Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Fall 2004

Problem Set #10

Due Date: Tuesday, November 16, 2004 at 5pm

	Time Spent (minutes)
M9	
M10	
M11	
F5	
F6	
F7	
Study Time	

Name: ________________

Unified Engineering Problem Set #10 **Units M2.2, M2.3**
Fall, 2004

10(M).1 Consider the stress equations of equilibrium.

(a) Write these equation in engineering notation.

(b) Reduce the full three-dimensional equations to their plane stress form.

10(M).2 Write out the succinct tensor equation that describes the following notation:

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} = \begin{bmatrix} B_{111} & B_{122} & 2B_{112} \\ B_{211} & B_{222} & 2B_{212} \\ B_{311} & B_{322} & 2B_{312} \end{bmatrix} \begin{Bmatrix} C_{11} \\ C_{22} \\ C_{12} \end{Bmatrix}$$

10(M).3 For each of the following two-dimensional displacement fields (no displacement in the x_3- direction), draw a neat sketch of a unit square (with the bottom left corner at the origin) before and after deformation (exaggerate deformation by a factor of 10). Calculate the associated two-dimensional strain field. Identify the "type" of deformation/strain that characterizes the displacement.

(a) $\underline{u} = (0.015\, x_1)\, \underline{i}_1 - (0.030\, x_2)\, \underline{i}_2$

(b) $\underline{u} = (0.030\, x_2)\, \underline{i}_1 + (0.020\, x_1)\, \underline{i}_2$

(c) $\underline{u} = (0.030)\, \underline{i}_1 - (0.015)\, \underline{i}_2$

(d) $\underline{u} = (0.040\, x_2)\, \underline{i}_1 - (0.040\, x_1)\, \underline{i}_2$

(e) $\underline{u} = (0.060\, x_1 - 0.040\, x_2)\, \underline{i}_1 + (-\, 0.040\, x_1 - 0.020\, x_2)\, \underline{i}_2$

(Adapted from Anderson Problem 10, page 82).

A Learjet flies at speed $V_1 = 250\,\mathrm{m/s}$, at an altitude of 10 km, where the atmospheric properties are $\rho_1 = 0.414\,\mathrm{kg/m^3}$, $T_1 = 223\,\mathrm{K}$.

a) Determine the ambient pressure p_1 and the flight Mach number M_1.

b) The flow about the Learjet is to be simulated using a 1/5 scale model in a wind tunnel, operated at sea-level atmospheric pressure $p_2 = 10^5\,\mathrm{Pa}$, Determine the operating conditions V_2, ρ_2, T_2 which are required if the flow about the model is to correctly represent the flow about the actual Learjet.

Note:
The speed of sound and viscosity of air depend only on the temperature. Suitable formulas are given below, with T in Kelvin.

$$\begin{aligned} a &= 20\,T^{1/2} \qquad (\mathrm{m/s}) \\ \mu &= 10^{-6}\,T^{1/2} \qquad (\mathrm{kg/m\cdot s}) \end{aligned}$$

The equation of state for air is

$$\begin{aligned} p &= \rho R T \\ \text{where} \qquad R &= 287\,\mathrm{J/kg\cdot K} \end{aligned}$$

Consider the 2-D velocity field $\vec{V} = u\hat{\imath} + v\hat{\jmath}$

$$u = x \qquad\qquad v = -y$$

whose hyperbolic-shape streamlines are sketched below. The density ρ is everywhere constant.

