Massachusetts Institute of Technology
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Unified Engineering
Fall 2004

Problem Set #11

Due Date: Tuesday, November 23, 2004 at 5pm

	Time Spent (minutes)
F8	
F9	
F10	
F11	
M12	
M13	
Study Time	

Name: ________________

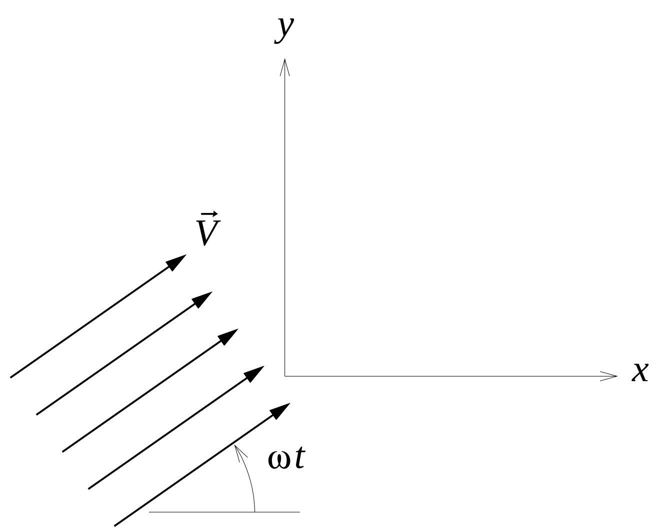
Unified Engineering Fall 2004
Fluids Problem F8

a) A velocity field has a constant unit magnitude and is uniform in space, but rotates in direction at a steady rate ω.

$$\vec{V}(x, y, t) = \vec{V}(t) \; = \; \hat{\imath} \cos \omega t \; + \; \hat{\jmath} \sin \omega t$$

a) Determine the pathline of a particle emitted at the origin $x, y = (0, 0)$ at time $t = 0$.

b) For the time interval $t = 0 \ldots \pi$, sketch the three pathlines corresponding to $\omega = 2$, $\omega = 1$, and $\omega = 0$



An airfoil flying above the ground has an aerodynamic force/span of $\vec{R}' = 0\hat{\imath} + L\hat{\jmath}$ acting on it. The airfoil is sufficiently close to the ground so that the airfoil's pressure field on the ground is significant. The figure shows the situation in the airfoil's frame of reference. The airfoil then appears stationary, and there's an apparent freestream velocity V_∞ opposite to the airfoil's motion.

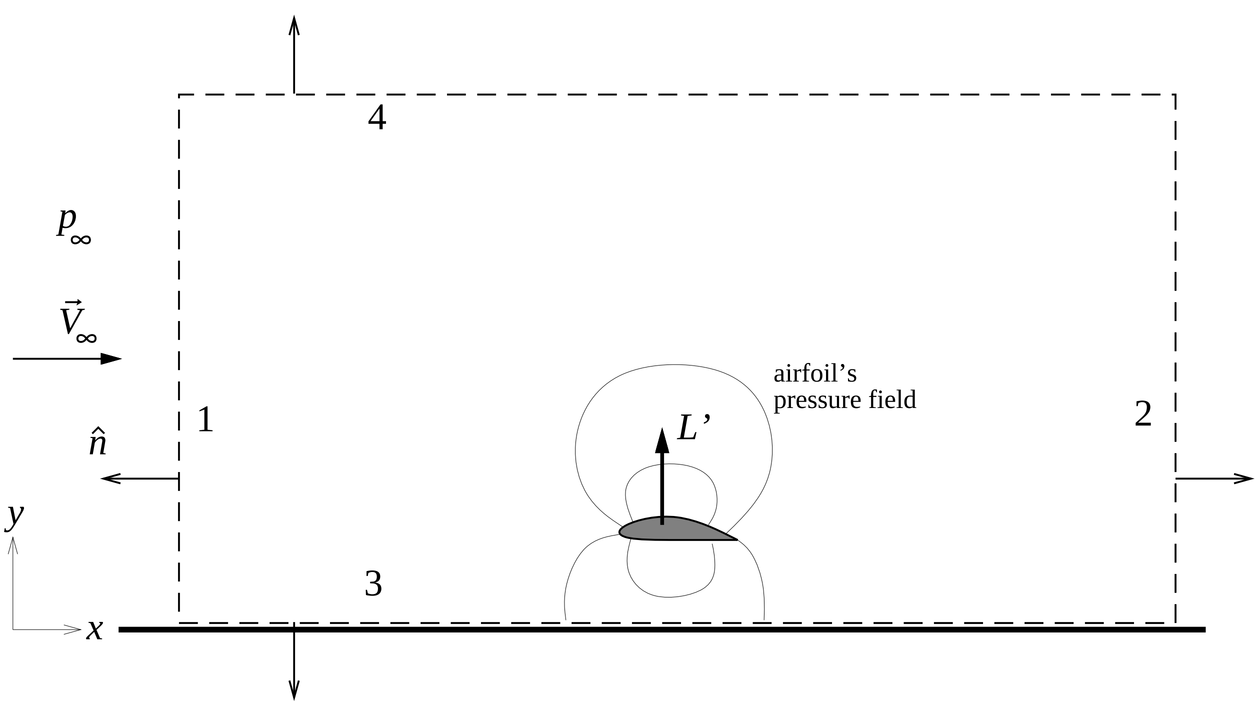
To analyze this problem, a rectangular control volume is drawn as shown. The y component of the integral momentum relation for this control volume is

