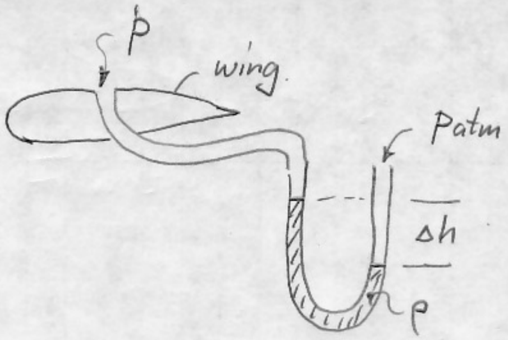
Problem F1	Solution	Fall '03

Part A. – Anderson Problem 1.11



U-tube manometer.

Given: $p_{atm} = 1.01 \times 10^5 \ N/m^2$

$\rho = 1.36 \times 10^4 \ kg/m^3$

$g = 9.81 \ m/s^2$

$\Delta h = 20 \text{ cm} = 0.2 m$

$\rightarrow \ p = p_{atm} - \rho g \Delta h = 7.43 \times 10^{-4} \ N/m^2$

Part B.

Measured weight = gravity force – buoyancy force

$$F = mg - \rho_{fluid} \cdot \mathcal{V}$$

volume of Al

Given: $m = 1 \ kg$, so $mg = 9.81 \ N$ same for all cases.

Also, $m = \rho_{Al} \cdot \mathcal{V}$, so $\mathcal{V} = m/\rho_{Al} = 1 \ kg / 2700 \ kg/m^3$

$\rightarrow \mathcal{V} = 3.70 \times 10^{-4} \ m^3$

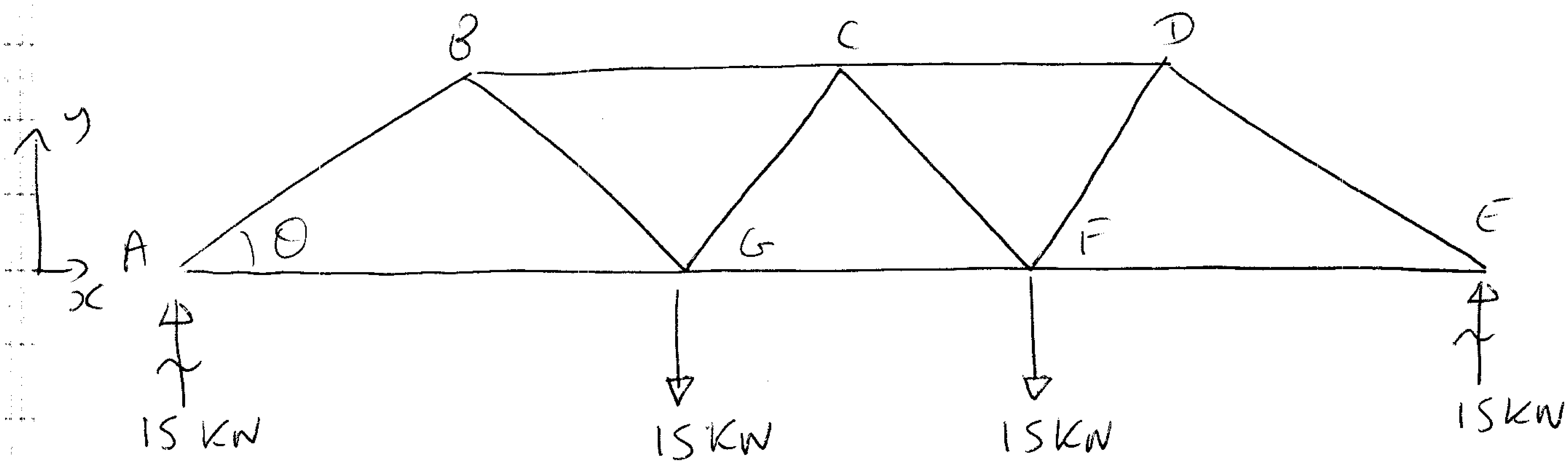
Vacuum: $\rho_{fluid} = 0 \rightarrow F = 9.81 \ N$

Air: $\rho_{fluid} = 1.226 \ kg/m^3 \rightarrow F = 9.8096 \ N$

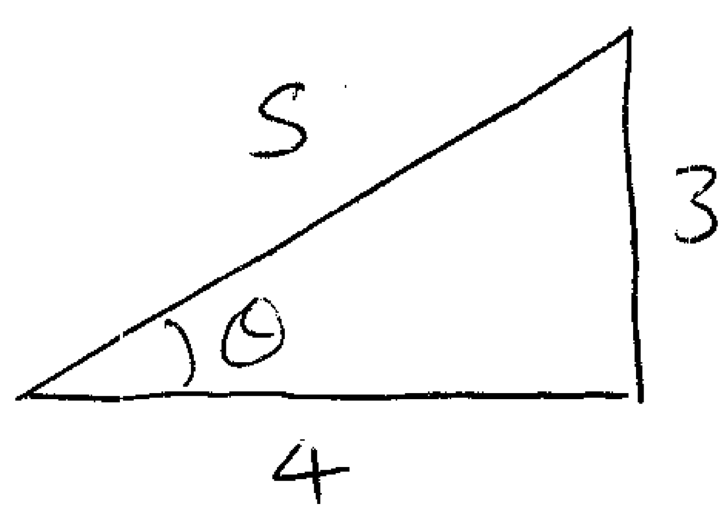
Water: $\rho_{fluid} = 1000 \ kg/m^3 \rightarrow F = 9.44 \ N$

Solutions M5.

