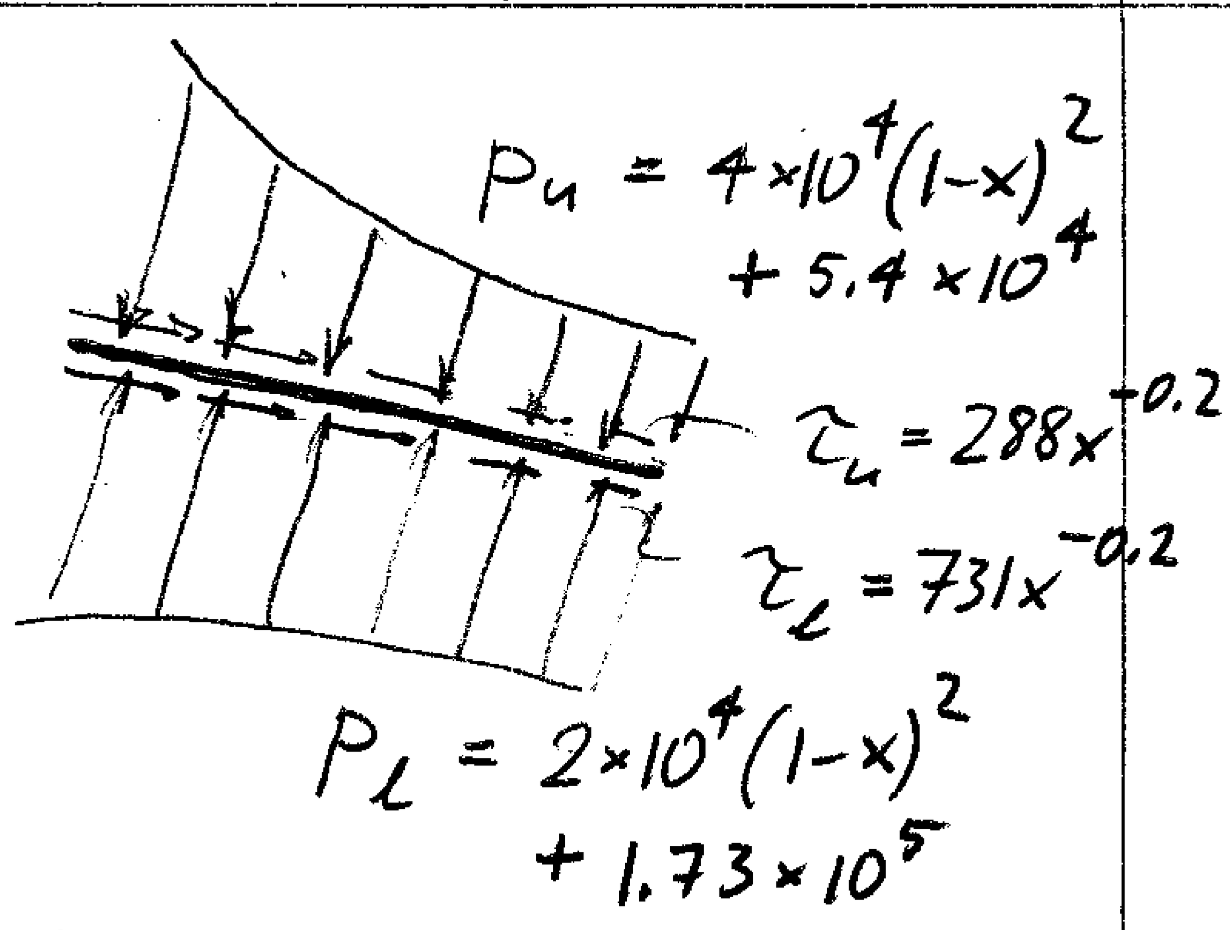
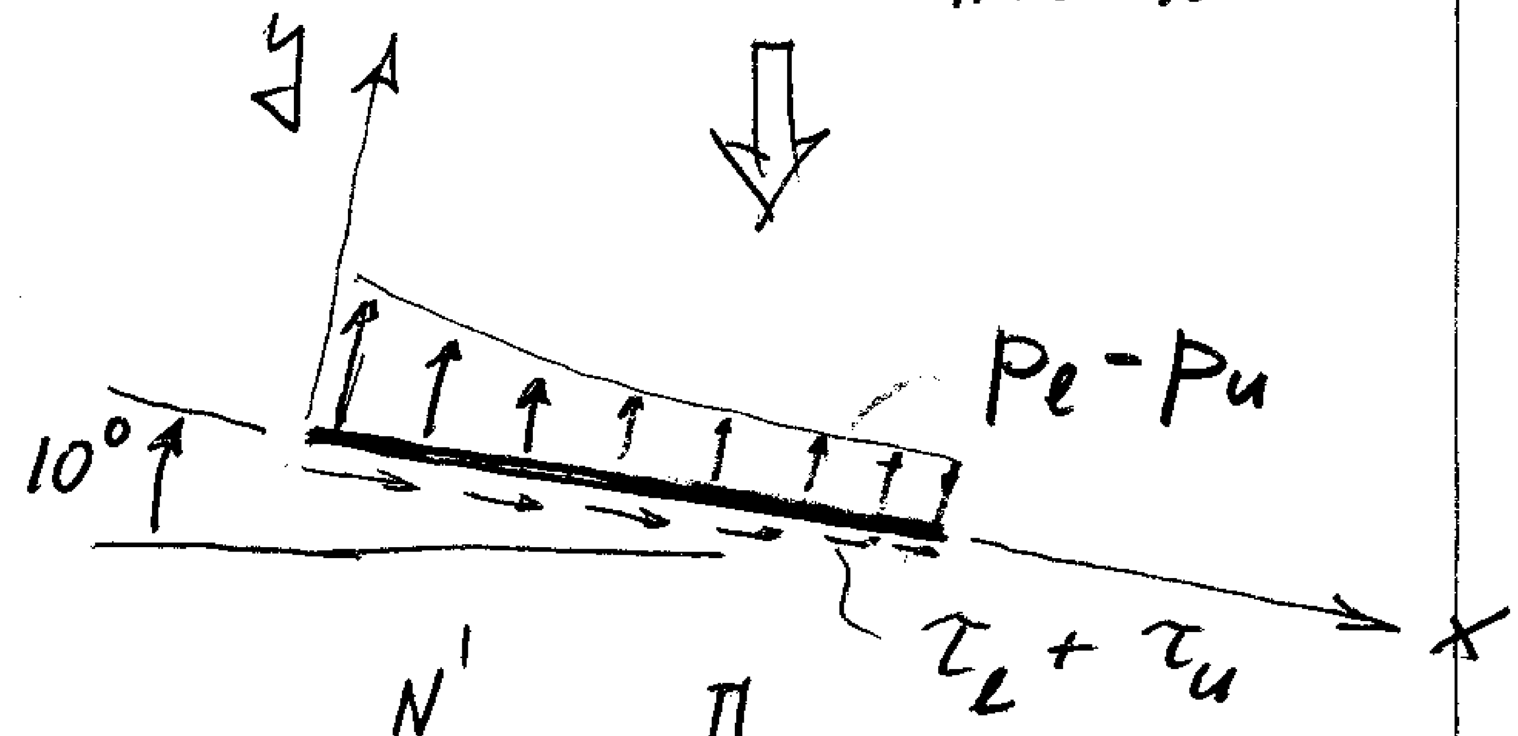
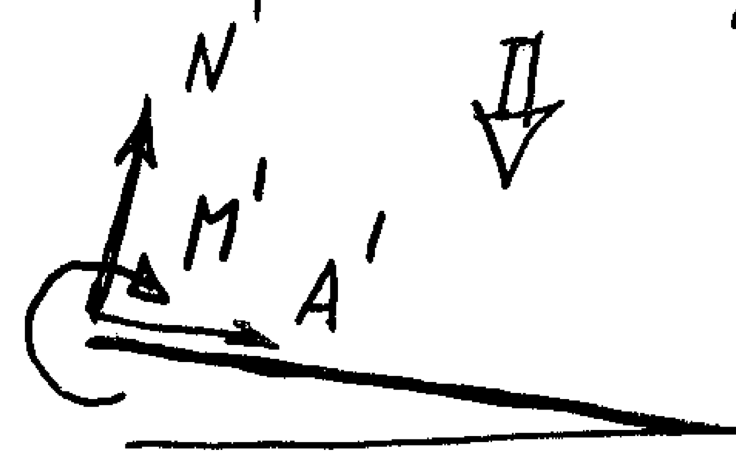
(Anderson 1.4)