$$\oint \rho \left(\vec{V}\cdot\hat{n}\right) v \, dA \; + \; \oint p \, \hat{n} \cdot \hat{\jmath} \, dA \; = \; -L' \qquad (*)$$

where $\vec{V} = \hat{\imath}u \, + \, \hat{\jmath}v$ as usual.

a) Each of the two integrals above can be broken up into four separate integrals for the four faces 1,2,3,4 (a total of eight integrals). To simplify the problem, we put faces 1,2,4 very far from the airfoil. Determine which of the eight integrals than then be neglected. Rewrite equation (*) above with only the significant parts remaining.

b) The airfoil's *overpressure* field $p - p_\infty$ imparts a force F' on the ground plane. Determine the magnitude and direction of this force.



A water rocket nozzle has an area distribution given by

$$\begin{aligned} A(x) &= A_0 + (A_1 - A_0)\frac{x}{\ell} \\ A_0 &= 40\,\mathrm{cm}^2 \\ A_1 &= 4\,\mathrm{cm}^2 \\ \ell &= 10\,\mathrm{cm} \end{aligned}$$

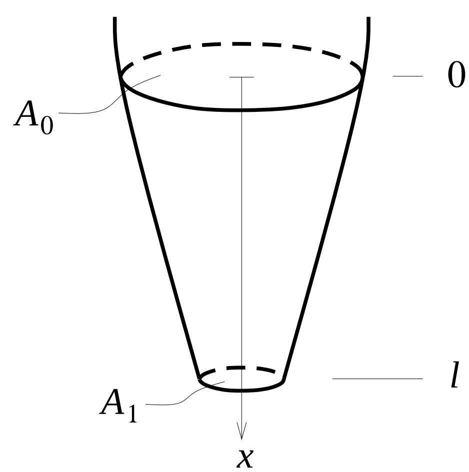
The rocket is partially filled with water ($\rho = 1000\,\mathrm{kg/m^3}$), and the air above the water is pressurized to a gauge pressure of $\Delta p = 4.5 \times 10^5\,\mathrm{Pa}$ (64 psi). According to Bernoulli, this produces a water exit velocity of

$$V_1 \equiv V(\ell) = \sqrt{2\,\Delta p/\rho} = 30\,\mathrm{m/s}$$

a) Using the mass continuity equation for a channel, determine and plot the water's velocity distribution $V(x)$ along the nozzle, for $x = 0 \ldots \ell$.

b) Determine and plot the water's acceleration $a(x)$ along the nozzle. Specify your units.

c) Is the gravitational acceleration significant here?



Unified Engineering Fall 2004
Fluids Problem F11

Consider two flows with circular streamlines.

$$\text{Flow A:} \qquad u = y \qquad\qquad v = -x$$

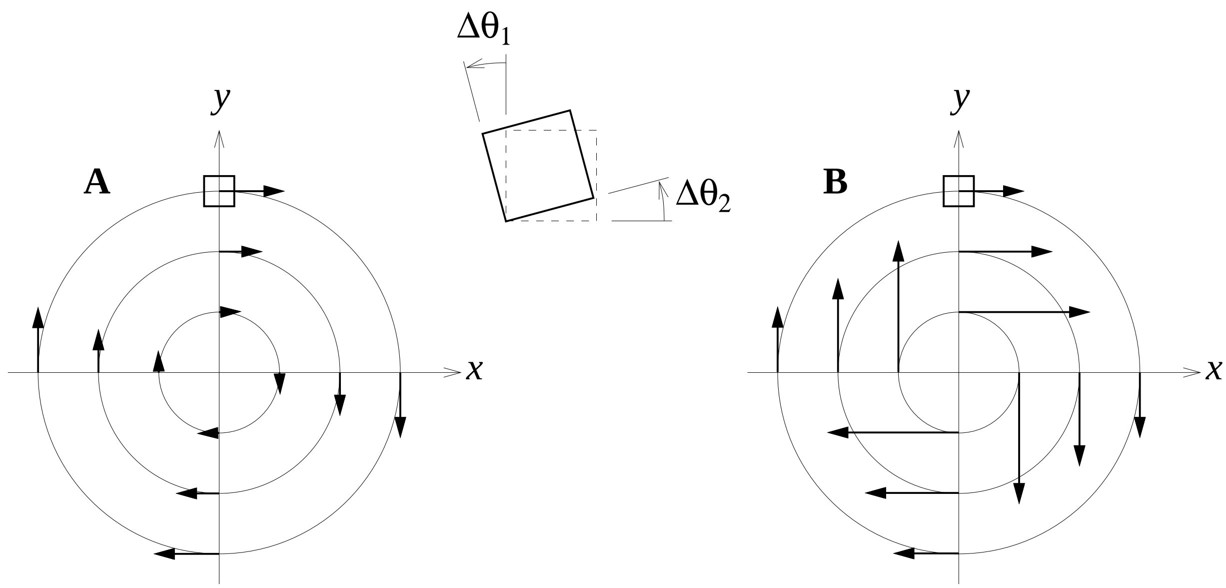
$$\text{Flow B:} \qquad u = \frac{y}{x^2+y^2} \qquad\qquad v = \frac{-x}{x^2+y^2}$$