From M4 FBD with reactions



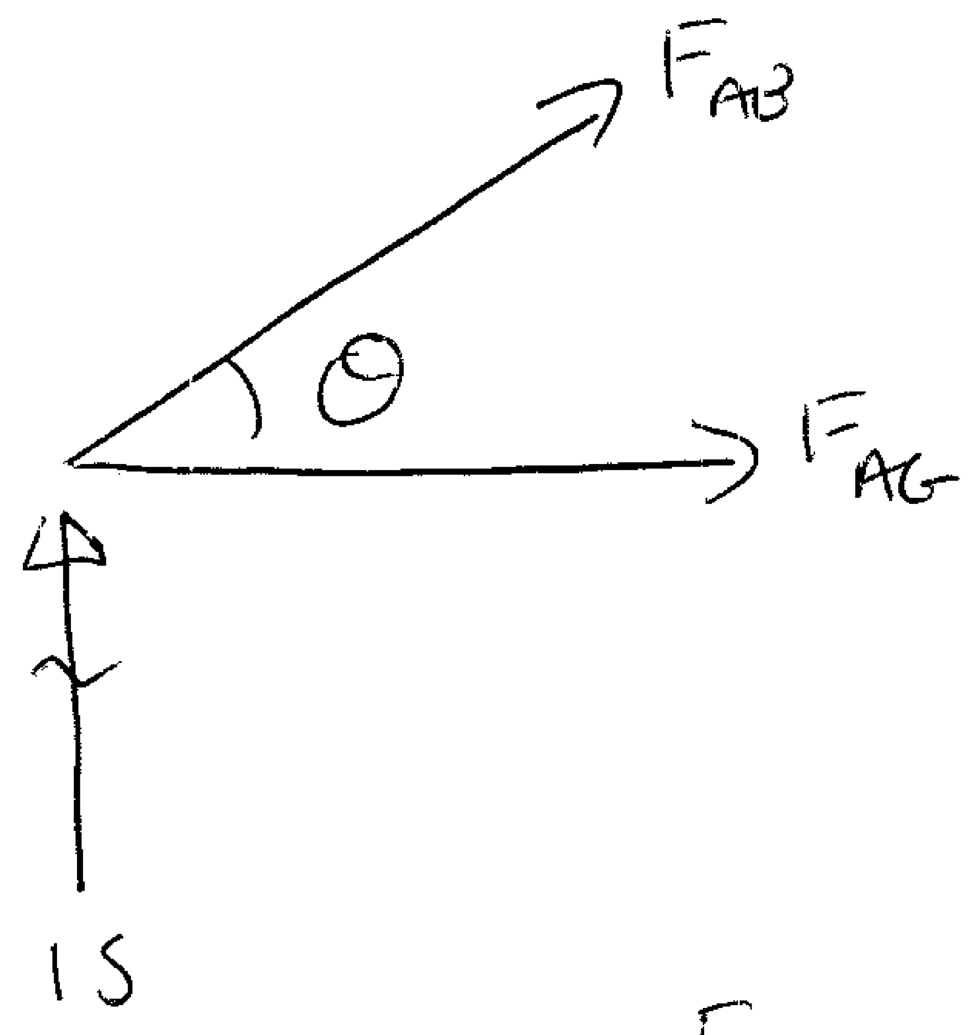
Note



$$\cos\theta = \frac{4}{5} \qquad \sin\theta = \frac{3}{5}$$

Use method of joints

Joint A



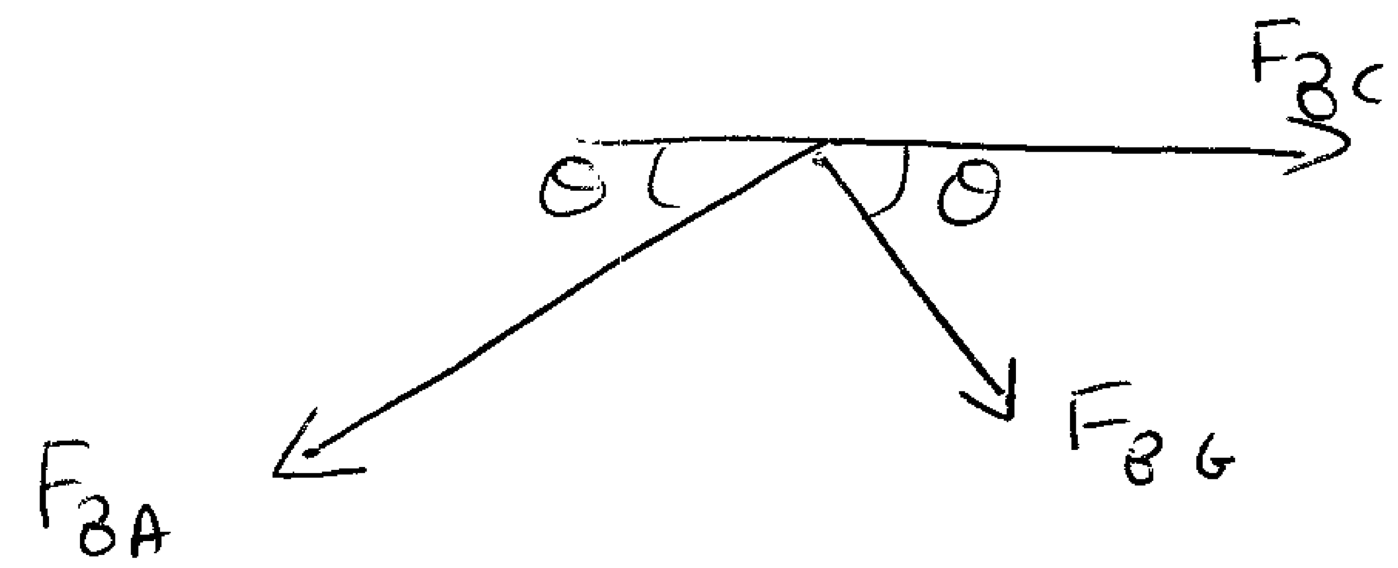
$$\sum \vec{F_x} = 0: \quad F_{AG} + F_{AB}\cos\theta = 0 \quad (1)$$

$$\sum F_y \uparrow = 0: \quad F_{AB}\sin\theta + 15 = 0 \quad (2)$$

$$F_{AB} = -\frac{15}{\sin\theta} = -15 \cdot \frac{5}{3} = \underline{\underline{-25\,\text{kN}}}$$

$$F_{AG} = -F_{AB}\cos\theta = +25 \cdot \frac{4}{5} = +20\,\text{kN}$$

Joint B



$$\Sigma \vec{F_x} = 0 \qquad F_{BC} + F_{BG} \cos\theta - F_{BA} \cos\theta = 0 \qquad (3)$$

$$\Sigma F_y \uparrow = 0 \qquad -F_{BA} \sin\theta - F_{BG} \sin\theta = 0 \qquad (4)$$

