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16.01/16.02 Unified Engineering I, II
Fall 2003

Problem Set 9

Name: _____

Due Date: 11/4/03

	Time Spent (min)
F3	
F4	
F5	
S12	
S13	
S14	
M7	
Study Time	

Announcements:

F3. Anderson Problem 1.4

F4. A rectangular wing of chord c and span b is operating in low-speed flow. Its drag depends on the following parameters:

$$D = f(\alpha, \rho_\infty, V_\infty, \mu_\infty, b, c)$$

Determine the dimensionless parameters (Pi products) which determine the drag coefficient C_D .

F5. An certain aircraft normally operates at 8km altitude, where the air properties compare to the sea level values as follows:

$$\rho = 0.50 \rho_{\text{SL}}$$

$$a = 0.95 a_{\text{SL}}$$

$$\mu = 0.95 \mu_{\text{SL}}$$

a) A 1/4 scale model of an aircraft is to be tested in an wind tunnel at sea level conditions. Is it possible to match both the Reynolds number and Mach number to those of the actual aircraft at altitude? Explain why or why not.

b) The Wright Brothers Wind Tunnel was designed to be pressurized during operation, as a means of increasing the air density ρ . The air temperature in the tunnel is still maintained at a normal $300K$, so that the speed of sound a and air viscosity μ inside the tunnel are unaffected by the pressurization, and are equal to their normal sea level values. Determine the pressure (in atmospheres) at which the tunnel must be operated to match the flight Reynolds and Mach numbers.

Problem S12 (Signals and Systems)

The longitudinal dynamics of an aircraft flying are given in state-space form as

$$\frac{dh}{dt} = V \sin \gamma \quad (1)$$

$$\frac{dV}{dt} = -g \sin \gamma + \frac{T}{m} - \frac{D}{m} \quad (2)$$

$$\frac{d\gamma}{dt} = \frac{L - mg}{mV} \quad (3)$$

where

h = altitude

V = velocity of aircraft

γ = flight path angle

g = acceleration due to gravity

L = lift

T = thrust

D = drag

m = mass of aircraft

We assume that (1) the thrust is constant, (2) the aircraft flies at constant coefficient of lift, and (3) the aircraft flies at constant drag coefficient. These assumptions are not bad for the case when the pilot releases all the controls, and makes no adjustments, say, for altitude variations. In this case, we can rewrite the equations as

$$\frac{dh}{dt} = V \sin \gamma \quad (4)$$

$$\frac{dV}{dt} = -g \sin \gamma + \frac{T}{m} - \frac{D_0}{m} \frac{V^2}{V_0^2} \quad (5)$$

$$\frac{d\gamma}{dt} = \frac{L_0(V/V_0)^2 - mg}{mV} \quad (6)$$

where L_0 and D_0 are the lift and drag at the nominal velocity, V_0 . If the aircraft is initially in trim, it must also be true that

$$T = D_0 \quad (7)$$

$$L_0 = mg \quad (8)$$

Equations (4)–(6) are the equations of motion in state-space form, although the equations are nonlinear. The equations may be *linearized*, by considering small perturbations δh , δV , and $\delta \gamma$ about the nominal trajectory. The linearized equations of

motion (after a little work) are

$$\frac{d}{dt} \delta h = V_0 \delta \gamma \quad (9)$$

$$\frac{d}{dt} \delta V = -g \delta \gamma - \frac{2g}{L_0/D_0} \frac{1}{V_0} \delta V \quad (10)$$

$$\frac{d}{dt} \delta \gamma = \frac{2g}{V_0^2} \delta V \quad (11)$$

So $\dot{\underline{x}} = A\underline{x}$, where

$$\underline{x} = \begin{bmatrix} \delta h \\ \delta V \\ \delta \gamma \end{bmatrix} \quad (12)$$

and

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{2g}{L_0/D_0} \frac{1}{V_0} & -g \\ 0 & \frac{2g}{V_0^2} & 0 \end{bmatrix} \quad (13)$$

For a light aircraft flying at a nominal velocity of 120 knots, with a lift-to-drag ratio of 10,

