

Massachusetts Institute of Technology
Department of Aeronautics and
Astronautics
Cambridge, MA 02139

Unified Engineering

Fall 2004

Problem Set #7

Due Date: Tuesday, October 26, 2004 at 5pm

	Time Spent (minutes)
T4	
T5	
T6	
S10	
S11	
S12	
Study Time	

Name: _____

Ian A. Waitz

Problem T4 (Unified Thermodynamics)

Consider a thermodynamic system containing air at $v_1=1 \text{ m}^3/\text{kg}$, $p_1=100 \text{ kPa}$. The system is compressed to $0.5 \text{ m}^3/\text{kg}$ via any one of three quasi-static processes: isobaric, isothermal, or adiabatic. Assume that $c_p = 1.0035 \text{ kJ/kg-K}$, $c_v = 0.7165 \text{ kJ/kg-K}$, and $R = 0.287 \text{ kJ/kg-K}$.
(LO#4, LO#5)

- a) Sketch all three processes on the same p-v diagram.
- b) For each process determine the pressure and temperature at the final state.
- c) For each process determine the work done by the system and the heat transferred to the system.
- d) For each process calculate the change in enthalpy of the air.

Ian A. Waitz

Problem T5 (Unified Thermodynamics)

A thermally-insulated cylinder holds a thermally perfect gas at $p = 4 \text{ atm}$ and $T = 300\text{K}$. The gas is contained by a thermally-insulated massless piston with a stack of many small weights on top of it. The surroundings are at $p = 1 \text{ atm}$ and $T = 300\text{K}$. Initially the system is in mechanical and thermal equilibrium. Consider the following three processes:

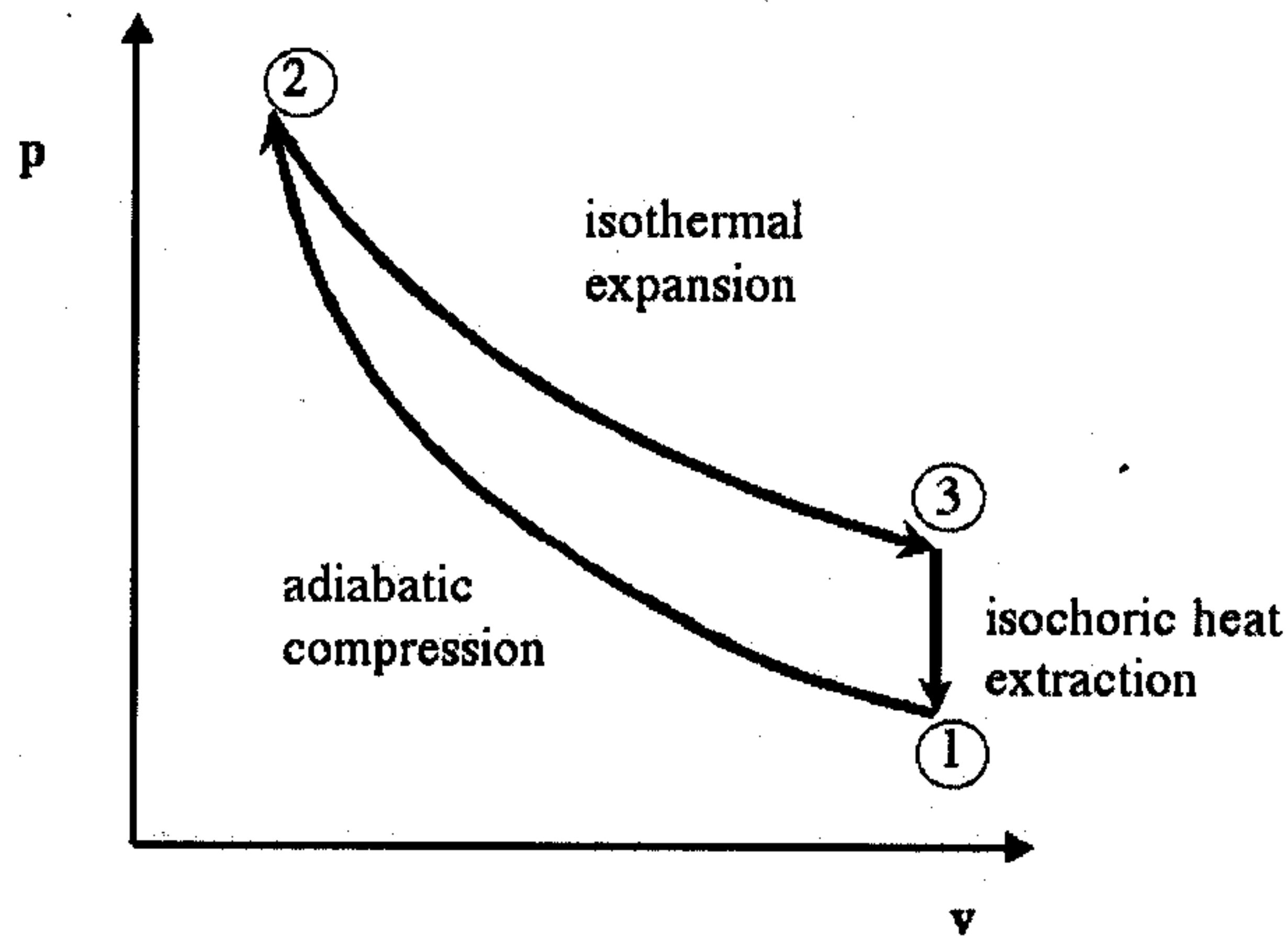
- a) All of the weights are removed from the piston instantaneously and the gas expands until the pressure matches that of the surroundings. How much work was done by the system? In the final state, what is the temperature and pressure of the system? Draw this process on a p-v diagram.
- b) Half of the weight is removed from the piston instantaneously, the system is allowed to come to equilibrium, and then the remaining half of the weight is removed from the piston and the gas expands until the pressure matches that of the surroundings. How much work was done by the system? During the intermediate state and the final state, what is the temperature and pressure of the system? Draw this process on a p-v diagram.
- c) Each small weight is removed from the piston one at a time, so that the pressure inside the cylinder can be assumed always to be in equilibrium with the weight on top of the piston. When the last weight is removed the gas has fully expanded to a pressure that matches that of the surroundings. How much work was done by the system? In the final state, what is the temperature and pressure of the system? Draw this process on a p-v diagram.
- d) If your goal is to get as much work out of a system as possible without adding heat, what type of a process would you use?