chord $c = 1$ m

Because airfoil is a flat plate, only the net top & bottom forces matter. (less stuff to integrate).





$$p_\ell - p_u = 1.19\times10^5 - 2\times10^4(1-x)^2 \text{ Pa}$$

$$\tau_\ell + \tau_u = 1019\,x^{-0.2} \text{ Pa}$$

$$N' = \int_0^{1m}(p_\ell - p_u)\,dx = \left(1.19\times10^5 x - \tfrac{2}{3}\times10^4(1-x)^3\right)\Big|_0^1$$

$$\boxed{N' = 1.19\times10^5 - 6.67\times10^3 = 1.123\times10^5 \text{ N/m}}$$

$$\boxed{A' = \int_0^{1m}(\tau_\ell + \tau_u)\,dx = \frac{1}{0.8}\,1019\,x^{0.8}\Big|_0^1 = 1.274\times10^3 \text{ N/m}}$$

$$M'_{LE} = \int_0^{1m} -(p_\ell - p_u)\,x\,dx = \int_0^1\left(-1.19\times10^5 x + 2\times10^4(x - 2x^2 + x^3)\right)dx$$

$$= \left(-\tfrac{1}{2}\,1.19\times10^5 x^2 + 2\times10^4\left(\tfrac{1}{2}x^2 - \tfrac{2}{3}x^3 + \tfrac{1}{4}x^4\right)\right)\Big|_0^1$$

$$= \left(-5.95\times10^4 + 2\times10^4\left(\tfrac{1}{2} - \tfrac{2}{3} + \tfrac{1}{4}\right)\right)$$

$$\boxed{M'_{LE} = -5.783\times10^4 \text{ N}}$$

$$L' = N'\cos\alpha - A'\sin\alpha = 1.104\times10^5 \text{ N/m}$$

$$D' = N'\sin\alpha + A'\cos\alpha = 2.076\times10^3 \text{ N/m}$$

$$M'_{c/4} = M'_{LE} + \tfrac{1}{4}N' = -3.02\times10^4 \text{ N}$$

$$x_{cp} = -M_{LE}/N' = 0.515 \text{ m}$$

Given: $D = f(\alpha, \rho, V, \mu, b, c)$

or $g(D, \alpha, \rho, V, \mu, b, c) = 0$

Parameter	Units
D	$m\ell t^{-2}$
α	—
ρ	$m\ell^{-3}$
V	ℓt^{-1}
μ	$m\ell^{-1}t^{-1}$
b	ℓ
c	ℓ
$N = 7$	$K = 3$

$\longrightarrow$ $7 - 3 = 4$ Pi products.

$$\Pi_1 = \frac{D}{\frac{1}{2}\rho V^2 bc} \equiv C_D$$

$$\Pi_2 = \alpha \equiv \alpha$$

$$\Pi_3 = \frac{\rho V c}{\mu} \equiv Re$$

$$\Pi_4 = \frac{b}{c} \equiv AR \quad \text{(aspect ratio)}$$

So $C_D = \bar{f}(\alpha, Re, AR)$

Alternative Pi products:

$$\Pi_1 = \frac{D}{\frac{1}{2}\rho V^2 b^2} = \frac{C_D}{AR}$$

$$\Pi_3 = \frac{\rho V b}{\mu} = Re \cdot AR$$

$$\Pi_4 = \frac{c}{b} = \frac{1}{AR}$$

etc.

These are valid alternative parameters which determine C_D, although a bit unconventional

Given: $\rho = 0.5\,\rho_{SL}$ $\quad l = 4\,l_{SL}$ (characteristic length) (span, chord, whatever)

$a = 0.95\,a_{SL}$

$\mu = 0.95\,\mu_{SL}$ → tunnel quantity

a) To match M_∞, must have $\dfrac{V}{a} = \dfrac{V_{SL}}{a_{SL}} \rightarrow V = 0.95\,V_{SL}$

To also match Re, must have $\dfrac{\rho V l}{\mu} \stackrel{?}{=} \dfrac{\rho_{SL} V_{SL} l_{SL}}{\mu_{SL}}$

or $\dfrac{\rho}{\rho_{SL}}\dfrac{V}{V_{SL}}\dfrac{l}{l_{SL}} \stackrel{?}{=} \dfrac{\mu}{\mu_{SL}}$

$0.5 \cdot 0.95 \cdot 4 \stackrel{?}{=} 0.95$ ✗ <u>not possible</u>

Cannot simultaneously match M_∞ and Re without being able to adjust another parameter (like ρ !)

b) Tunnel quantities: $\rho_T\ a_T\ \mu_T\ V_T\ l_T$

Unknown: ρ_T

Given: $a_T = a_{SL} = \dfrac{1}{0.95}\,a$ $\quad$ because $T_T = T_{SL}$ as given

$\mu_T = \mu_{SL} = \dfrac{1}{0.95}\,\mu$

$l_T = \dfrac{1}{4}\,l$

Require $M = M_T \rightarrow \dfrac{V}{a} = \dfrac{V_T}{a_T} \rightarrow V = 0.95\,V_T$

