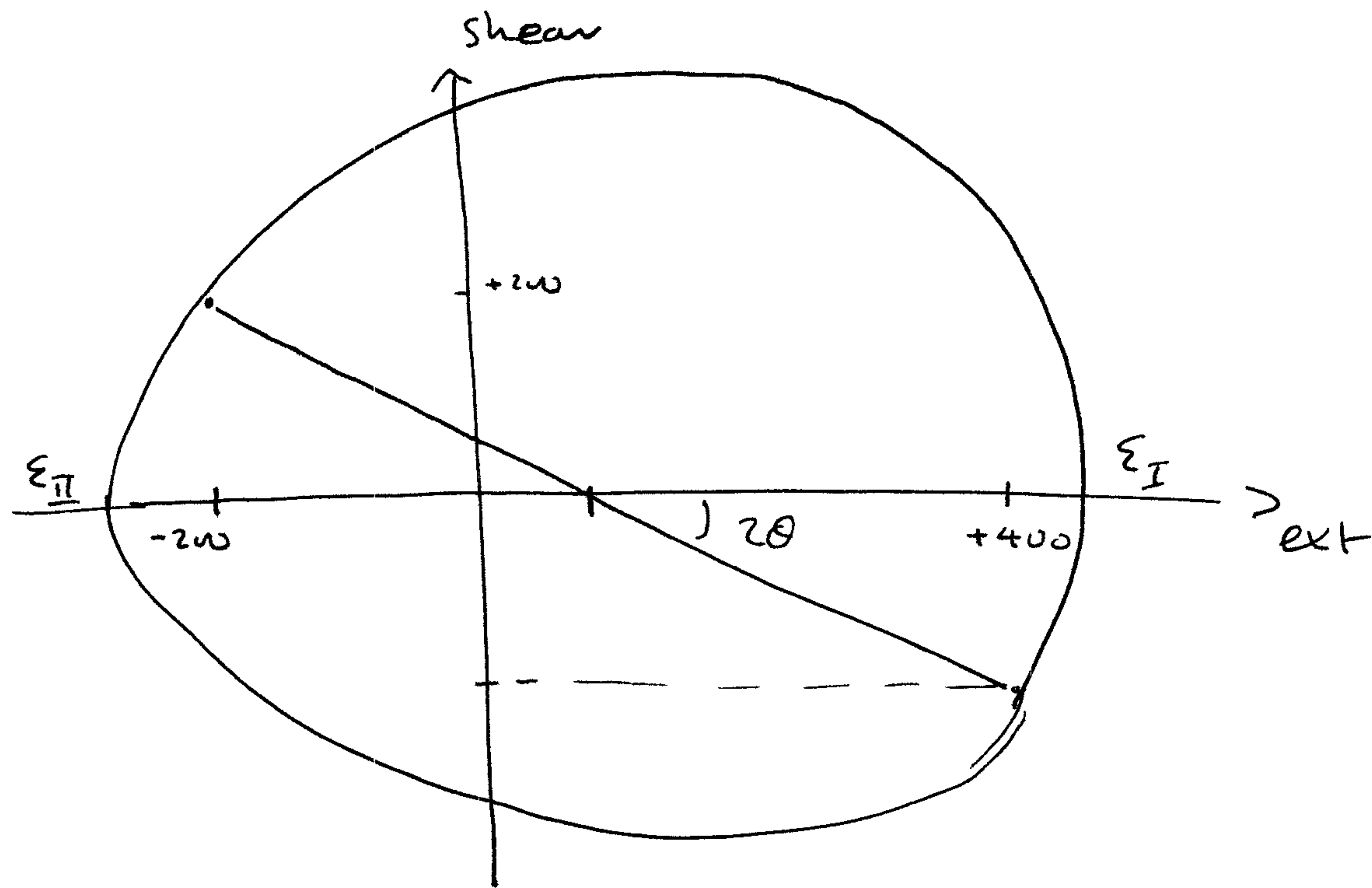
$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{4.5}{5.5} \right) = 19.6^\circ \text{ clockwise from } x,$$

d) Volumetric strain =

$$10.11 + (-4.11) + 0 = 6 \times 10^{-3} \text{ } \equiv$$

M16

a)



center @  $+100 \mu \epsilon$ .