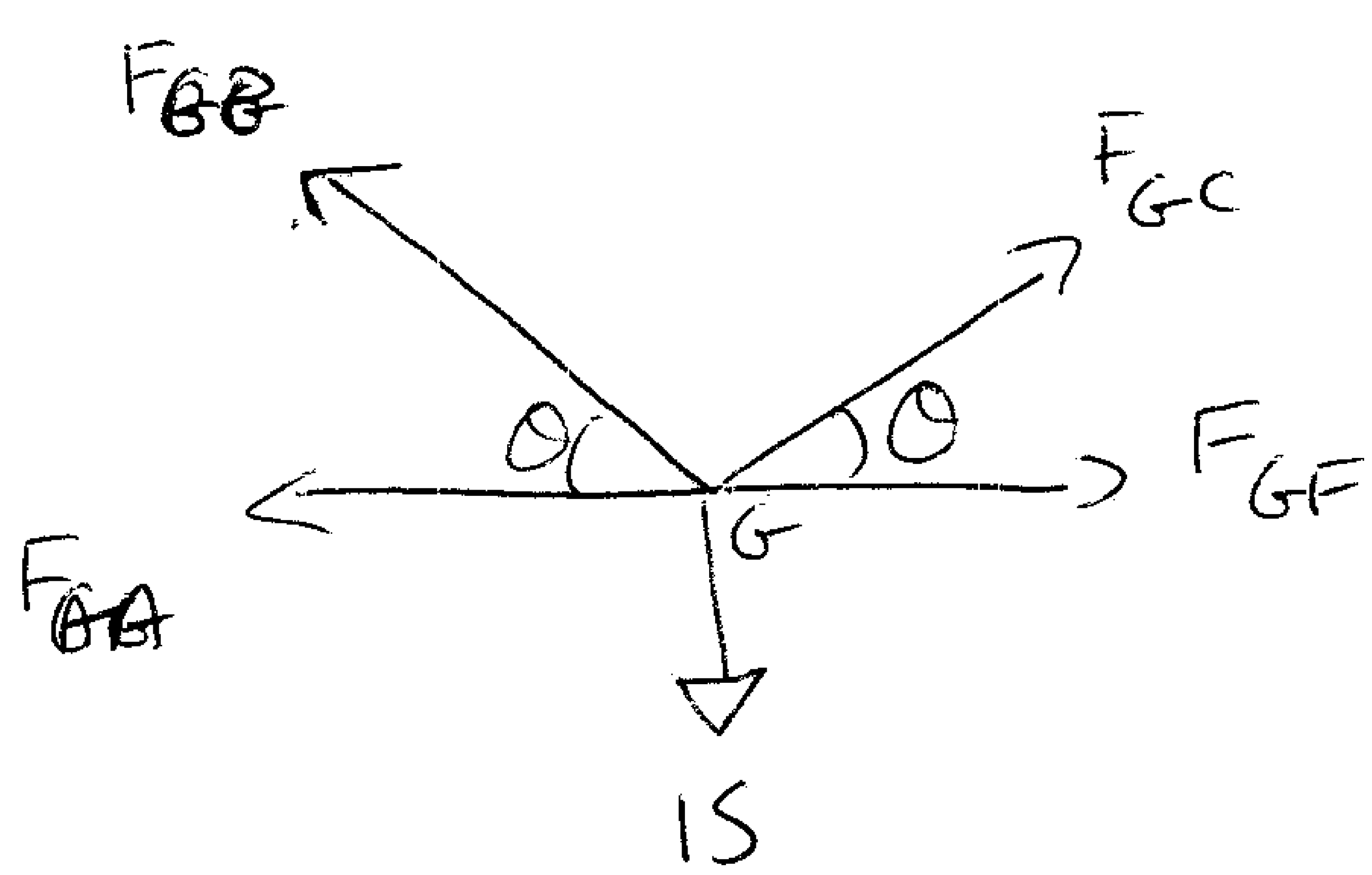
$$F_{BA} = -F_{BG}$$

from Joint A $\quad F_{BA} = -25 kN \quad \Rightarrow \quad F_{BG} = +25 kN$

From (3) $\quad F_{BC} + 25 \cos\theta - (-25 \cos\theta) = 0$

$$F_{BC} + 50 \frac{4}{5} = 0 \qquad F_{BC} = -40 kN \Leftarrow$$

From Joint G



Σ

$$\Sigma \vec{F_x} = 0: \quad -F_{GA} - F_{GB} \cos\theta + F_{GC} \cos\theta + F_{GF} = 0 \qquad (5)$$

$$\Sigma F_y \uparrow = 0 \qquad F_{GB} \sin\theta + F_{GC} \sin\theta - 15 = 0 \qquad (6)$$

From joint B $\quad F_{GB} = +25\,KN$

From joint A $\quad F_{GA} = +20\,KN$

$$\therefore \quad \cancel{25}^{15}\frac{3}{\cancel{5}} + F_{GC}\times\frac{3}{5} - 15 = 0$$

$$F_{GC} = 0 \Leftarrow$$

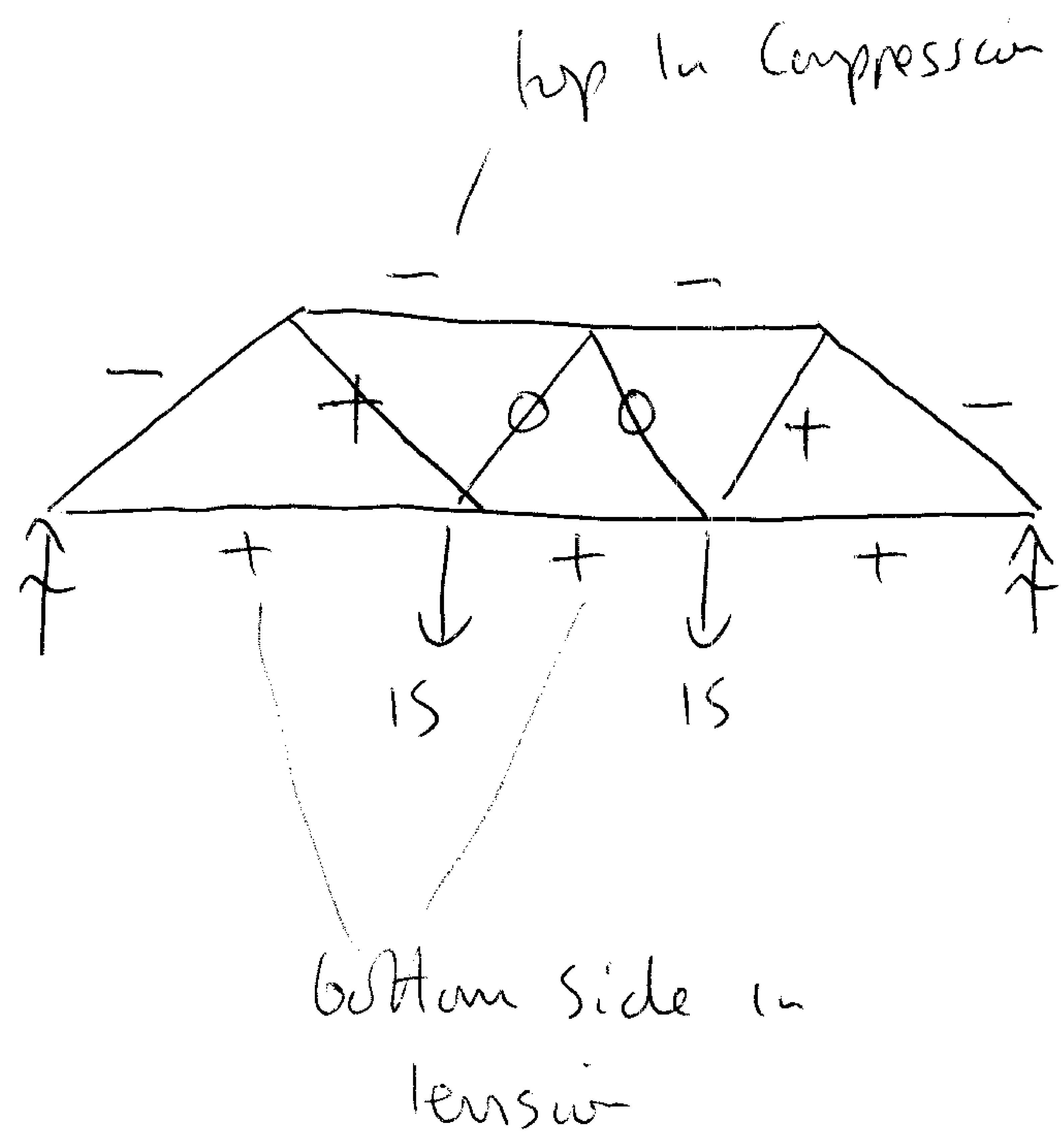
$$\text{from } G: \; -20 - 25\times\frac{4}{5} + 0 + F_{GF} = 0$$

$$F_{GF} = +40\,KN$$

Have solved for bar forces in LH half of truss. By symmetry, forces in RHS must be identical

