

**Massachusetts Institute of Technology**  
**Department of Aeronautics and**  
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**Cambridge, MA 02139**

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**16.01/16.02 Unified Engineering I, II**  
**Fall 2003**

**Problem Set 14**

Name: \_\_\_\_\_

Due Date: 12/9/03

	<b>Time Spent (min)</b>
<b>F18</b>	
<b>F19</b>	
<b>F20</b>	
<b>M19</b>	
<b>M20</b>	
<b>M21</b>	
<b>M22</b>	
<b>U4</b>	
<b>Study Time</b>	

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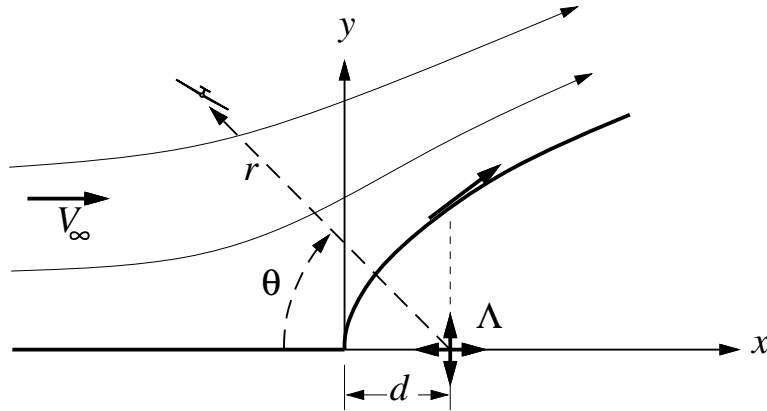
Announcements:

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F18. Wind with velocity  $V_\infty$  is flowing over a mountain ridge have the shape  $Y(x) = \sqrt{Cx}$ . The flow is to be modeled by superimposing a uniform flow with a source located at some location  $x, y = (d, 0)$ .

$$\phi(x, y) = V_\infty x + \frac{\Lambda}{4\pi} \ln [(x - d)^2 + y^2]$$



a) Determine both the source's location  $d$ , and the strength  $\Lambda$ , with the conditions:

$$\begin{aligned} u &= 0 && \text{at } x, y = (0, 0) \\ v/u &= dY/dx && \text{at } x, y = (d, \sqrt{Cd}) \end{aligned}$$

The second condition simply requires that the flow direction on the ridge surface directly above the source is parallel to the ridge surface.

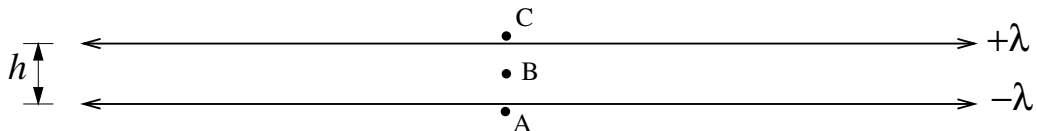
b) A sailplane flying in the slope lift upwind of the ridge requires a vertical velocity of at least  $v \geq 1\text{m/s}$  to stay aloft. For a wind speed of  $V_\infty = 15\text{m/s}$  (33 mph) and ridge size scale  $C = 500\text{m}$ , determine the maximum flyable radius  $r(\theta)$  inside which the sailplane can sustain flight. Plot the  $r(\theta)$  boundary superimposed on a plot of the ridge.

F19. Determine the  $C_p$  variation along the  $-x$  and  $+y$  directions for

- a) Freestream  $V_\infty$  with source of strength  $\Lambda$
- b) Freestream  $V_\infty$  with vortex of strength  $\Gamma$
- c) Freestream  $V_\infty$  with doublet of strength  $\kappa$ .