Require $Re = Re_T \rightarrow \dfrac{\rho V l}{\mu} = \dfrac{\rho_T V_T l_T}{\mu_T}$

$$\rho_T = \rho \cdot \frac{V}{V_T}\frac{l}{l_T}\frac{\mu_T}{\mu} = \rho \cdot 0.95 \cdot 4\,\frac{1}{0.95} = 4\,\rho$$

or $\boxed{\rho_T = 2\,\rho_{SL}}$

From equation of state: $\boxed{p_T = p_{SL}\left(\dfrac{\rho_T}{\rho_{SL}}\right)^{\nearrow 2}\left(\dfrac{T_T}{T_{SL}}\right)^{\nearrow 1} = 2\,p_{SL} = 2\,\text{atm.}}$

1. The aircraft is flying at 120 knots. Therefore,

$$V_0 = 120 \text{ Kn} \times \frac{6080 \text{ ft}}{\text{NM}} \times \frac{1}{3600 \text{ s/hr}} \times 0.3048 \frac{\text{m}}{\text{ft}}$$

$$= 61.77 \text{ m/s}$$

Also,

$$g = 9.82 \text{ m/s}^2, \quad L_0/D_0 = 10$$

Therefore, the matrix A is given by

$$A = \begin{bmatrix} 0 & 0 & 61.77 \\ 0 & -0.03180 & -9.82 \\ 0 & 0.005147 & 0 \end{bmatrix}$$

The eigenvalues are the roots of

$$\det(sI - A) = 0$$

$$= s\left[(s + 0.0318)s + (0.005147)(9.82)\right]$$

$$= s(s^2 + 0.03180 s + 0.05055)$$

The roots can be found using the quadratics formula, so

$$s_1 = 0, \quad s_2 = -0.01590 + 0.2243j$$

$$s_3 = -0.01590 - 0.2243j$$

The eigenvectors are found by solving

$$(s_i I - A)\,\underline{X}_i = \underline{0}$$

Do each in turn:

$\underline{s_1 = 0}$: $\quad s_1 I - A = \begin{bmatrix} 0 & 0 & -61.77 \\ 0 & +0.03180 & +9.82 \\ 0 & -0.005147 & 0 \end{bmatrix}$

Since the 1st column is all zeros, a solution is

$$\underline{X}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{s_2 = -0.01590 + 0.2243j}$:

$s_2 I - A =$

$$\begin{bmatrix} -0.01590 + 0.2243j & 0 & -61.77 \\ 0 & 0.01590 + 0.2243j & 9.82 \\ 0 & -0.005147 & -0.1590 + 0.2243j \end{bmatrix}$$

Row reduction proceeds as normal, but is messy. The result is

$$\begin{bmatrix} 1 & 0 & -19.43 - 274.1j \\ 0 & 1 & 3.0885 - 43.57j \\ 0 & 0 & 0 \end{bmatrix}$$

Note that one row is zero, as it should be, if $\det(s_2 I - A) = 0$!

Arbitrarily take 3rd element of $\underline{X}_2 = 1$.
The

$$\underline{X}_2 = \begin{bmatrix} 19.43 + 274.1j \\ -3.0885 + 43.57j \\ 1 \end{bmatrix}$$

$s_3 = -0.0159 - 0.2243j$:

Because $s_3 = s_2^*$ (complex conjugate),

$$\underline{X}_3 = \underline{X}_2^* = \begin{bmatrix} 19.43 - 274.1j \\ -3.0885 - 43.57j \\ 1 \end{bmatrix}$$

2. The general solution is

$$\underline{x}(t) = a_1 \underline{X}_1 e^{s_1 t} + a_2 \underline{X}_2 e^{s_2 t} + a_3 \underline{X}_3 e^{s_3 t}$$

The initial condition is

$$\underline{x}(0) = a_1 \underline{X}_1 + a_2 \underline{X}_2 + a_3 \underline{X}_3$$

$$= [\underline{X}_1 \ \underline{X}_2 \ \underline{X}_3] \underline{a} \equiv V \underline{a}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

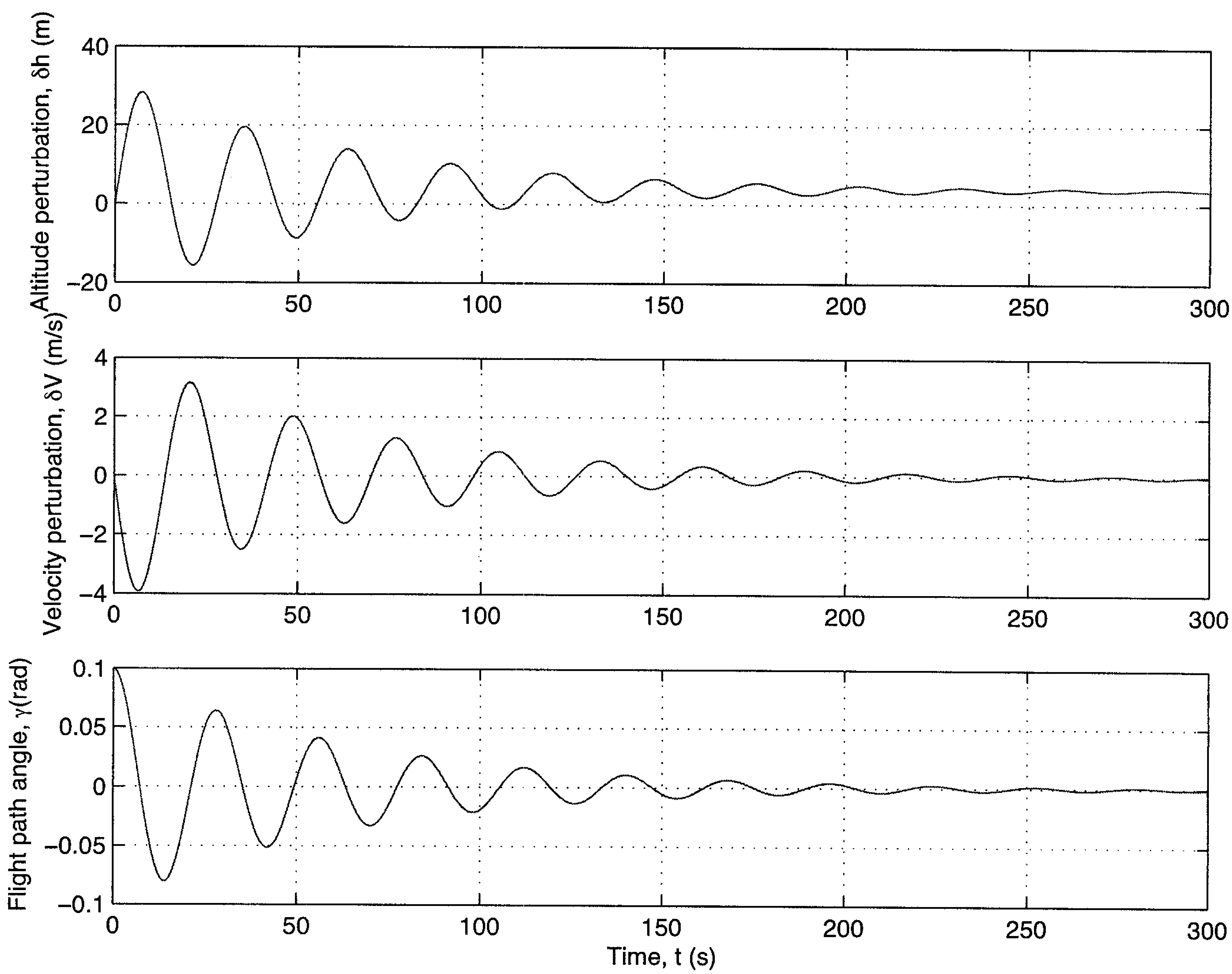
Therefore,

$$\underline{a} = \begin{bmatrix} 3.885 + 0j \\ 0.05 - 0.003544j \\ 0.05 + 0.003544j \end{bmatrix}$$

I found this solution using Matlab, but it could easily be done with a calculator.