$$\text{Radius} = \sqrt{300^2 + 200^2} = 360 \mu \epsilon.$$

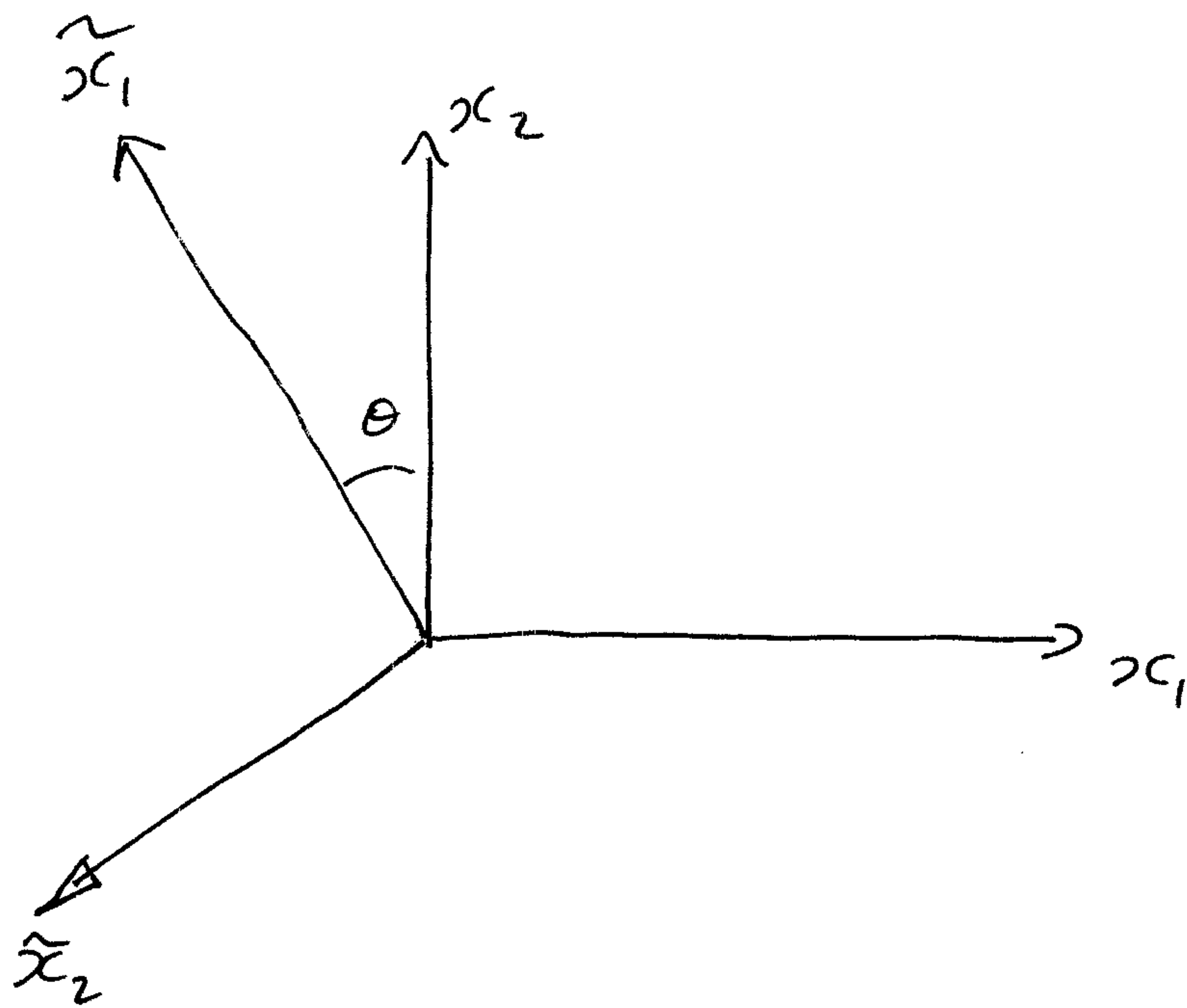
$$\epsilon_I = 100 + 360 = 460 \mu \epsilon$$