1. Find the eigenvalues and eigenvectors of the system.
2. For an initial perturbation

$$\delta h(0) = 0 \text{ m}$$

$$\delta V(0) = 0 \text{ m/sec}$$

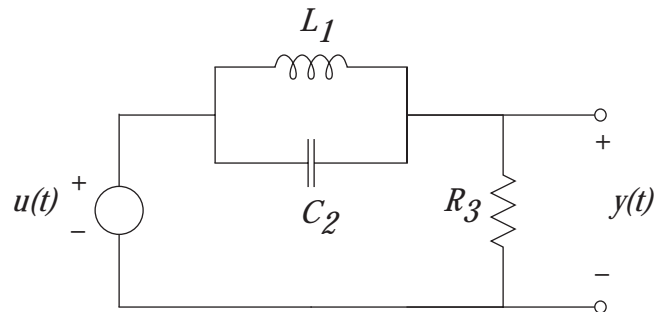
$$\delta \gamma(0) = 0.1 \text{ rad}$$

find and plot the perturbations $\delta h(t)$, $\delta V(t)$, and $\delta \gamma(t)$ as functions of time. This perturbation corresponds to a perturbation caused by a vertical gust which puts the aircraft momentarily into a 0.1 rad (6 deg) climb.

Note: Some of the eigenvalues and eigenvectors are complex. As in the notes, treat a complex eigenvalue / eigenvector just as you would a real eigenvalue / eigenvector. You may find it useful to use Matlab, a spreadsheet, or Ada to calculate the results numerically.

Problem S13 (Signals and Systems)

Consider the RLC circuit below:



This circuit is a *notch filter*, meaning that the output $y(t)$ is almost the same as the input $u(t)$, except that the circuit “filters out” frequencies in a narrow range, determined by the component values. For example, this circuit might be used to filter out 60 Hz noise caused by electrical wiring from the input to an audio system, to prevent 60 Hz “hum.”

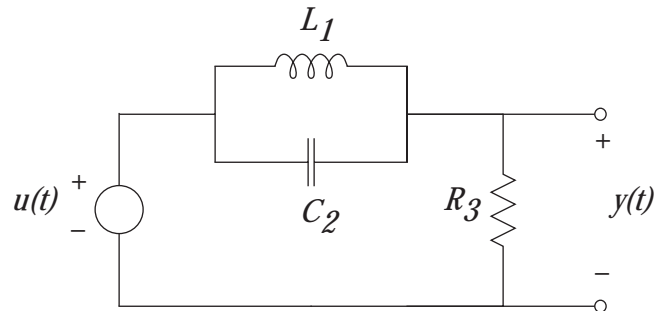
For this circuit, find a state-space description of the system, in the form

$$\begin{aligned}\frac{d\mathbf{x}(t)}{dt} &= A\mathbf{x}(t) + Bu(t) \\ y(t) &= C\mathbf{x}(t) + Du(t)\end{aligned}$$

No component values are given, so just find the matrices A , B , C , and D in symbolic form.

Problem S14 (Signals and Systems)

Consider the RLC circuit of Problem S13, shown below:



1. Find the transfer function, $G(s)$, of the system, using

$$G(s) = C(sI - A)^{-1}B + D \quad (1)$$

2. Find the transfer function using impedance methods. Show that your result agrees with the result in part (1).
3. For component values

$$L_1 = 1 \text{ H}, \quad C_2 = 0.25 \text{ F}, \quad R_3 = 10 \text{ } \Omega$$

plot the magnitude of the transfer function $G(j\omega)$ vs. ω . Explain why the filter is called a notch filter.

Note: You may find it useful to use Matlab or a spreadsheet to calculate values of the transfer function, since there is a fair amount of complex arithmetic.

Problem M7

For the truss shown below calculate the forces in all the bars. The bars have length L or $\sqrt{2}L$ and all angles are either 90° or 45° . You will be conducting experiments on a truss of this configuration beginning on October 30th. It is strongly encouraged that you complete this problem set question before you begin performing the experiments. We also encourage you to use the method of joints to solve this truss as you will be required to write an Ada code as part of the truss lab, which will require use of the method of joints.