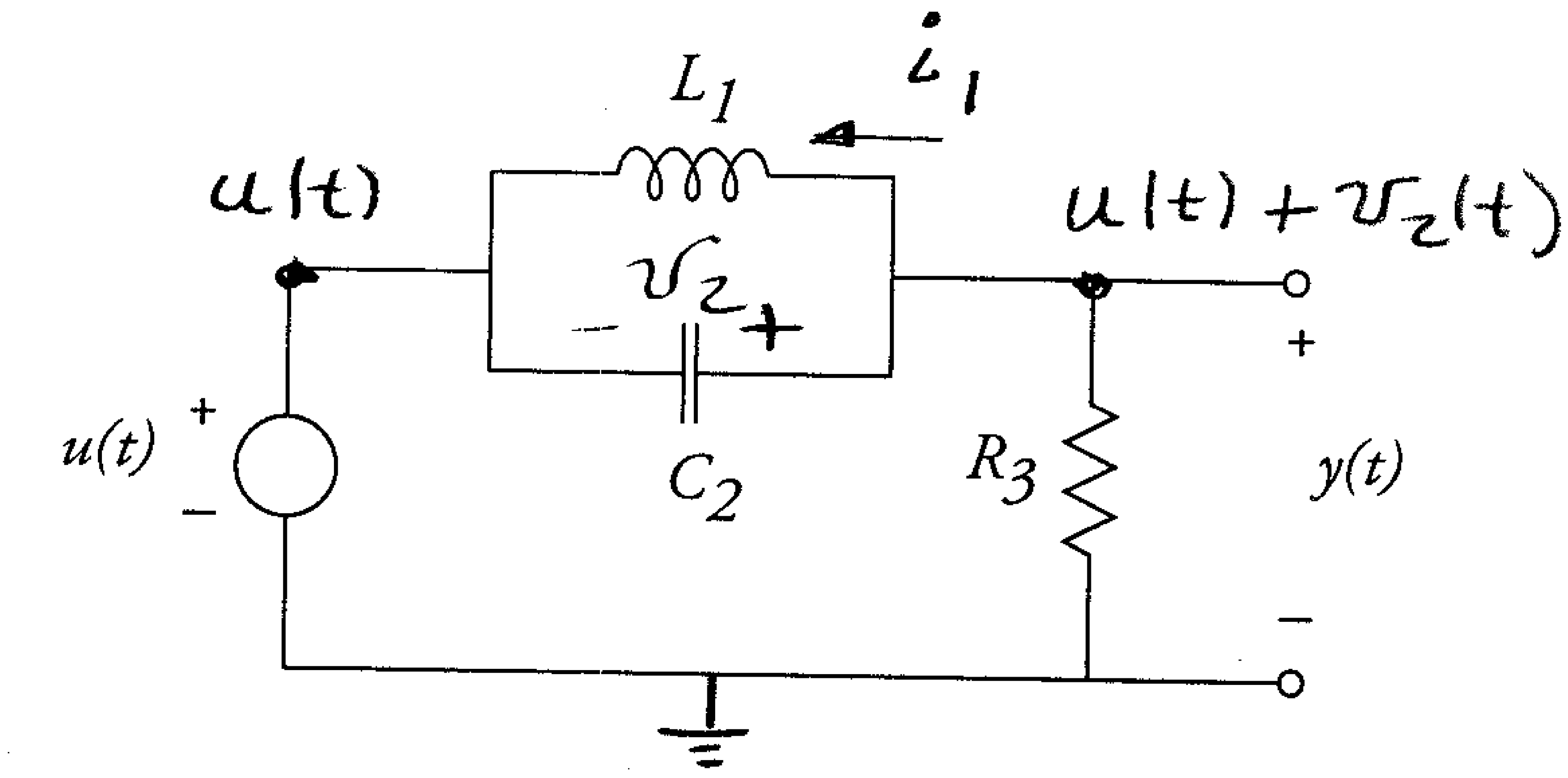
The result can now be plotted, since Matlab does complex exponentials.

```
s1 = 0;
s2 = -0.0159 + 0.2243j;
s3 = -0.0159 - 0.2243j;
X1 = [1;0;0];
X2 = [-19.43-274.1j;-3.0885+43.57j;1];
X3 = [-19.43+274.1j;-3.0885-43.57j;1];
a1 = 3.885;
a2 = 0.05-0.003544j;

a3 = 0.05+0.003544j;
t = 0:0.5:300;
x = a1*X1*exp(s1*t)+a2*X2*exp(s2*t)+a3*X3*exp(s3*t);

subplot(311)
plot(t,real(x(1,:)))
ylabel('Altitude perturbation, \delta{}h (m)')
grid
subplot(312)
plot(t,real(x(2,:)))
ylabel('Velocity perturbation, \delta{}V (m/s)')
grid
subplot(313)
plot(t,real(x(3,:)))
ylabel('Flight path angle, \gamma{}(rad)')
grid
xlabel('Time, t (s)')

print -depsc phugoid.eps
```



Altitude perturbation, δh (m)
40
20
0
−20
Velocity perturbation, δV (m/s)
4
2
0
−2
−4
Flight path angle, γ(rad)
0.1
0.05
0
−0.05
−0.1
0
50
100
150
200
250
300
Time, t (s)

To solve the circuit, use the node method:



Note that there are no unknown nodes, which simplifies things!