$$\epsilon_{II} = 100 - 360 = -260 \mu \epsilon.$$

$$2\theta = \frac{1}{2} \tan^{-1} \left( \frac{200}{300} \right) = 16.85^\circ$$

$$\therefore \hat{x}_1 = 90 + \theta = 106.85^\circ \text{ CCW from } x_1$$

$$\hat{x}_2 = \theta = 16.85^\circ \text{ CCW from } x_1, \quad (108.85^\circ \text{ CCW from } x_2)$$



$$l_{11}^{\sim} = \cos(106.8) = -0.290 \quad \Leftarrow$$

$$l_{12}^{\sim} = \cos(16.85) = 0.957 \quad \Leftarrow$$

$$l_{22}^{\sim} = \cos(106.8) = -0.290 \quad \Leftarrow$$

$$l_{21}^{\sim} = \cos(180+16.85) = -0.957 \quad \Leftarrow$$

$$l_{33}^{\sim} = 1, \quad l_{13}^{\sim} = l_{23}^{\sim} = l_{31}^{\sim} = l_{32}^{\sim} = 0. \quad \Leftarrow$$

$$d) \quad \tilde{\Sigma}_{mn} = l_{\tilde{m}p} l_{\tilde{n}q} \Sigma_{pq}$$

$$\begin{aligned} \tilde{\Sigma}_{11} &= l_{\tilde{1}1} l_{\tilde{1}1} \Sigma_{11} + l_{\tilde{1}1} l_{\tilde{1}2} \Sigma_{12} + l_{\tilde{1}2} l_{\tilde{1}2} \Sigma_{21} \\ &\quad + l_{\tilde{1}2} l_{\tilde{1}2} \Sigma_{22} + 0 + 0 + 0 \\ &\quad + (0.957)^2 (4\omega) \\ &= 460.5 \mu \Sigma \end{aligned}$$

$$\begin{aligned} \tilde{\Sigma}_{22} &= l_{\tilde{2}1} l_{\tilde{2}1} \Sigma_{11} + l_{\tilde{2}1} l_{\tilde{2}2} \Sigma_{12} + l_{\tilde{2}2} l_{\tilde{2}1} \Sigma_{21} + l_{\tilde{2}2} l_{\tilde{2}2} \Sigma_{22} \\ &\quad + (-0.957)^2 (-2\omega) + (-0.957)(-0.290)(-2\omega) + (-0.290)(-0.957)(-2\omega) + (-0.290)^2 (+4\omega) \\ &= -261 \mu \Sigma \end{aligned}$$

$$\begin{aligned} \tilde{\Sigma}_{12} &= l_{\tilde{1}1} l_{\tilde{2}1} \Sigma_{11} + l_{\tilde{1}2} l_{\tilde{2}1} \Sigma_{21} + l_{\tilde{1}1} l_{\tilde{2}2} \Sigma_{12} + l_{\tilde{1}2} l_{\tilde{2}2} \Sigma_{22} + 0 \\ &= (-0.290)(-0.957)(-2\omega) + (0.957)(-0.957)(-2\omega) + (-0.290)(-0.290)(-2\omega) + \\ &\quad + (0.957)(-0.290)(4\omega) \\ &= -0.2 \approx 0 \end{aligned}$$

$$\tilde{\Sigma}_{11} = +460.5, \quad \tilde{\Sigma}_{22} = -261, \quad \tilde{\Sigma}_{12} = 0$$

$\therefore$  agrees with Mohr's circle.