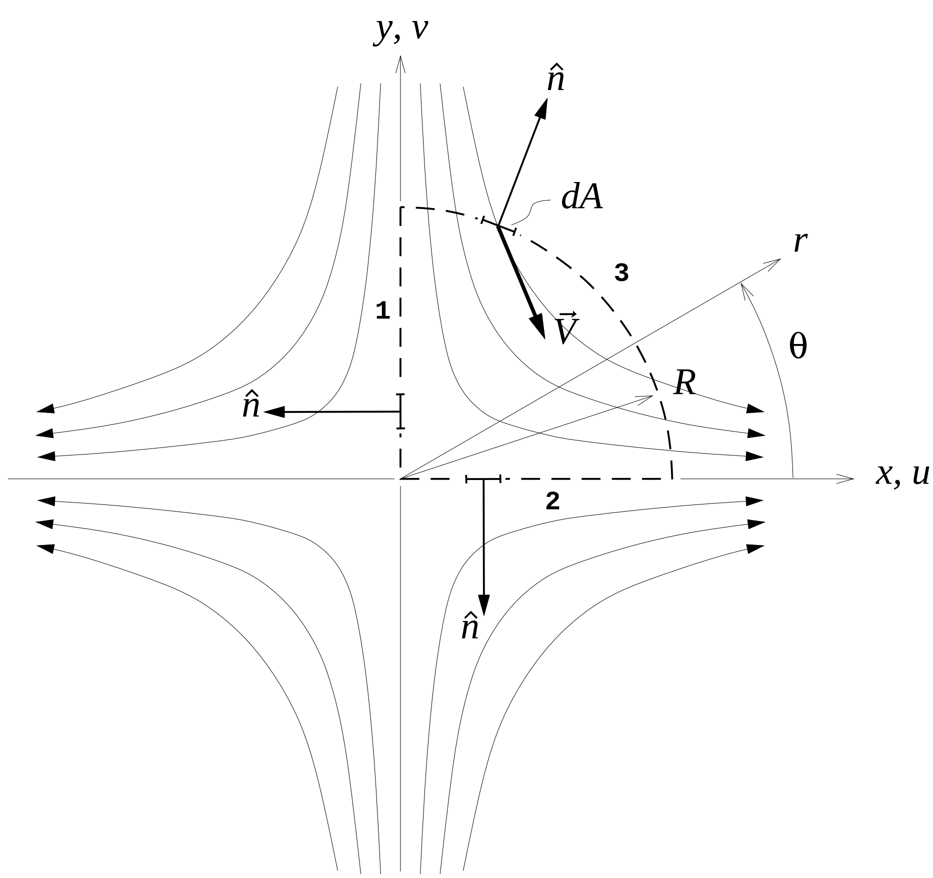
a) Evaluate the mass-flow integral

$$I \;=\; \oint \rho\,(\vec{V}\cdot\hat{n})\,dA$$

for the pie-slice control volume of radius R shown in the figure.

Hint: First express I as three separate integrals I_1, I_2, I_3 over each of the three segments of the C.V. boundary, and simplify where possible. Also, I_3 is most easily evaluated by first writing $\vec{V}$, $\hat{n}$, and dA as functions of the polar coordinates r, θ. Note that dA is a length in 2-D.

b) Does this flow satisfy the mass conservation law? Explain.



Unified Engineering Fall 2004
Fluids Problem F7

Air flows into a channel of height h with a linear velocity distribution.

$$u_1(y) \;=\; \bar{u}_1 \, \frac{y}{h}$$

The air then mixes in the channel into a uniform velocity u_2 at the outlet. The density ρ is constant everywhere. There is a uniform pressure p_1 over the inlet plane, and a uniform pressure p_2 over the outlet plane.

a) Apply the mass conservation integral to the dashed control volume shown in the figure,

$$\oint \rho \, (\vec{V} \cdot \hat{n}) \, dA \;=\; 0$$

and thus determine the exit velocity u_2 in terms of the known inlet maximum velocity $\bar{u}_1$.

b) Apply the momentum conservation integral to the dashed control volume,

$$\oint \left[\rho \, (\vec{V} \cdot \hat{n}) \, \vec{V} \;+\; p\hat{n} \right] dA \;=\; 0$$

and thus determine the pressure difference $p_2 - p_1$ in terms of ρ and $\bar{u}_1$. State whether the mixing causes the pressure to increase or decrease downstream.