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# Unified Engineering Fall 2004 <br> Problem Set \#12 

Due D ate: Tuesday, November 30, 2004 at 5pm

|  | Time <br> Spent <br> (minutes) |
| :--- | :--- |
| F12 |  |
| F13 |  |
| F14 |  |
| M14 |  |
| M15 |  |
| Study <br> Time |  |

A flow is defined by the following velocity potential function:

$$
\phi(x, y)=\frac{1}{2}\left(x^{2}-y^{2}\right)
$$

a) Does this flow satisfy mass conservation?
b) Determine the velocity components, and sketch the streamlines in the upper right $x, y$ quadrant.
c) Determine the stream function $\psi(x, y)$ which describes the same flow.
d) Determine the mass flow passing through a line connecting $x, y=(0,0)$ and $(1,1)$. i.e.

$$
\dot{m}=\int_{0,0}^{1,1} \rho \vec{V} \cdot \hat{n} d A
$$

Assume $\rho=1$.

The tunnel in the Bldg 33 hangar has a centrifugal blower, followed by a conical diffuser, followed by a large constant-area stilling section roughly $6 \times 6 \mathrm{ft}$ in cross-section, followed by a contraction into the $1 \times 1 \mathrm{ft}$ open-jet test section. The ambient atmospheric pressure in the hangar is $p_{\text {atm }}=10^{5} \mathrm{~Pa}$. The air density is approximately $\rho=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ everywhere.
a) What do expect the difference $p_{\text {test }}-p_{\text {atm }}$ to be? Explain your reasoning.
b) The tunnel has a practical maximum velocity of about $V_{\text {test }}=45 \mathrm{~m} / \mathrm{s}$ (about 100 mph ). Determine the stilling-section pressure rise $p_{1}-p_{\text {atm }}$ that the blower/diffuser combination must supply in this case. Hint: First relate $V_{1}$ and $V_{\text {test }}$.
c) The stilling section is about 15 ft long (not to scale on the drawing). Determine the force on one of its walls in this case.

a) A flow in a channel has a simple linear velocity profile

$$
u=C y \quad v=0
$$

where $C$ is some constant. Determine the stream function $\psi(x, y)$ for this flow.
b) Can this flow be given in terms of a velocity potential $\phi(x, y)$ ? Explain.

c) The channel is now given a height constriction, from $h$ to $h / 2$. It is hypothesized that the flow after the constriction is

$$
u=2 C y \quad v=0
$$

Is this proposed flow physically possible? Examine using mass conservation, and the Helmholtz equation.


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M14 For the following cases, briefly (in one or two sentences) state the primary functional requirement that has to be met, the loads (e.g. tension, compression, shear, impact, cyclic, thermal, electrical), and list the five material properties that you think will be most relevant to meeting this requirement (confine your choices to the list give in Table 1.1 of Ashby and Jones or other properties discussed in class). Indicate for each property whether it should have a high, medium, or low value.
(a) Components of a space truss
(b) Components of a truss of a radio tower
(c) Kitchen countertop
(d) Electrical wires strung on telephone poles
(e) Reentry shield on the space shuttle

M15 A bar of a constant length has a solid circular cross-section and must carry a constant load P. This bar might be used for a variety of purposes but such are currently not specified. The design criteria at this point are that the bar is to deform as little as possible and weigh as little as possible. Cost is also a consideration. The design variables are the bar diameter and the material used to make the bar.
(a) Determine the figure(s) of merit that is/are pertinent in this case.
(b) For the materials listed in the accompanying table, indicate which you would choose for the bar depending upon which of the three design considerations are most important: minimization of deformation, minimization of weight, minimization of cost. Be sure to clearly explain your reasoning. Use figures as appropriate.

| Material | Density <br> $\left[\mathrm{lb} / \mathrm{in}^{3}\right]$ | Modulus <br> $[\mathrm{Msi}]$ | Acquisition <br> Cost, $[\$ / \mathrm{lb}]$ |
| :--- | :---: | :---: | :---: |
| Steel (low carbon) | 0.285 | 30.3 | 1.05 |
| Aluminum alloy (2000 series) | 0.101 | 10.4 | 5.25 |
| Titanium alloy (TI-6Al-4V) | 0.162 | 17.8 | 19.50 |
| Wood | 0.022 | 1.81 | 0.75 |
| Carbon fiber Composite | 0.054 | 24.2 | 65.00 |
| Silicon Carbide | 0.108 | 60.5 | 122.00 |