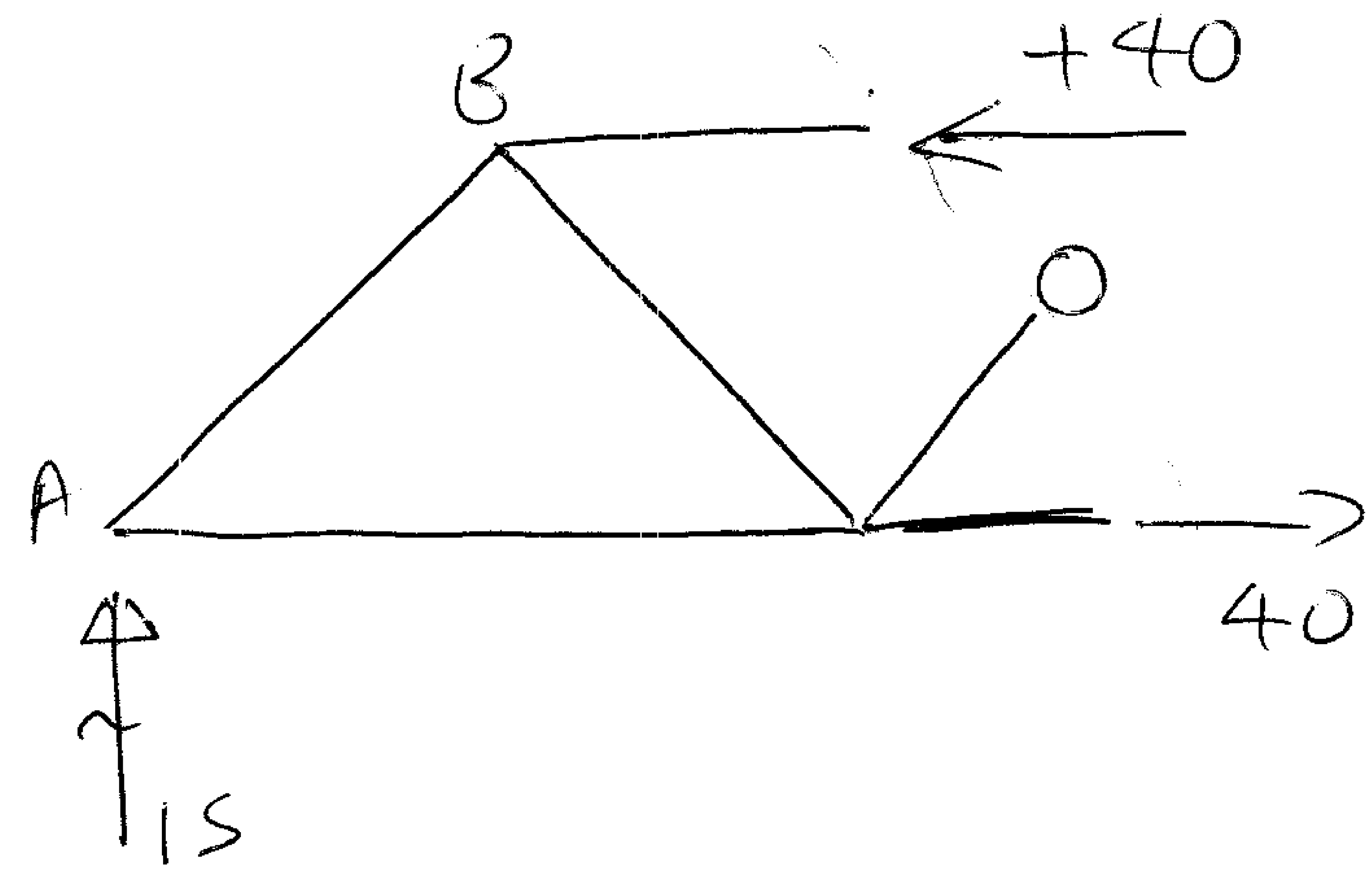
Tabulate

Bar	Force /KN
F_{AB}	-25
F_{AG}	$+20$
F_{BC}	-40
F_{BG}	$+25$
F_{GC}	0
F_{CF}	0
F_{CD}	-40
F_{GF}	$+40$
F_{FD}	$+25$
F_{DE}	-25
F_{EF}	$+20$



