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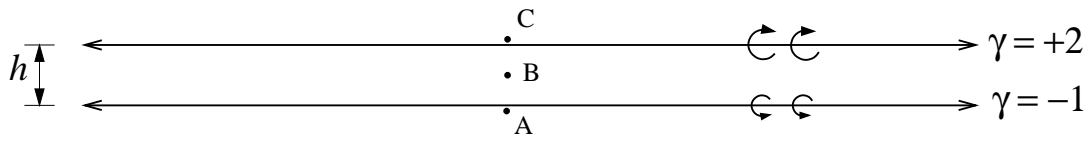
**Unified Engineering**  
**Fall 2004**  
**Problem Set #14**  
**(Optional)**

<b>F18</b>
<b>F19</b>
<b>F20</b>
<b>F21</b>
<b>M19</b>
<b>M20</b>
<b>M21</b>

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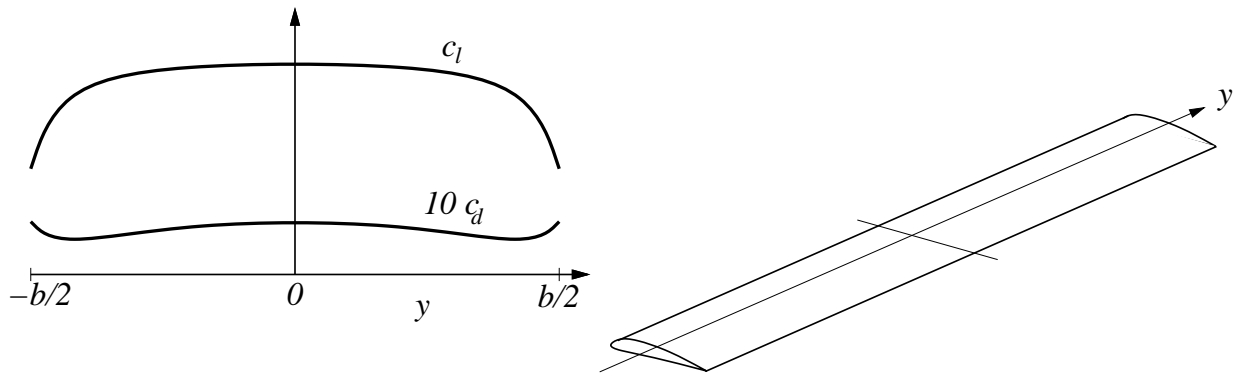
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Two very long vortex panels of strengths  $\gamma = +2$ ,  $\gamma = -1$ , as shown, are located at distance  $h$  apart. Determine the velocities at points A, B, C.



A long rectangular wing has span  $b$  and constant chord  $c$ , and hence the wing area is  $S = bc$ . Because of “tip effects”, the local lift and drag coefficients  $c_\ell(y)$  and  $c_d(y)$  tend to vary across the span. Determine how these local  $c_\ell(y)$  and  $c_d(y)$  distributions relate to the wing’s overall  $C_L$  and  $C_D$ .

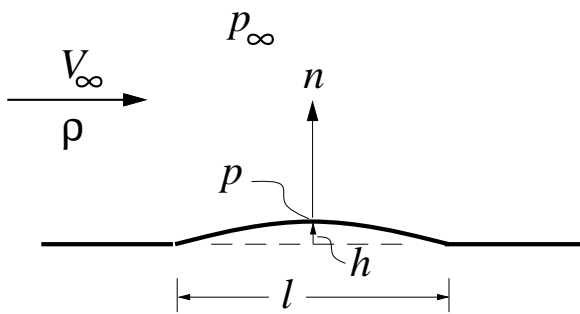
(Hint: Determine  $L'$  and  $D'$ , then get  $L$  and  $D$ , then from these determine  $C_L$  and  $C_D$ ).



(Note: Using only the airfoil’s  $c_d$  ignores other contributions such as induced drag, which become especially significant at low flight speeds!)

A wall with air flowing over it has a shallow parabolic bump of length  $l$  and height  $h$ .

- Determine the curvature  $\kappa$  of the bump, and the resulting normal pressure gradient  $\partial p/\partial n$  over the center of the bump.
- Assume that the pressure gradient extends some height  $\Delta n$  into the flow. What is the resulting overpressure  $\Delta p \equiv p - p_\infty$  at the center of the bump? Specify the sign of  $\Delta p$ .
- Experiments indicate that  $\Delta p$  is very nearly proportional to the bump's  $h/l$  ratio. How must  $\Delta n$  relate to the bump geometry? (i.e. how far does the bump's pressure disturbance reach into the flow?)



a) The flight power needed to overcome drag is

$$P = V_\infty D$$

Derive a formula for  $P$  in level flight, in terms of the following quantities:

$\rho$  air density  
 $W$  aircraft weight  
 $S$  wing area  
 $C_L$  lift coefficient  
 $C_D$  drag coefficient

b) How does flight power vary with air density (e.g. via changed altitude)? Explain your seemingly paradoxical result.

c) If  $C_D$  is reduced by 1%, how much does flight power decrease percentagewise?

d) If  $W$  is reduced by 1%, how much does flight power decrease percentagewise?

**Unified Engineering Problem Set #14**  
**Fall, 2004**

**Units M3.3, M3.4**

- M19** A material has a rectangular crystal lattice. The potential energy of the two atoms in the lattice, a distance  $r$  apart, is :

$$U = -A/(r^m) + B/(r^n)$$

with values of the two exponents of 2 for  $m$  and 10 for  $n$ . It is known that the atoms form a stable pair at a separation of 0.3 nm with an energy of -4 eV.

- (a) Determine the values of the constants  $A$  and  $B$ .
  - (b) Making no further assumptions (i.e. just use this single bond model), estimate the extensional modulus,  $E$ , of the material.
  - (c) Comment on possible discrepancies between the estimated value of the modulus and an actual measured value for the material
- M20** A unidirectional composite material is to be made of graphite fibers with a fiber modulus of 230 GPa and an epoxy matrix with a modulus of 10 GPa. Determine estimates for the composite ply modulus along and perpendicular to the fiber direction as a function of the fiber volume fraction  $v_f$ . Plot these estimates.
- M21** Crystalline aluminum and titanium have face-centered cubic and close-packed hexagonal structures, respectively.
- (a) Assuming that the atoms can be represented as hard spheres, calculate the percentage of the volume occupied by atoms in each material.
  - (b) Starting from basic principles, determine the dimensions of the unit cell for each material. Note that the density of aluminum is  $2.70 \text{ Mg/m}^3$ ; that of titanium is  $4.51 \text{ Mg/m}^3$ .