The states are

$$x_1 = i_1$$

$$x_2 = v_2$$

To find $\dot{x}_1 = di_1/dt$, need v_1:

$$\dot{x}_1 = \frac{di_1}{dt} = \frac{1}{L} v_1(t)$$

$$= \frac{1}{L} \underbrace{\left[(u + v_2) - u \right]}_{\text{potential across } L}$$

$$= \frac{1}{L} v_2$$

To find $\dot{x}_2 = dv_2/dt$, need i_2. To find i_2, apply KCL at $u + v_2$ node:

$$\frac{u + v_2 - 0}{R} + i_1 + i_2 = 0$$

Therefore,

$$i_2 = -i_1 - \frac{1}{R} v_2 - \frac{1}{R} u$$

and

$$\dot{x}_2 = \frac{dv_2}{dt} = \frac{1}{C} i_2$$

$$= -\frac{1}{C} i_1 - \frac{1}{RC} v_2 - \frac{1}{RC} u$$

Therefore, the state equation is given by

$$\dot{\underline{x}} = \underbrace{\begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix}}_{A} \underline{x} + \underbrace{\begin{bmatrix} 0 \\ -1/RC \end{bmatrix}}_{B} u$$

To find the measurement equation, note that

$$y(t) = v_2 + u = x_2 + u$$

Therefore,

$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \underline{x} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{D} u$$

N.B.:

There are other possible labellings for v_2 and i_1. If you used a different labelling, some of the signs may be different

In particular,

1) If v_2 labelled opposite mine,

$$C = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ +1/RC \end{bmatrix}$$

2) If v_2 or i_1 labelled opposite mine (but not both),

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix}$$

3) If both v_2 and i_1 labelled opposite mine, A remains the same.

1. From S13,

$$A = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -1/RC \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 \end{bmatrix}$$

First find $(sI-A)^{-1}$:

$$sI - A = \begin{bmatrix} s & -1/L \\ 1/C & s + 1/RC \end{bmatrix}$$

For any matrix M, $M^{-1} = \dfrac{\mathrm{adj}(M)}{\det(M)}$. For 2×2 matrices, this gives:

$$(sI-A)^{-1} = \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} s + 1/RC & +1/L \\ -1/C & s \end{bmatrix}$$

Now find $G(s) = C(sI-A)^{-1}B + D$:

$$C(sI-A)^{-1} =$$

$$\frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s + 1/RC & 1/L \\ -1/C & s \end{bmatrix}$$

$$= \frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}} \begin{bmatrix} -1/C & s \end{bmatrix}$$

Then

$$C(sI-A)^{-1}B = \frac{1}{s^2+\frac{s}{RC}+\frac{1}{LC}}\begin{bmatrix}-1/C & s\end{bmatrix}\begin{bmatrix}0\\ -1/RC\end{bmatrix}$$

$$= \frac{-s/RC}{s^2+\frac{s}{RC}+\frac{1}{LC}}$$

Finally,

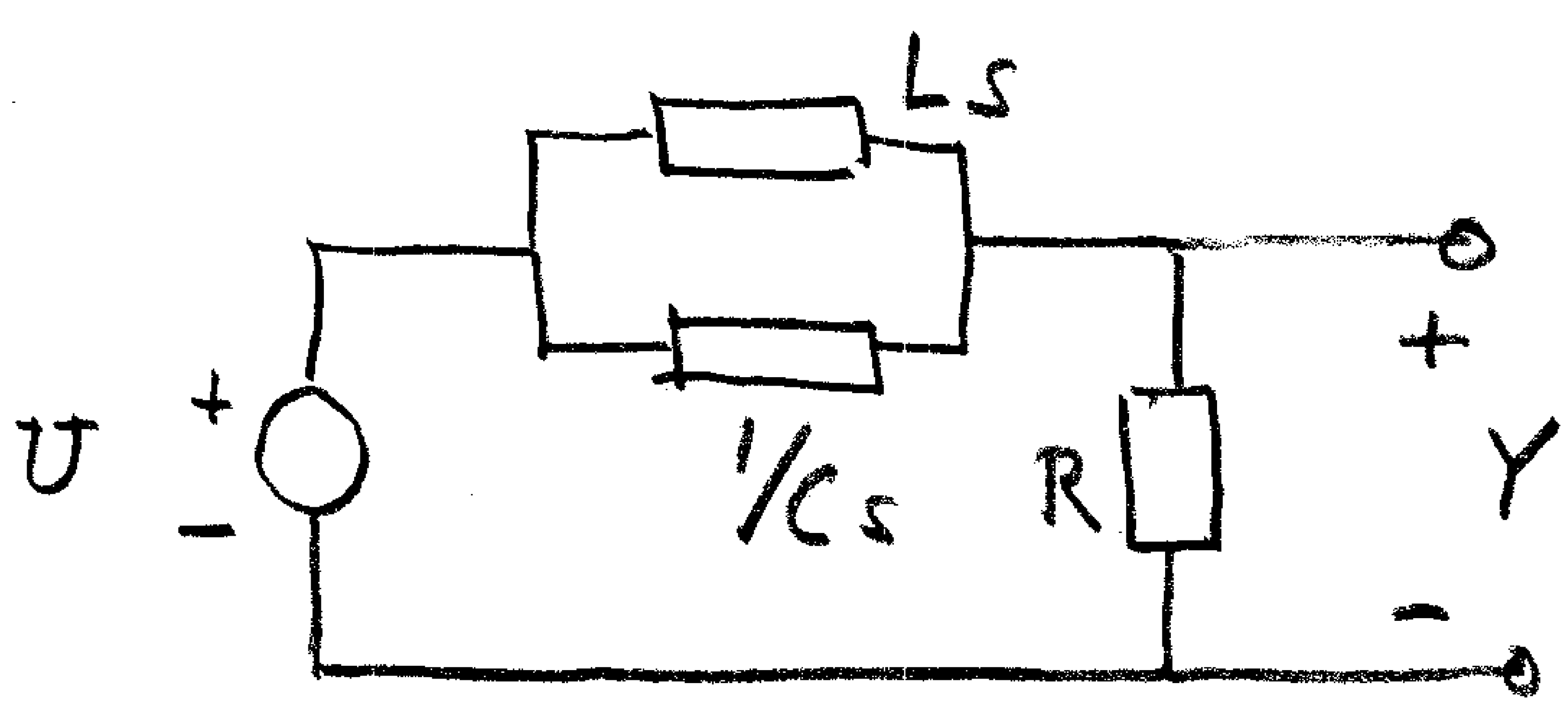
$$G(s) = C(sI-A)^{-1}B + D$$

$$= \frac{-s/RC}{s^2+s/RC+1/LC} + 1$$

$$= \frac{s^2+1/LC}{s^2+s/RC+1/LC}$$

$$\boxed{G(s) = \frac{s^2+1/LC}{s^2+s/RC+1/LC}}$$

2. We can also find $G(s)$ by impedance methods. Redraw the circuit:



The inductor and capacitor are in parallel. The combined impedance is

$$Ls \parallel \frac{1}{Cs} = \frac{(Ls)(1/Cs)}{Ls + 1/Cs}$$

$$= \frac{Ls}{LCs^2+1}$$

With this impedance, the circuit becomes a voltage divider:

$$Y = \frac{R}{R + \frac{Ls}{LCs^2+1}} \cdot v$$

$$= \frac{RLCs^2 + R}{RLCs^2 + R + Ls} \, v$$

$$= \underbrace{\frac{s^2 + 1/LC}{s^2 + \frac{s}{RC} + \frac{1}{LC}}}_{G(s)} \, v$$

So we get the same G(s) as before.