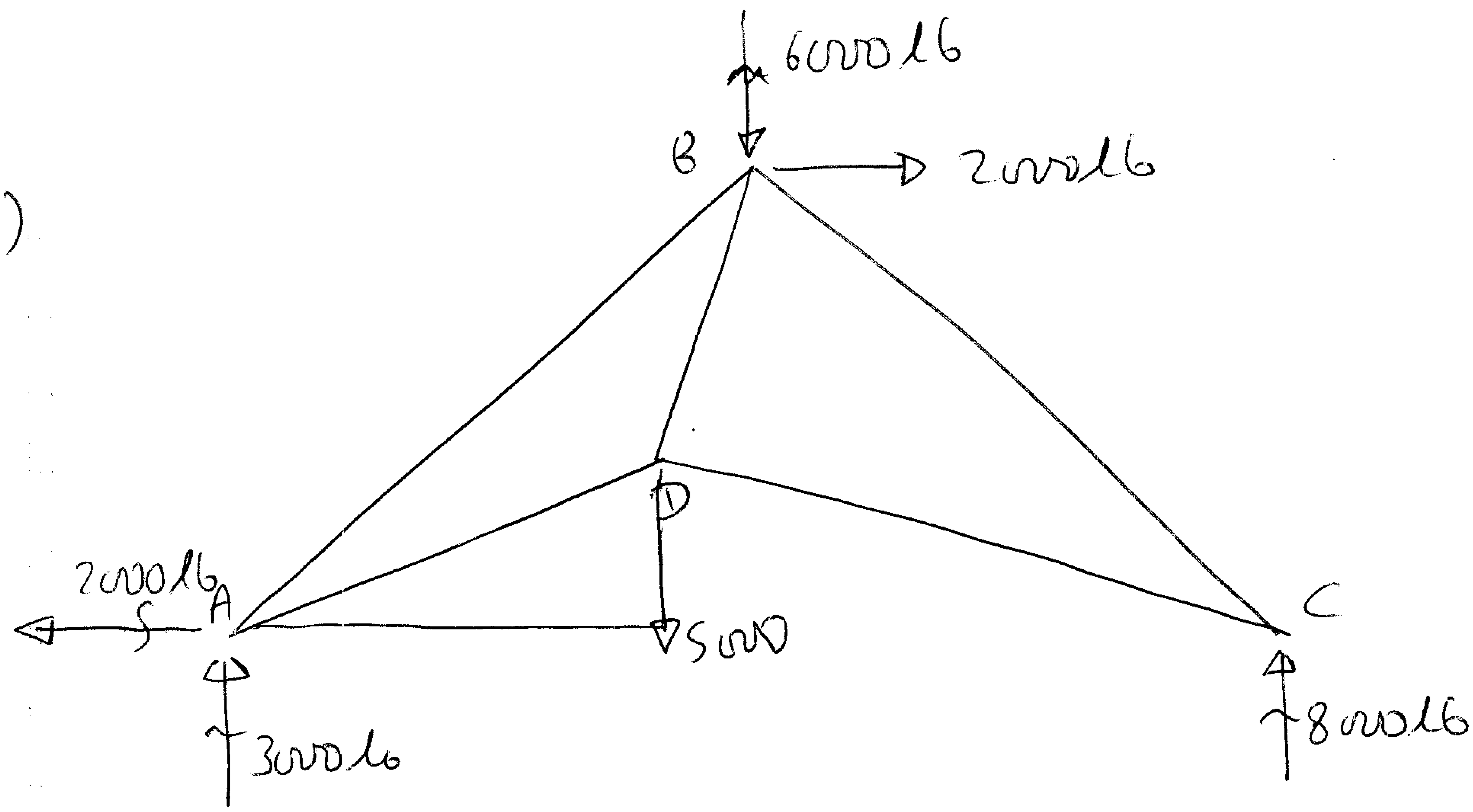
Check

Apply method of sections



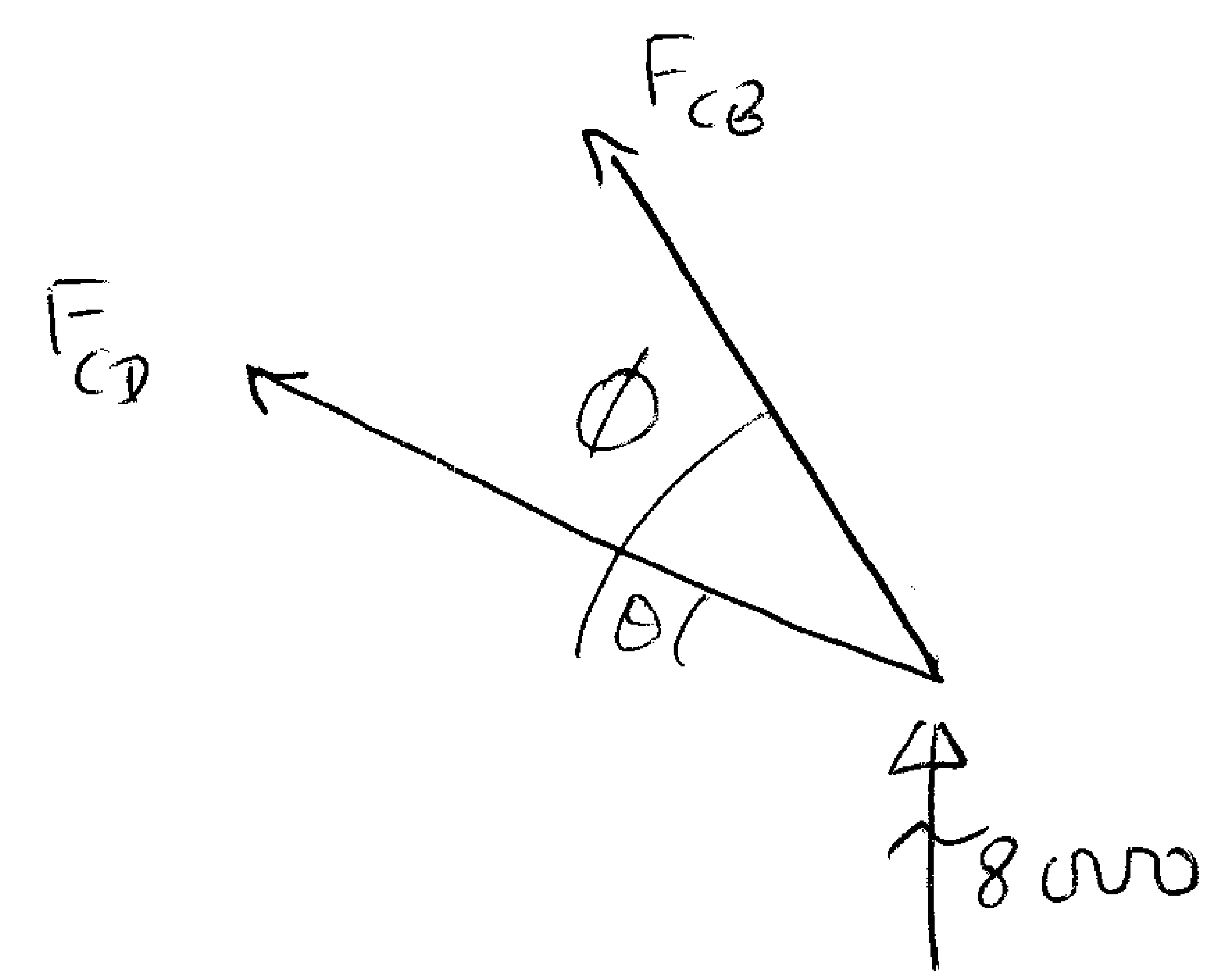
looks right

M6 a)



Use method of joints

at C



$\tan\theta = \frac{6}{12} = \frac{1}{2}$

$\tan\phi = \frac{12}{9} = \frac{4}{3}$

$\Sigma \vec{F}_x = 0: \quad -F_{CD}\cos\theta - F_{CB}\cos\phi = 0$ (1)

$\Sigma F_y \uparrow = 0 \quad \Rightarrow \quad 8000 + F_{CD}\sin\theta + F_{CB}\sin\phi = 0$ (2)

$\theta = 26.56$ $\quad \cos\theta = 0.894 = \frac{2}{\sqrt{5}}$ $\quad \sin\theta = 0.447 = \frac{1}{\sqrt{5}}$ | $\cos\phi = \frac{4}{5}$ $\quad \sin\phi = \frac{3}{5}$

$-\ 0.894F_{CD} - \frac{4}{5}F_{CB} = 0$ (1)

$0.447\,F_{CD} + \frac{3}{5}F_{CB} = -8000$ (2)

