Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Fall 2004

Problem Set #13

Due Date: Tuesday, December 7, 2004 at 5pm

	Time Spent (minutes)
F15	
F16	
F17	
M16	
M17	
M18	
Study Time	

Name: ________________

Unified Engineering Fall 2004
Fluids Problem F15

As given in the F14 class notes, the equations and boundary conditions which must hold for an external aerodynamic flow are:

1) at all points in the flowfield: $\nabla^2 \phi = 0$ (mass conservation requirement)

2) at all points on the body surface: $\nabla \phi \cdot \hat{n} \equiv \dfrac{\partial \phi}{\partial n} = 0$ (surface flow tangency requirement)

3) at all points "at infinity": $\dfrac{\partial \phi}{\partial x} = V_\infty$ (consistency with freestream flow)

The flow about a circular cylinder of radius R centered on the origin is supposedly given by the following velocity potential:

$$\phi(r, \theta) \;=\; V_\infty \cos\theta \left(r - \frac{R^2}{r} \right)$$

or equivalently

$$\phi(x, y) \;=\; V_\infty \left(x - \frac{x R^2}{x^2 + y^2} \right)$$

Does this flow meet the necessary equations and boundary conditions 1, 2, 3 ?

Unified Engineering Fall 2004
Fluids Problem F16

1) An airfoil of chord c is flying at some speed V_∞, at a lift coefficient of $c_\ell = 0.8$. What must be the circulation Γ about this airfoil?

2) We can define average upper and lower surface velocities by

$$\bar{V}_u = \frac{1}{s_{u_{\max}}} \int_0^{s_{u_{\max}}} V_u \, ds \qquad \qquad \bar{V}_l = \frac{1}{s_{l_{\max}}} \int_0^{s_{l_{\max}}} V_l \, ds$$

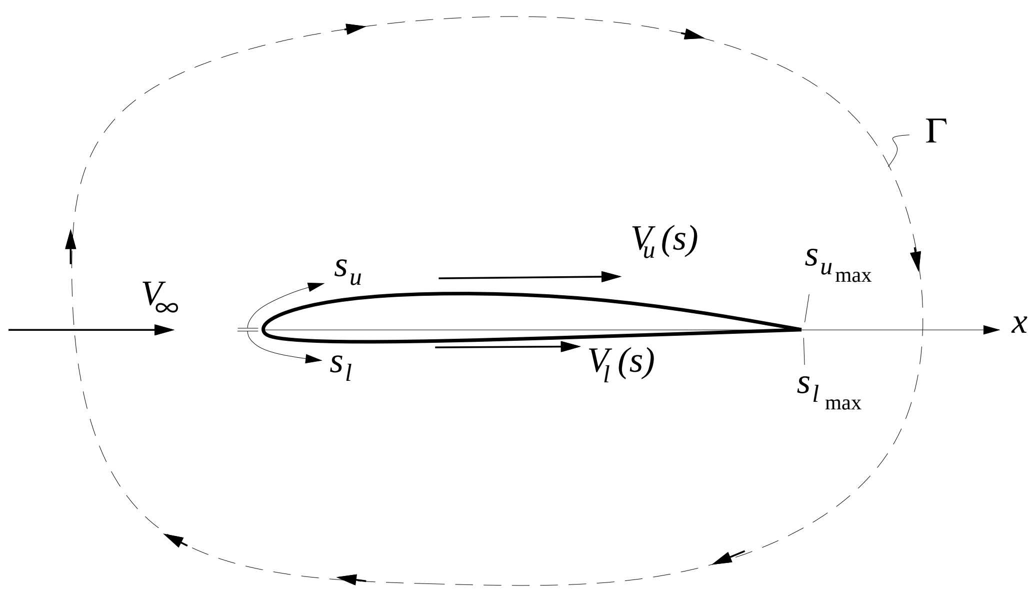
Express Γ about the airfoil in terms of $\bar{V}_u$ and $\bar{V}_l$.