3. For $L = 1\,H$, $C = 0.25\,F$, $R = 10\,\Omega$, the transfer function is

$$G(s) = \frac{s^2 + 4}{s^2 + 0.4s + 4}$$

For sinusoidal input, we can write

$$u(t) = \cos \omega t = \text{Real}\left[e^{j\omega t}\right]$$

Thus,

$$U = 1$$
$$s = j\omega$$

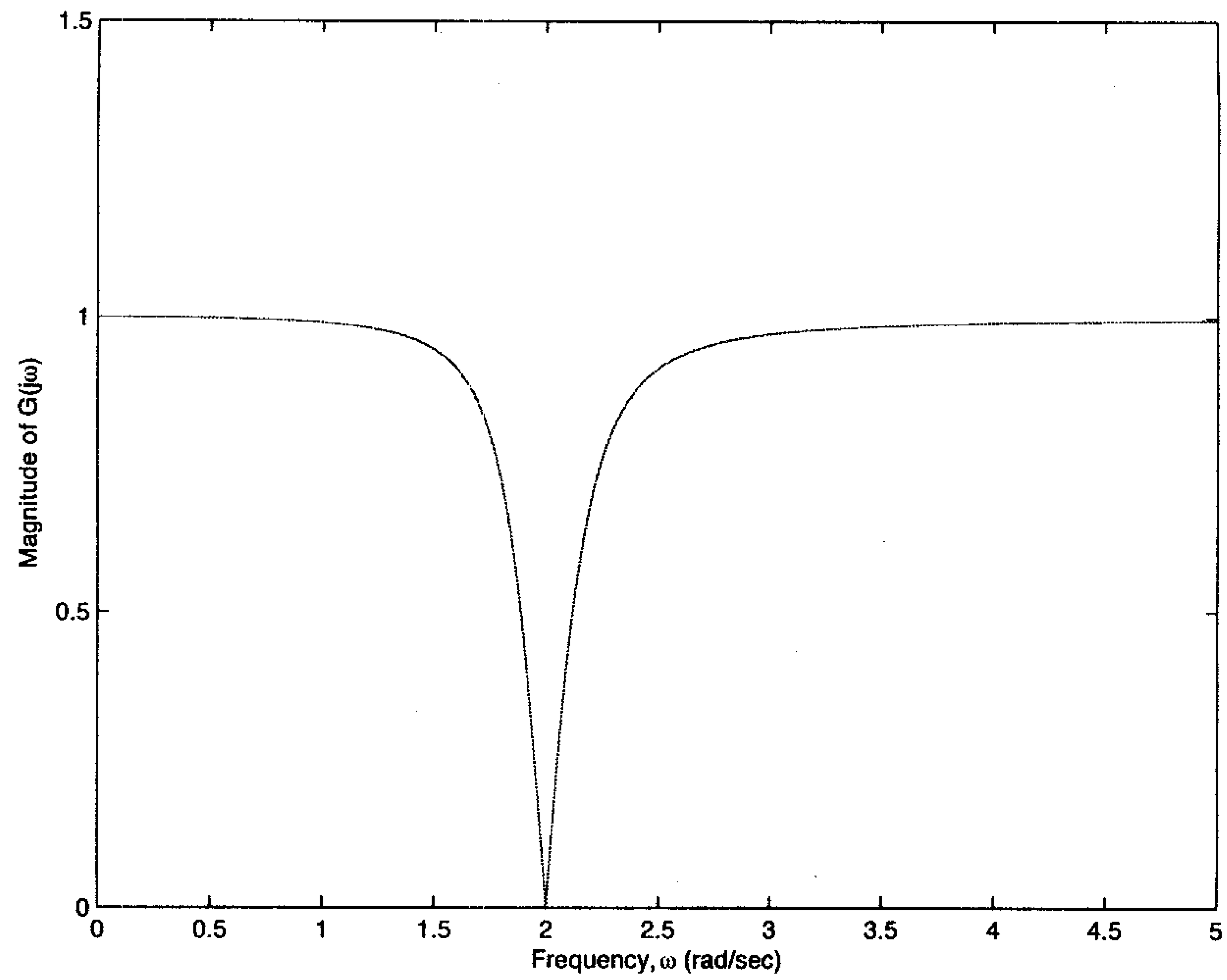
So the ratio of output to input amplitudes is

$$|G(j\omega)| = \left|\frac{-\omega^2 + 4}{-\omega^2 + 0.4j\omega + 4}\right|$$

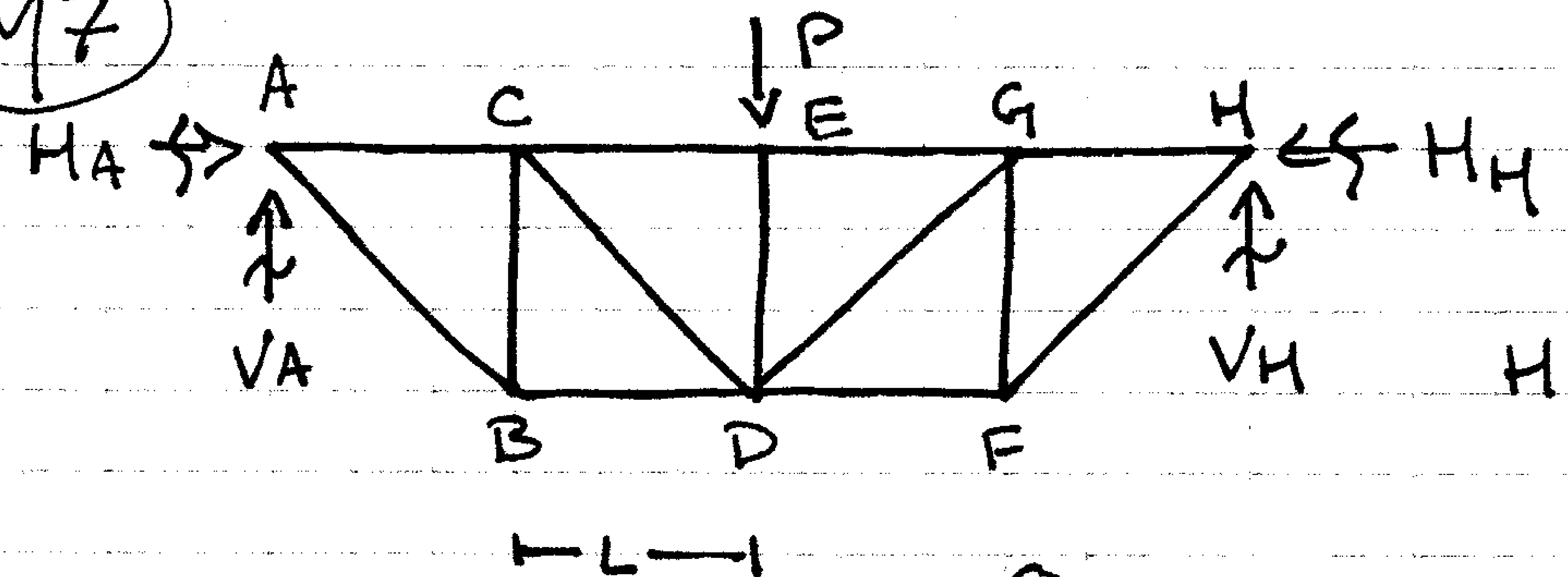
This transfer function magnitude can be plotted by hand, or by using, say, Matlab. My Matlab code is below:

```
>> w = 0:.001:5;
>> G = (-w.^2+4)./(-w.^2+0.4j*w+4);
>> plot(w,abs(G))
>> axis([0 5 0 1.5]); ylabel('Magnitude of G(j\omega)'); xlabel('Frequency, \omega (rad/sec)');
>> print -depsc notch.eps
```

The resulting plot is on the next page. You can see why it is called a notch filter — the plot has a notch at the resonant frequency.