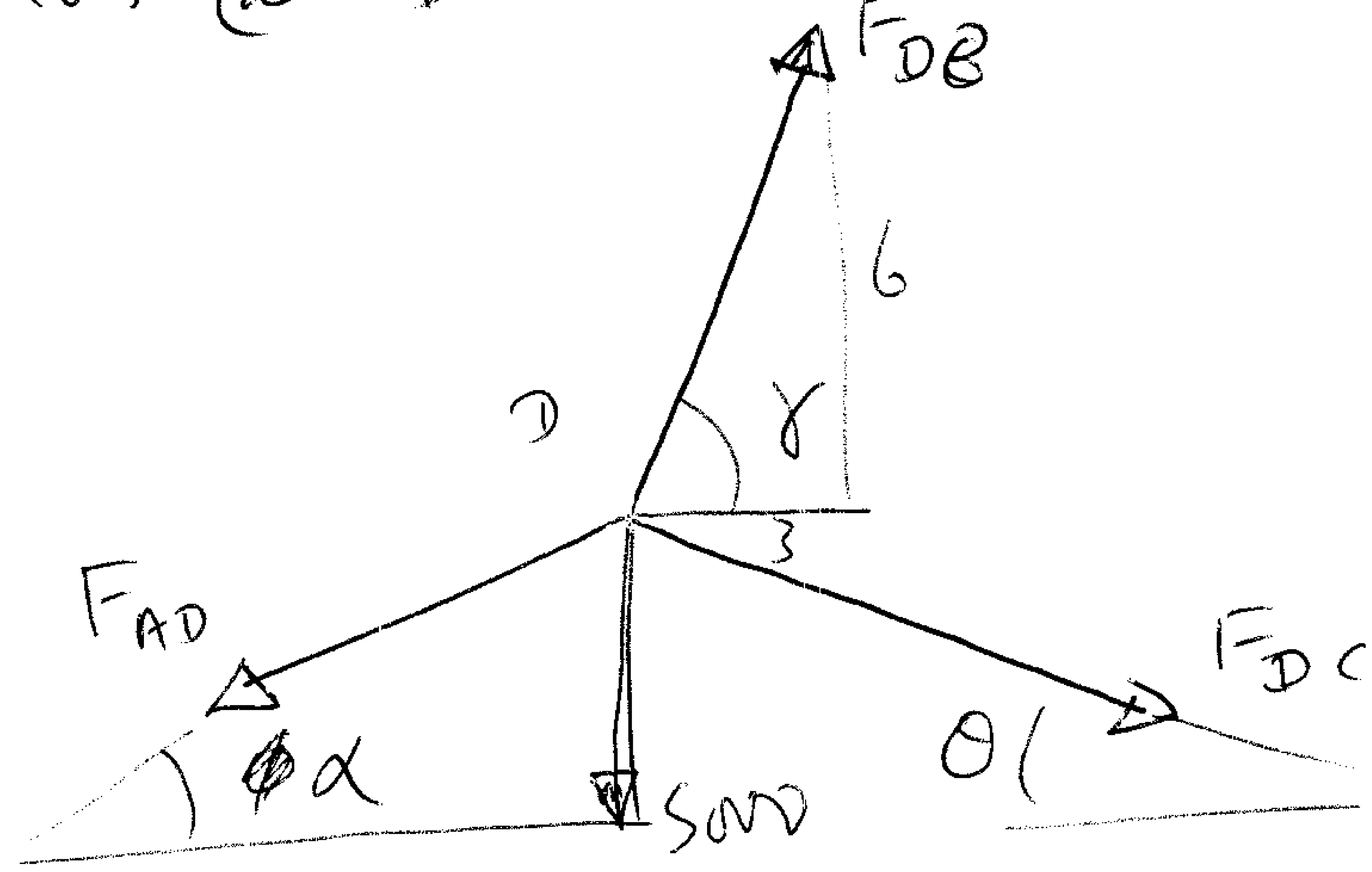
(2) $\times \frac{4}{3}$ + (1) $\Rightarrow$ $0.596\,F_{CD} = -10666.7$

$F_{CD} = -17897$ lb

fm (1) $F_{CB} = -\frac{5}{4} \times 0.894\,(-17897)$

$F_{CB} = +20000$ lb

MoJ @ D



$\cancel{\cos}\tan\alpha = \frac{6}{18} = \frac{1}{3}$ $\alpha = 18^\circ$

$\cos\alpha = 0.949$

$\sin\alpha = 0.316$

$\tan\gamma = \frac{6}{3} = 2 \Rightarrow \cancel{\alpha}\ \gamma = 63.4^\circ$

$\cos\gamma = 0.447$

$\sin\gamma = 0.894$

$\sum \vec{F}_x = 0 \quad -F_{AD} \cos\alpha + F_{DC} \cos\theta + F_{DB} \cos\gamma = 0$ (3)

$\sum F_y = 0 \quad -F_{AD} \sin\alpha - 5000 - F_{DC} \sin\theta + F_{DB} \sin\gamma = 0$ (4)

Substitute for $F_{DC} = -17897$ kN

(3) $\Rightarrow \quad -0.949 F_{AD} - 0.894 \times 17897 + F_{DB} \times 0.447 = 0$

$\Rightarrow 0.447 F_{DB} - 0.949 F_{AD} = 16008$ (3)