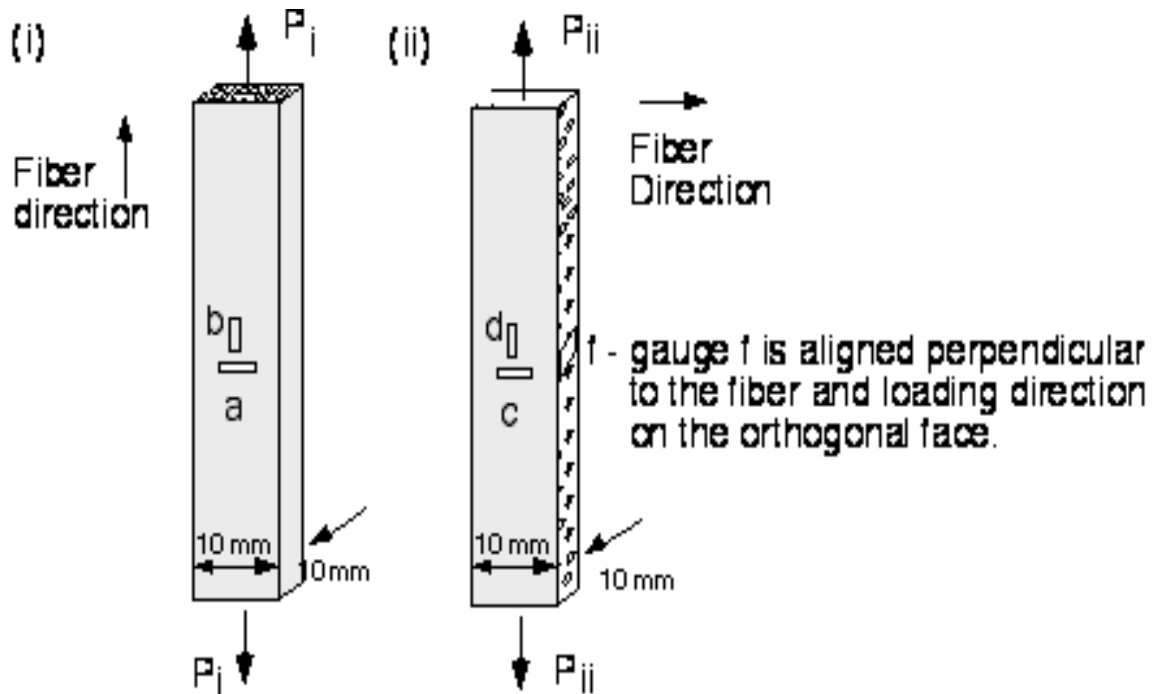
At large distances, where does a lifting airfoil cause the strongest disturbance to the flow (as measured by  $C_p$ ), above and below, or fore and aft? Explain.

F20. Two very long source panels of strength  $\pm\lambda$  are located at distance  $h$  apart. Determine the velocities at points A, B, C.

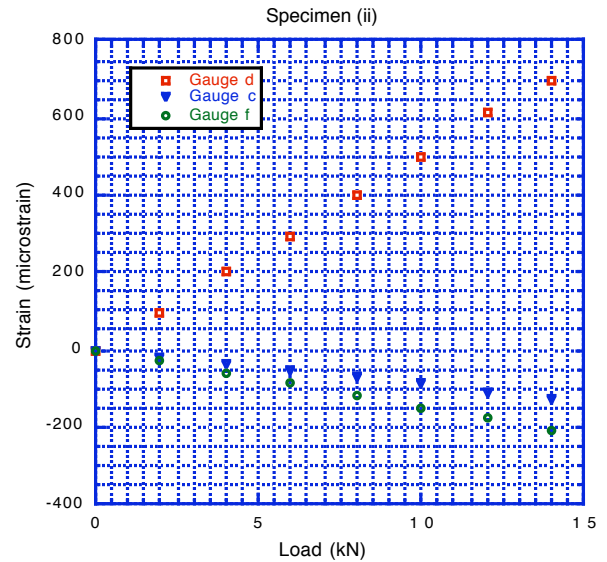
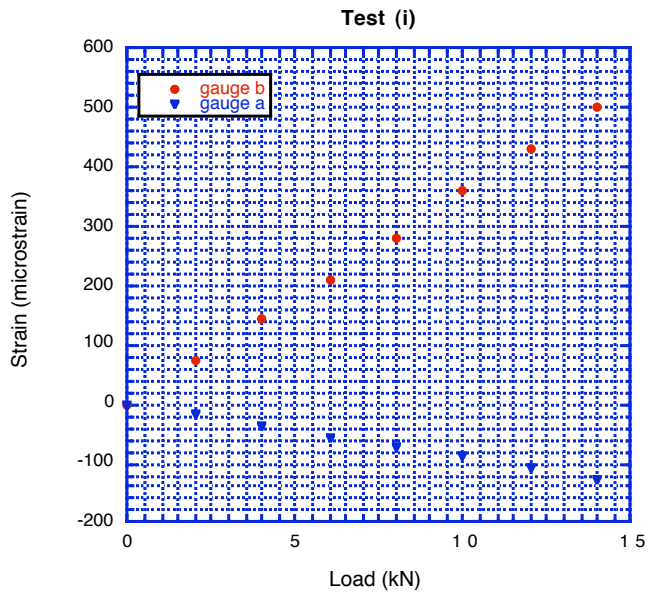