A small square fluid element is placed at the point $x, y = (0, 1)$ in each flow at time $t = 0$. This element then moves with the flow and possibly distorts.

a) For flow A, determine the angles of the two sides $\Delta\theta_1$ and $\Delta\theta_2$ at some small Δt later, and sketch the new fluid element shape.

b) For flow B, determine the angles of the two sides $\Delta\theta_1$ and $\Delta\theta_2$ at some small Δt later, and sketch the new fluid element shape.

c) Determine whether each flow is rotational or irrotational.

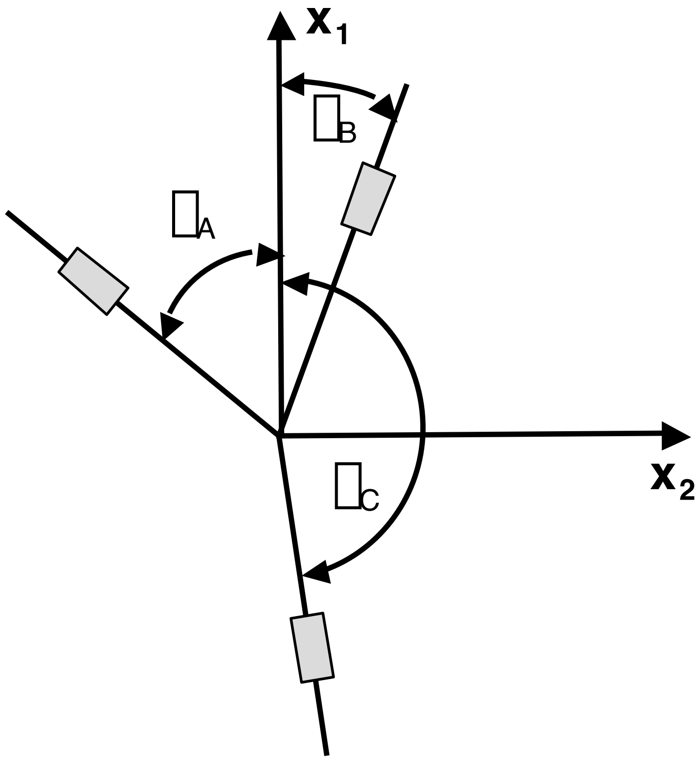


Unified Engineering Problem Set #11 **Units M2.3, M2.4**
Fall, 2004

M12 A strain gage rosette is a set of strain gages (generally three) placed at specified angles on the surface of a material/structure. The measurement of elongational strain in three independent directions on the surface allows complete characterization of the in-plane strains.

(a) Explain in words and with a few equations, as needed, why this is possible.

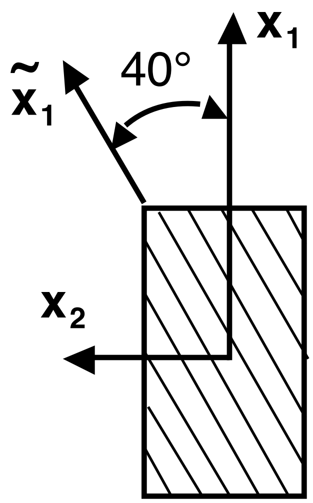
(b) If the three strain gages are oriented at three angles (θ_A, θ_B, θ_C) as illustrated below, show how one would determine the in-plane strain state in the loading axes (x_1, x_2) based on the measured strains from the three gages (ε_A, ε_B, ε_C). Use equations as needed but DO NOT ACTUALLY SOLVE FOR THE IN-PLANE STRAINS.



M13 A structure made of a unidirectional composite material has all its fibers oriented at an angle of 40° relative to the loading axes. The structural loading produces the following state of plane stress in the loading axis system:

$$\sigma_{11} = 30 \text{ MPa}$$
$$\sigma_{22} = -20 \text{ MPa}$$
$$\sigma_{12} = 15 \text{ MPa}$$

This situation is illustrated in the accompanying figure.



(a) Find the stress state in the "composite fiber axes". These axes are defined by aligning the 1-direction along the fiber direction.

(b) Determine the principal stresses and the associated directions.

(c) Find the maximum shear stresses and their associated planes.

(d) Draw the Mohr's circle for this situation and check the answers to parts b and c.