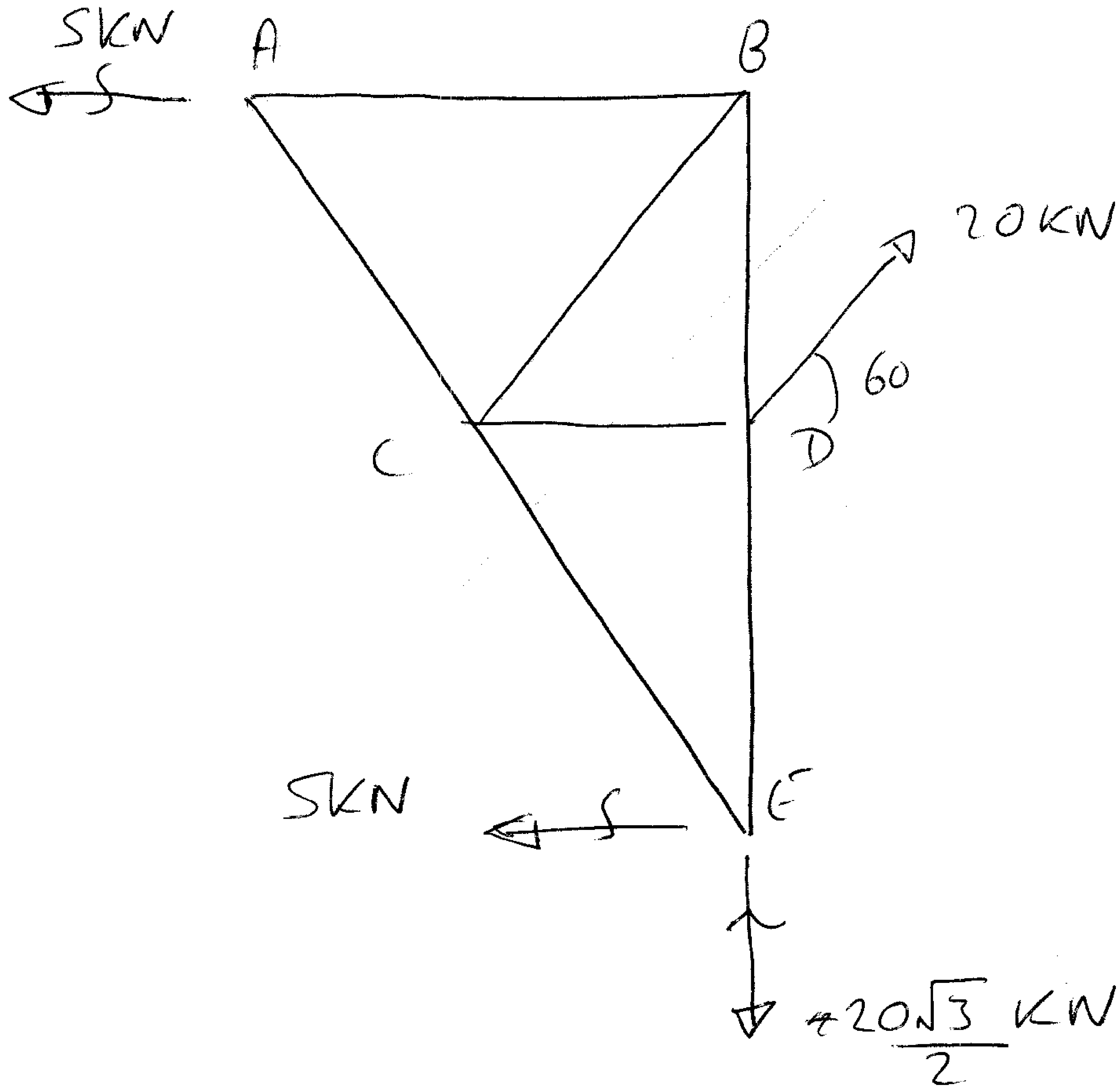
$-0.316 F_{AD} + 0.894 F_{DB} = +3000$ (4)

(4) $\times \dfrac{0.949}{0.316}$ $-$ (3)

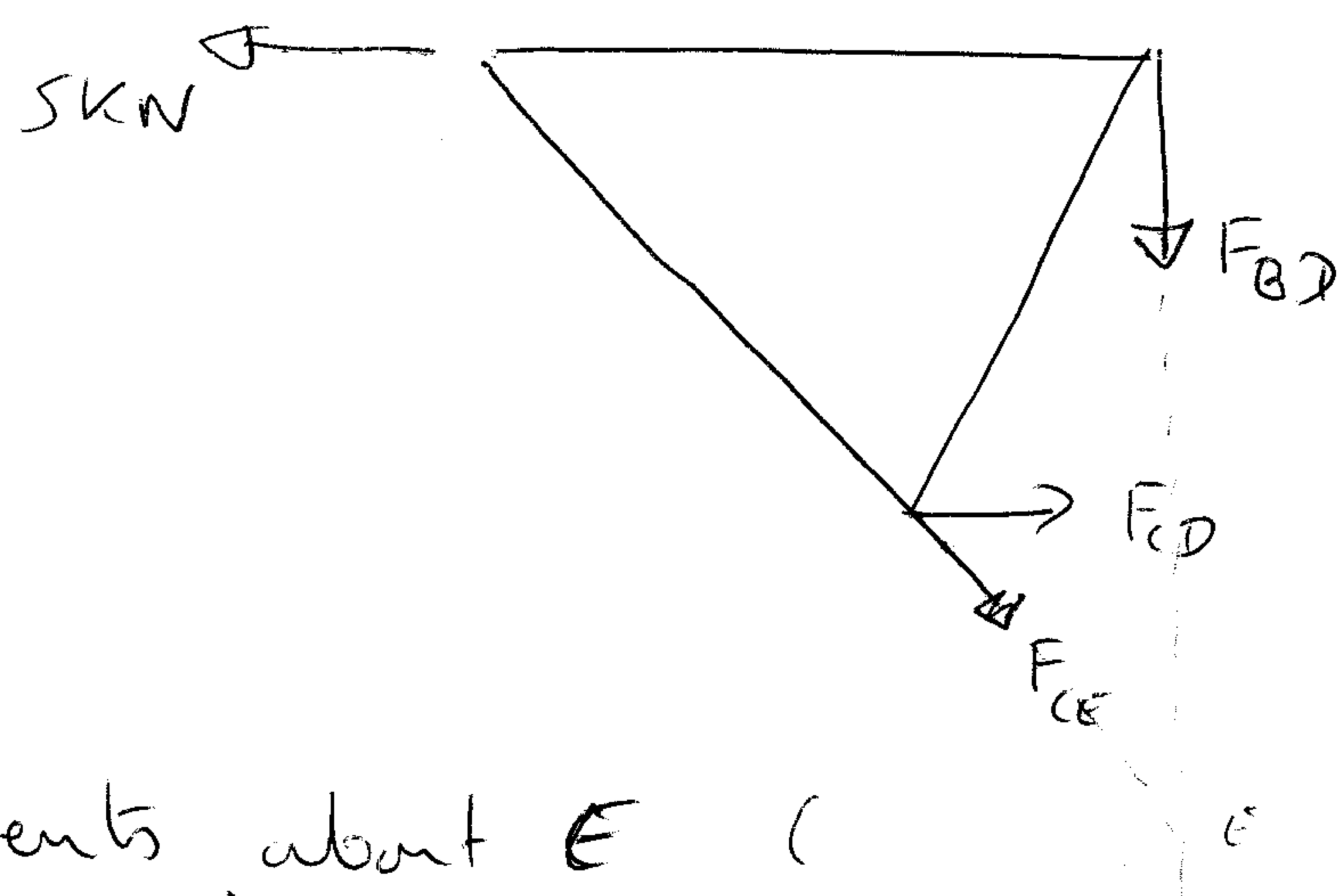
$\Rightarrow \quad 2.238 F_{DB} = -7000$ lb.

$F_{DB} = -3127$ lb.