3) For the airfoil in 1), what is the fractional velocity difference $(\bar{V}_u - \bar{V}_l)/V_\infty$?

4) One popular explanation about how lift is created goes something like this:

The upper surface of a cambered airfoil is longer than the bottom surface. Therefore, the upper surface flow velocity must be proportionally faster so it can "keep up" with the lower surface flow. According to Bernoulli, the upper-lower velocity difference implies an opposite pressure difference, which results in lift.

Can this theory be even remotely correct? Using your result from 3), estimate the upper-lower surface length difference which would be required for the modest c_ℓ value.

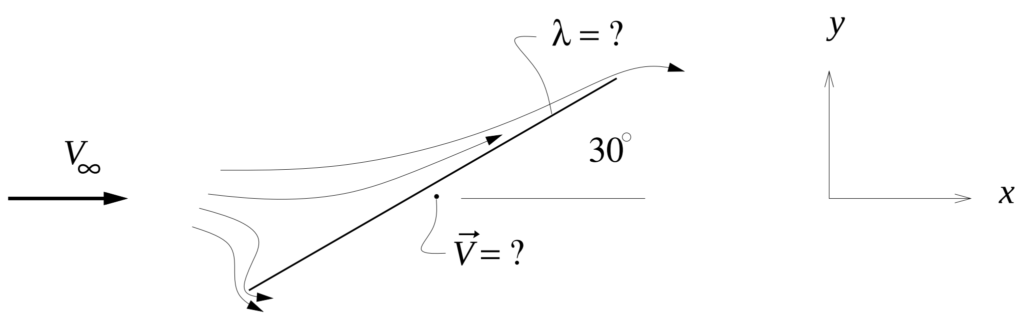


Unified Engineering Fall 2004
Fluids Problem F17

A source panel is set at an angle of 30° from the x axis. There is a freestream velocity V_∞ parallel to the x axis.

1) What must the panel strength λ be set to, so that the flow is tangent to the panel on the upstream face?

2) What is the velocity $\vec{V}$ just behind the panel at its midpoint? Specify both the u and v components. Hint: First determine the normal and tangential components.



Unified Engineering Problem Set #13 **Unit M3.2**
Fall, 2004

M16 Write out in full the tensorial version of the full three-dimensional compliance relations (full anisotropic form). Group the components of the compliance tensor into the three groups (as done for the elasticity tensor in class).

M17 Write the generalized Hooke's law (full anisotropic form) in engineering notation.

M18 A certain polymer material is reinforced with particles and fibers of unknown directions. The material has three experiments performed on it with a single stress applied in each case and various strains measured. The stresses and strains for the three experiments are:

<ins>Experiment A</ins>

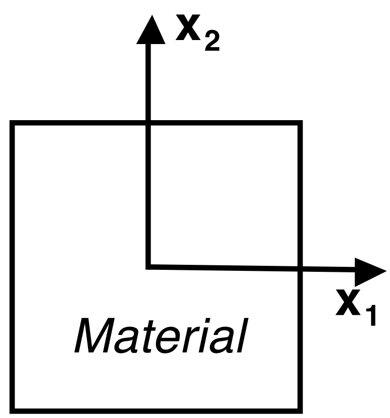
σ_{11} = 125 MPa
ε_{11} = 6220 μstrain
ε_{22} = - 1990 μstrain

<ins>Experiment B</ins>

σ_{12} = 75 MPa
ε_{12} = 4900 μstrain

<ins>Experiment C</ins>

σ_{22} = 100 MPa
ε_{11} = - 1600 μstrain
ε_{22} = 5000 μstrain



Note that any stresses or strain not specified are equal to zero.

(a) Determine the in-plane engineering constants (all possible) and characterize the stress-strain behavior of the material.

(b) Determine as many components of the compliance tensor as possible and put this in matrix form.