**Problem M19**

Silicon carbide fiber reinforced titanium matrix composites are being considered for use in the compressor stages of next generation gas turbine engines. It is required to obtain the elastic properties of a batch of material under development. To do this, two test specimens are machined from the batch, each 200 mm long and 10 mm by 10 mm in width and thickness. Two strain gauges are attached to specimens (i) in which the load is applied parallel to the fiber direction and three are applied to specimen (ii) which is loaded perpendicular to the fiber direction as shown below. A uniaxial load,  $P$  is applied through a mechanical testing machine, which is varied from 0 to 14 kN.



- a) What is the fewest number of independent engineering elastic constants that you would expect to be required to describe the elastic constitutive behavior of this material?. State any assumptions that you are making in giving this number.



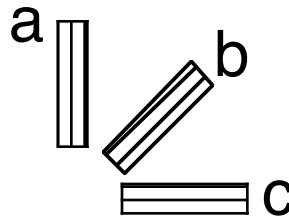
- b) The graphs above show the strain readings plotted vs. applied load for the two tests. Using the information contained in the graphs calculate **all** the engineering elastic constants that you can obtain. If there are any elastic constants that you cannot obtain/estimate from this data suggest additional tests that you might perform in order to obtain them. In describing the elastic constants use x, y and z to indicate the orthogonal material directions, with x aligned parallel to the fibers.
- c) Estimate the fiber volume fraction for this composite, given that the Young's modulus of SiC is 450 GPa, and the Poisson's ratio is 0.16 and the corresponding values for the titanium alloy used for a matrix are 110 GPa and 0.34 respectively. Is this consistent with the data provided?

**Problem M20**

A 45° strain gauge rosette attached to the surface of an aluminum alloy wing skin panel measures the following strains:

$$\epsilon_a = -0.0025, \quad \epsilon_b = 0.0020, \quad \epsilon_c = -0.0040$$

The orientation of the gauges is shown below



a) What are the extensional stresses in the a, b and c directions?

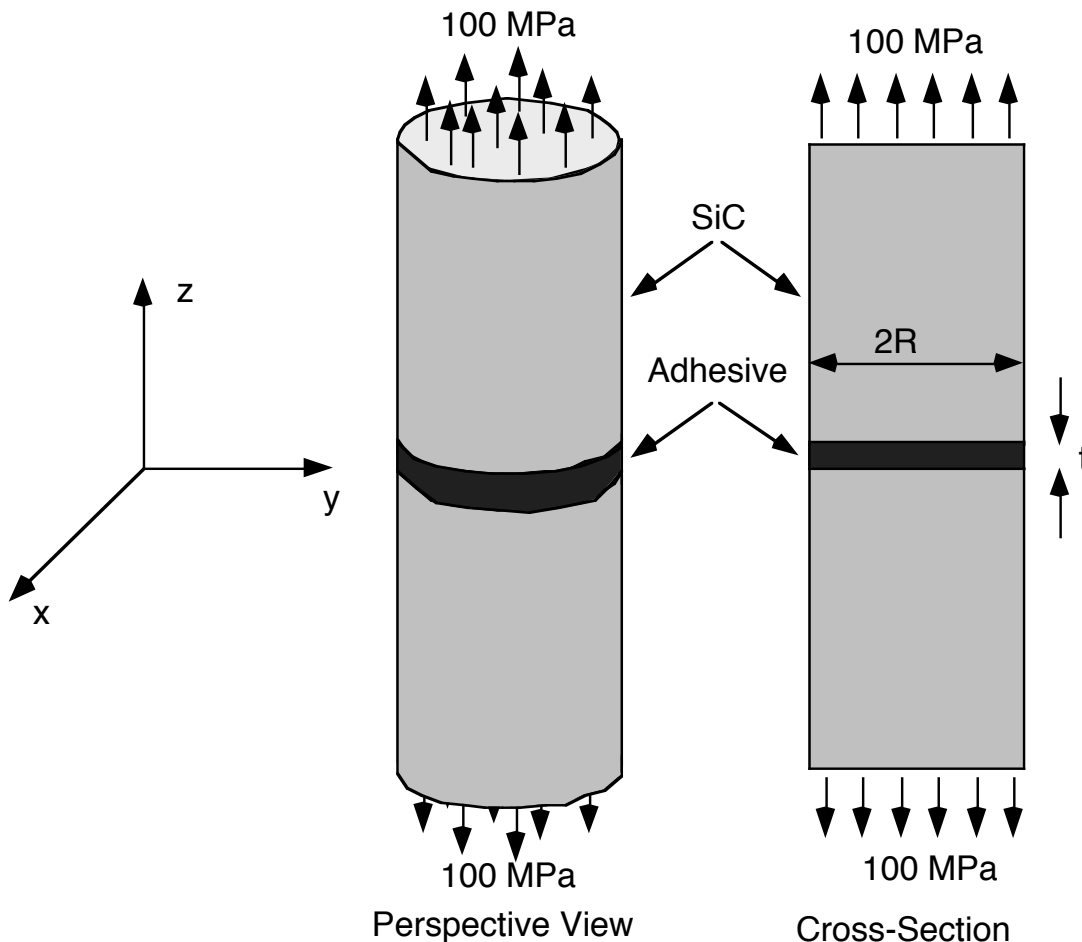
b) What are the in-plane principal stresses?