M6 b)



Method of sections



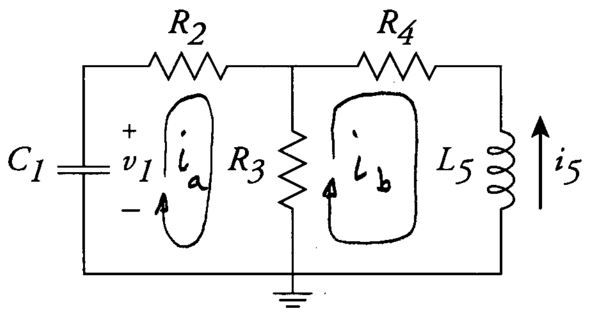
Take moments about E

(F_{BD} + F_{CE} intersect)

$$\sum M_E = 0: \; +5\text{KN}\, 2\cos 30^\circ - F_{CD}\cos 30^\circ = 0$$

$$F_{CD} = +10 \text{ KN}$$

To solve, you can use the node method or loop method. It's easier with loop method. To solve, write KVL around 2 loops, plus capacitor constitutive law:



$$i_a: \ (R_2 + R_3)\, i_a - R_3 i_b - v_1 = 0$$

$$i_b: \ -R_3 i_a + \left(R_3 + R_4 + L_5 \frac{d}{dt}\right) i_b = 0$$

$$C_1: \ i_a + C_1 \frac{dv_1}{dt} = 0$$

(Note that $i_a = -C_1 dv_1/dt$, because $i_1 = -i_a$)

Plugging in numbers,

$$8 i_a - 4 i_b - v_1 = 0$$

$$-4 i_a + \left(2\frac{d}{dt} + 5\right) i_b = 0$$

$$i_a + 0.5 \frac{dv_1}{dt} = 0$$

If we assume that

$$i_a(t) = I_a e^{st}$$
$$i_b(t) = I_b e^{st}$$
$$v_1(t) = V_1 e^{st}$$

then the above equations become

$$8 I_a \quad - 4 I_b \quad - V_1 = 0$$
$$-4 I_a + (2s + 5) I_b = 0$$
$$I_a \quad + 0.5s = 0$$

In matrix form,

$$\underbrace{\begin{bmatrix} 8 & -4 & -1 \\ -4 & 2s+5 & 0 \\ 1 & 0 & 0.5s \end{bmatrix}}_{M(s)} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = 0$$

For this equation to have a solution,

$$\det(M(s)) = 0$$
$$= 8\left[(2s+5)(0.5s) - (0)(0)\right]$$
$$+ 4\left[(-4)(0.5s) - (1)(0)\right]$$
$$- 1\left[(-4)(0) - (1)(2s+5)\right]$$
$$= \left(4s(2s+5)\right) - 8s + 2s + 5$$
$$= 8s^2 + 14s + 5 = 0$$

The roots are

$$s_1 = -1.25 \ \text{sec}^{-1}$$
$$s_2 = -0.5 \ \text{sec}^{-1}$$

Now find the characteristics vectors:

$s_1 = -1.25$:

$$M(s_1) = \begin{bmatrix} 8 & -4 & -1 \\ -4 & 2.5 & 0 \\ 1 & 0 & -0.625 \end{bmatrix}$$

$M(s_1)$ can be row-reduced to obtain

$$\begin{bmatrix} 1 & -1/2 & -1/8 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \underline{0}$$

One solution is

$$\begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \begin{bmatrix} 5/8 \\ 1 \\ 1 \end{bmatrix}$$

Similary, for $s_2 = -0.5$,

$$M(s_2) = \begin{bmatrix} 8 & -4 & -1 \\ 4 & 4 & 0 \\ -1 & 0 & -0.25 \end{bmatrix}$$

which can be row-reduced to obtain

$$\begin{bmatrix} 1 & -1/2 & -1/8 \\ 0 & 1 & -1/4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \underline{0}$$

A solution is

$$\begin{bmatrix} I_a \\ I_b \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

The general solution is then

$$\begin{pmatrix} i_a(t) \\ i_b(t) \\ v_1(t) \end{pmatrix} = a \begin{pmatrix} 5/8 \\ 1 \\ 1 \end{pmatrix} e^{-1.25t} + b \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} e^{-0.5t}$$

The initial conditions are

$$v_1(0) = 2V = a + 4b$$

$$\Rightarrow a + 4b = 2$$

$$i_5(0) = 1A = -i_b(0) = -a - b$$

$$\Rightarrow -a - b = 1$$

In matrix form,

$$\begin{bmatrix} 1 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The solution is

$$a = -2$$

$$b = 1$$

No. 5505 Engineer's Computation Pad

Therefore,

$$v_1(t) = a e^{-1.25t} + 4b e^{-0.5t}$$

$$= \left(-2e^{-1.25t} + 4e^{-0.5t}\right) \text{ volts}$$

$$i_5(t) = -i_6(t)$$

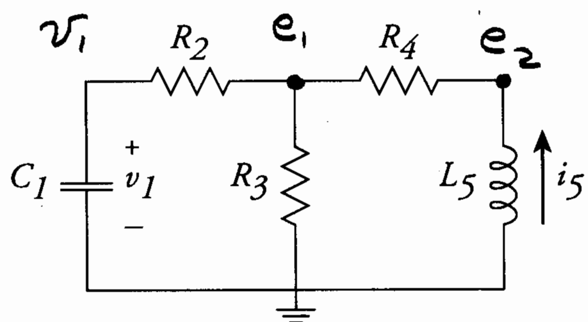
$$= -a e^{-1.25t} - b e^{-0.5t}$$

$$= \left(2e^{-1.25t} - e^{-0.5t}\right) \text{ amps}$$

To find state-space equations for this system,

1. Treat v_1, i_5 as sources
2. Find i_1, v_5 in terms of v_1, i_5
3. Use constitutive laws to find
   $\frac{d}{dt} v_1$, $\frac{d}{dt} i_5$

We can use the loop method or node method. I will use the node method (even though loop method would have one fewer equation)



The node equations are:

$$e_1: \quad (G_2 + G_3 + G_4)\, e_1 - G_4 e_2 = G_2 v_1$$
$$e_2: \quad -G_4 e_1 + G_4 e_2 = i_5$$

Plugging in numbers,

$$1.5\, e_1 - e_2 = 0.25\, v_1$$
$$-e_2 + e_2 = i_5$$

Solving (by row reduction or matrix inverse),

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0.5v_1 + 2i_5 \\ 0.5v_1 + 3i_5 \end{pmatrix}$$

Now, find $\dot{v}_1$, $\dot{i}_5$

$$\dot{v}_1 = \frac{1}{C_1} i_1 = 2i_1$$

i_1 can be found by applying KCL @ v_1:

$$i_1 + \frac{v_1 - e_1}{R_2} = 0$$

$$\Rightarrow i_1 = \frac{e_1 - v_1}{R_2} = \frac{1}{4}\left[(0.5v_1 + 2i_5) - v_1\right]$$

$$= -0.125 v_1 + 0.5 i_5$$

$$\Rightarrow \dot{v}_1 = 2i_1 = -0.25 v_1 + i_5$$

To find $\dot{i}_5$, use

$$\dot{i}_5 = \frac{1}{L_5} v_5 = \frac{1}{L_5}(-e_2)$$

$$= \frac{1}{2}\left(-0.5v_1 - 3i_5\right)$$

$$= -0.25 v_1 - 1.5 i_5$$

Therefore,

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ i_5 \end{bmatrix} = \begin{bmatrix} -0.25 & 1 \\ -0.25 & -1.5 \end{bmatrix}\begin{bmatrix} v_1 \\ i_5 \end{bmatrix}$$