1.5
1
0.5
0
Magnitude of G(jω)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency, ω (rad/sec)

M7



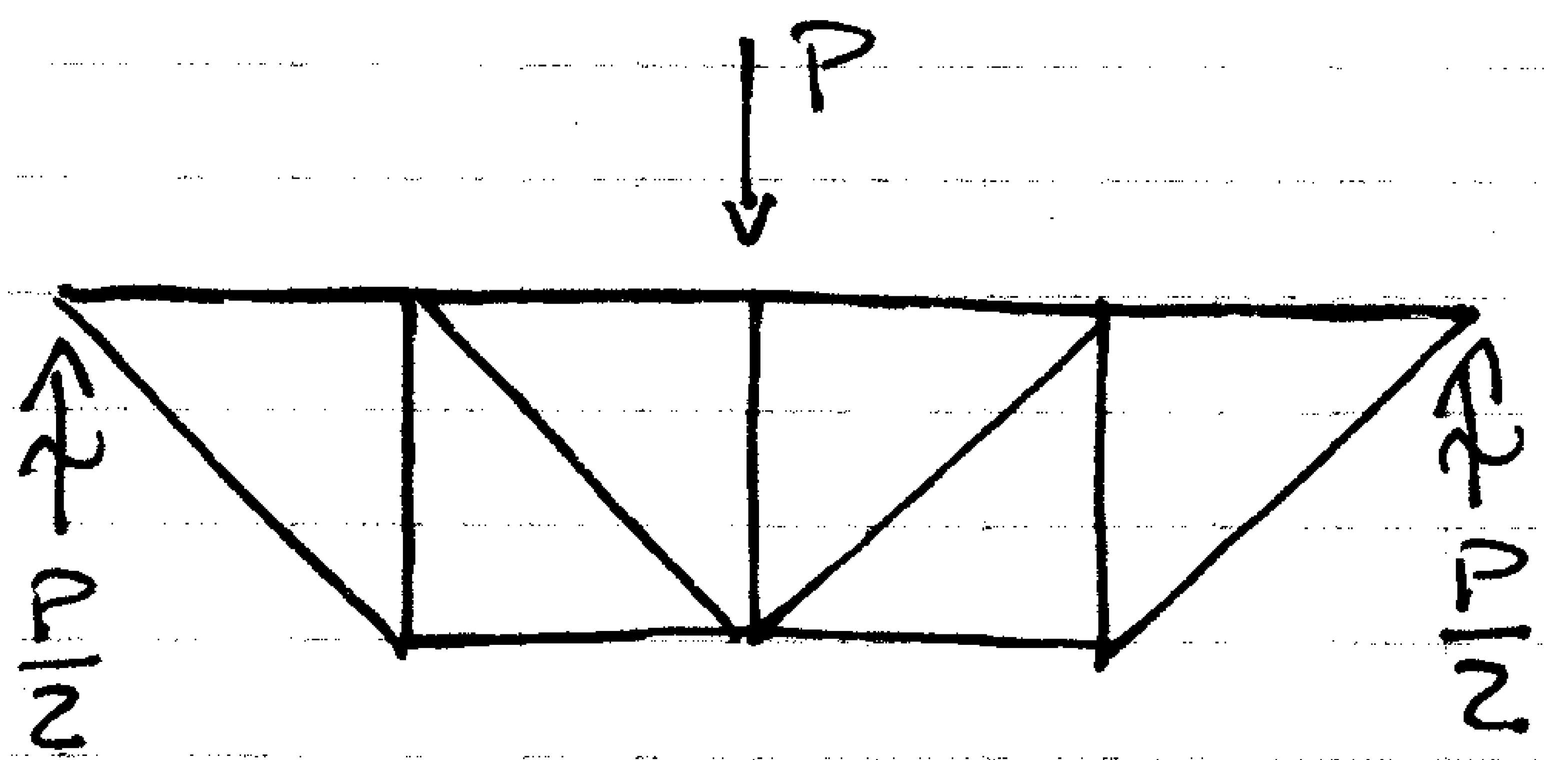
$H_H = 0$

$\sum \vec{F}_x = 0 \quad H_A + \cancel{H_H}^{\,0} = 0 \quad H_A = 0$

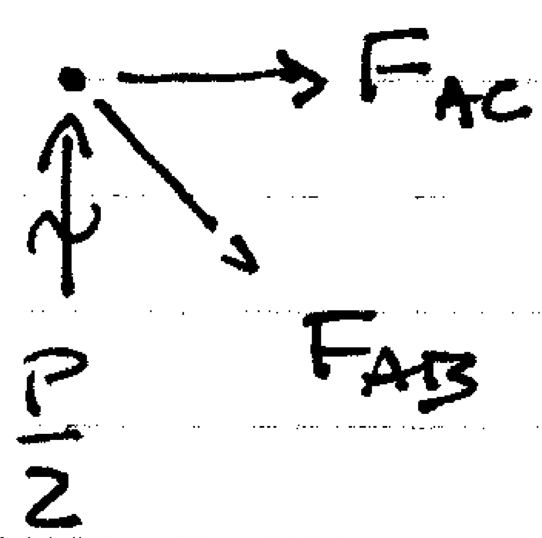
$\sum \vec{F}_y = 0 \quad V_A + V_H - P = 0$

$\sum M_A = 0 \quad V_H(4L) - P(2L) = 0$

$V_H = \frac{P}{2} \quad \& \quad V_A = \frac{P}{2}$ (also from symmetry)



Using the method of joints @ A:



$\sum F_y \uparrow = 0$

$\frac{P}{2} - F_{AB} \cos 45° = 0$

$F_{AB} = \frac{\sqrt{2}\,P}{2}$

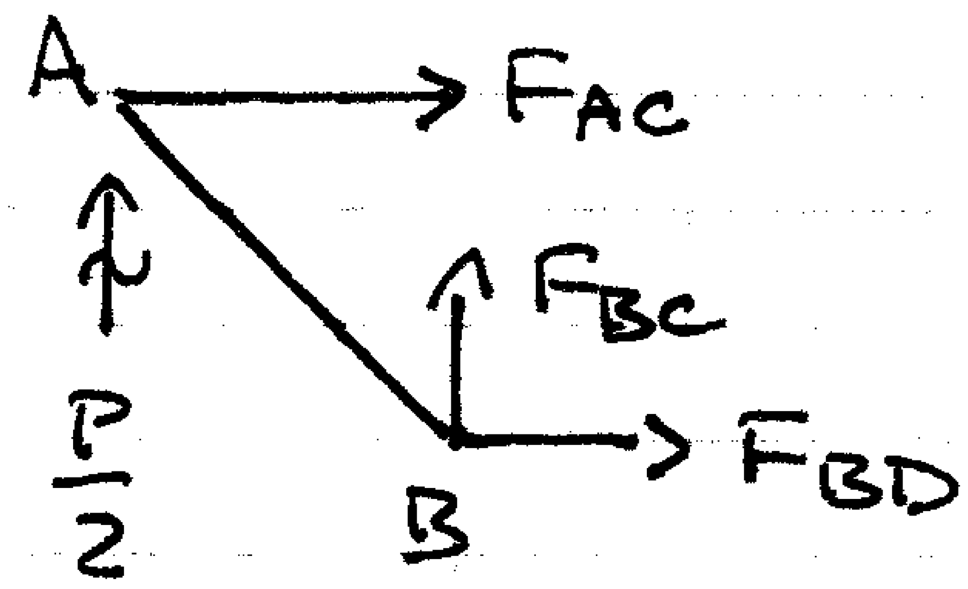
$\sum \vec{F}_x = 0$

$F_{AC} + \frac{\sqrt{2}\,P}{2} \cos 45° = 0$

$F_{AC} = \frac{-P}{2}$

→

Method of Sections :



$$\Sigma F_y \uparrow = 0$$

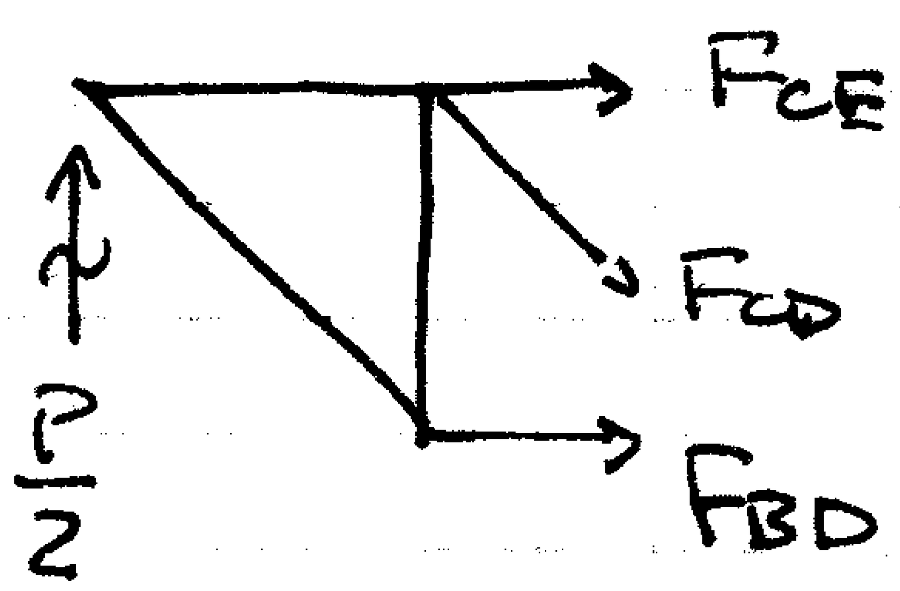
$$F_{BC} + \frac{P}{2} = 0$$

$$F_{BC} = \frac{-P}{2}$$

Method of Sections : $\circlearrowright \Sigma M_C = 0$

$$-\frac{P}{2}(L) + F_{BD}(L) = 0$$

$$F_{BD} = \frac{P}{2}$$



$$\Sigma F_y \uparrow = 0 \qquad \frac{P}{2} - F_{CD} \sin 45^\circ = 0$$

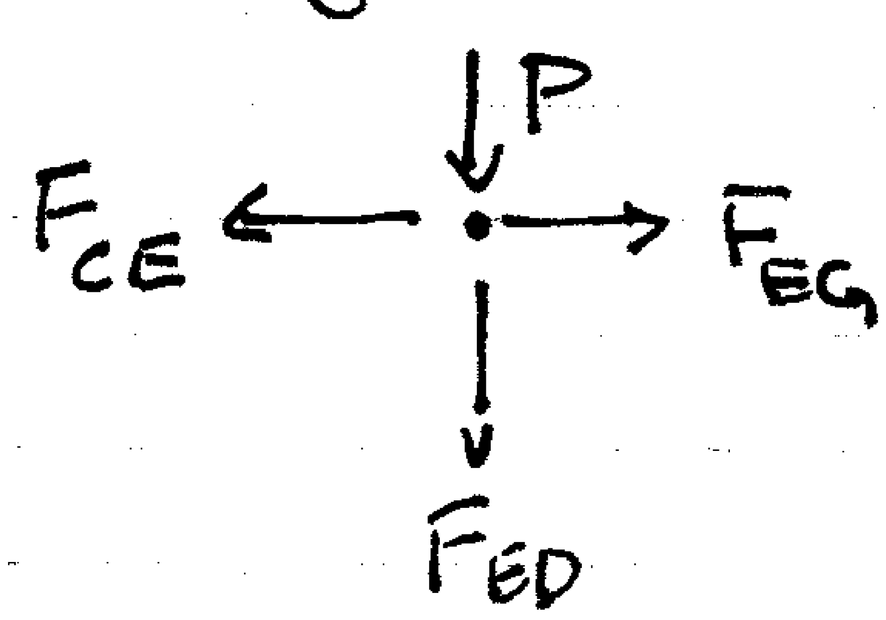
$$F_{CD} = \frac{P}{2}\sqrt{2} = \frac{\sqrt{2}P}{2}$$

$$\circlearrowright \Sigma M_D = 0$$

$$-\frac{P}{2}(2L) - F_{CE}(L) = 0$$

$$F_{CE} = -P$$

Method of Joints @ E :



$$\Sigma F_y \uparrow = 0$$

$$-P - F_{ED} = 0$$

$$F_{ED} = -P$$

By symmetry all other bar forces are the same on the right hand side.

Bar	Force
AB	$\frac{\sqrt{2}}{2} P$
AC	$-P/2$
BC	$-P/2$
CE	$-P$
CD	$\frac{\sqrt{2}}{2} P$
BD	$P/2$
ED	$-P$
EG	$-P$
DG	$\frac{\sqrt{2}}{2} P$
DF	$P/2$
GF	$-P/2$
GH	$-P/2$
FH	$\sqrt{2}P/2$