The Young's modulus of the aluminum alloy is 70 GPa, the Poisson's ratio is 0.33 (note. the a, b, and c directions are numbered in the counterclockwise direction)

### Problem M21

a) A tensile specimen is made of an epoxy (a thermosetting polymer). It is loaded in the  $z$  direction to a stress of 100 MPa. Assuming that the epoxy remains elastic, what are the resulting strains in the  $x$ ,  $y$  and  $z$  directions? The Young's modulus of the epoxy is 3 GPa and the Poisson's ratio is 0.3

b) A thin ( $t \ll R$ ) layer of an epoxy adhesive is used to join two silicon carbide bars of circular cross-section, as shown below. An axial ( $z$  direction) tensile stress of 100 MPa is applied to the joint as shown. Calculate the axial strain ( $\epsilon_{zz}$ ) in the adhesive. The Young's modulus of the epoxy adhesive is 2 GPa, the Poisson's ratio is 0.3. The Young's modulus of the SiC is 450 GPa and the Poisson's ratio is 0.16. State any assumptions/approximations that you make in solving this problem - note that you may not be able to achieve an exact solution but you should aim to obtain a good approximation. Hint: think about the transverse strains in the adhesive ( $\epsilon_{xx}$  and  $\epsilon_{yy}$ ).



## Problem M22

A cantilever beam is to be made with a length  $L$  and circular cross section, radius  $R$  (to be specified by the designer), to support an applied moment,  $M$ .

- a) What combination of material properties must be minimized to obtain the highest bending stiffness (i.e. minimum deflection for a given applied moment) beam for a given mass of material? (show your working). The deflection,  $\delta$  of a cantilever beam of this shape is given by:

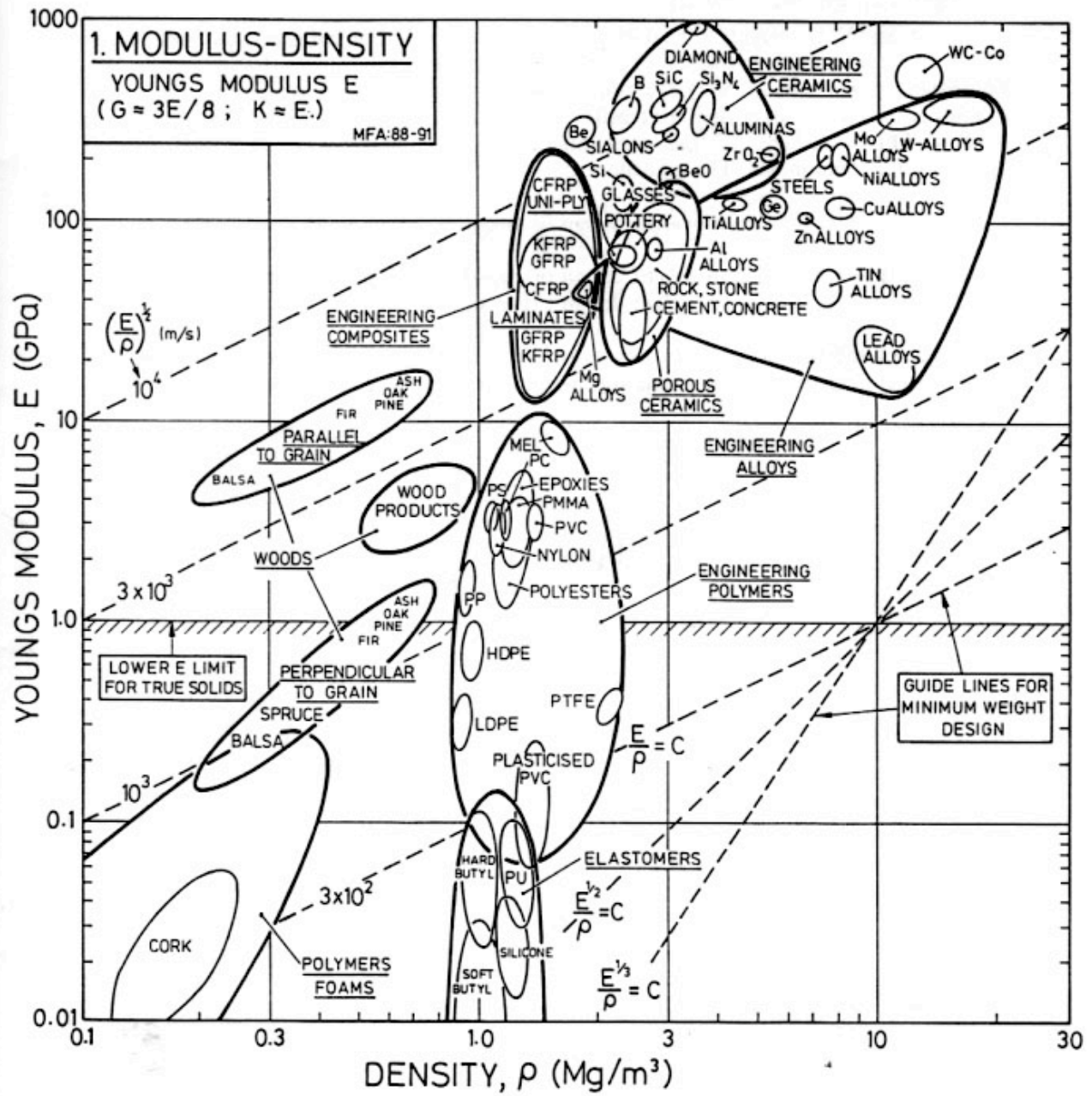
$$\delta = \frac{2ML^2}{\rho R^4 E}$$

where  $E$  is the Young's modulus.

- b) Choose between the following material choices to select the most promising material to construct a wing for a small Unmanned Aerial Vehicle (which will carry load as a cantilever beam)

Material	Density, $\rho$ (Mg/m <sup>3</sup> )	Modulus, $E$ , (GPa)	Poisson's Ratio	Yield Stress, $\sigma_y$ , (MPa)	Price , $p$ , (\$/Mg)
Mild Steel	7.9	203	0.31	220	375
Aluminum alloy (2024)	2.8	71	0.33	350	1650
Titanium alloy Ti-6Al4V	4.5	120	0.34	850	30000
Carbon fiber composite	1.5	230	0.3	1050	70000
Polyethelene (High Density)	0.96	1.1	0.39	30	1000
Wood (pine)	0.6	12	0.35	300	300
Silicon Carbide (SiC)	3.0	410	0.17	300 (Flexural)	30000

- c) To a first approximation the human hip joint carries forces like a cantilever beam. In hip replacement surgery it is important to use material with the same total mass and bending stiffness as the bone that is removed. Bone has a modulus of 18 GPa and a density of 1.55 Mg/m<sup>3</sup>. By reference to the attached materials selection chart suggest two materials that could be used for hip replacements. Include the chart, and your working with your answers.



From: Material Selection in Mechanical Design, M.F Ashby, Pergamon Press, Oxford, 1992



## **Unified Engineering Problem U4**

Please fill out the course evaluation survey available online at <http://paperairplane.mit.edu>. You will receive a 10 for problem U4 if the survey is filled out by Wednesday, 12/10 (the last day of classes).