Assume that $c_p = 1.0035 \text{ kJ/kg-K}$ and $c_v = 0.7165 \text{ kJ/kg-K}$ are constants, and that $R = 0.287 \text{ kJ/kg-K}$.

Ian A. Waitz

Problem T6. (Thermodynamics)

Consider the following thermodynamic cycle. Assume all processes are quasi-static.

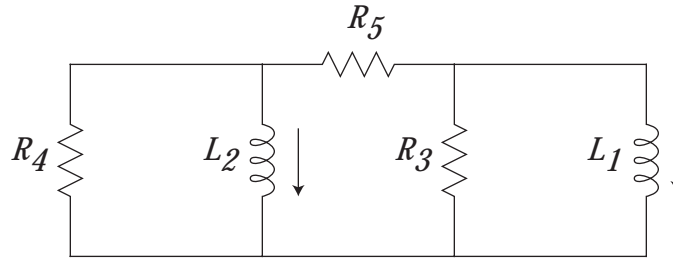


Air undergoes a quasi-static thermodynamic cycle 1-2-3-1 as shown below. Process 1-2 is adiabatic compression, 2-3 is isothermal expansion, and process 3-1 is constant volume heat removal. The conditions at state 1 are $p_1 = 100\text{kPa}$, $T_1 = 300\text{K}$. The compression ratio of the cycle (v_1/v_2) is 8. Assume that $c_p = 1.0035\text{ kJ/kg-K}$ and $c_v = 0.7165\text{ kJ/kg-K}$ are constants, and that $R = 0.287\text{ kJ/kg-K}$.

- For each leg of the cycle identify whether the heat added to the system, Q , and the work done by the system, W , are positive, negative or zero.
- For each leg of the cycle calculate the work and heat transfer, the change in internal energy and the change in enthalpy.
- What is the net work of the cycle?
- What is the thermal efficiency of the cycle?



Problem S10 (Signals and Systems)



Consider the network above, with

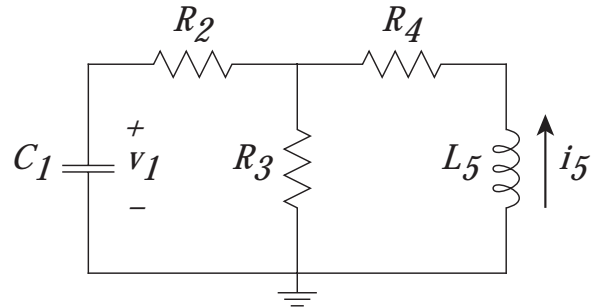
$$L_1 = 1 \text{ H}, \quad L_2 = 1 \text{ H}, \quad R_3 = 1 \text{ } \Omega, \quad R_4 = 1 \text{ } \Omega, \quad R_5 = 0.5 \text{ } \Omega$$

The inductor currents at time $t = 0$ are

$$i_1(0) = 10 \text{ A}, \quad i_2(0) = 0 \text{ A}$$

Find the inductor currents ($i_1(t)$, $i_2(t)$) as a function of time.

Problem S11 (Signals and Systems)



Consider the circuit above, with

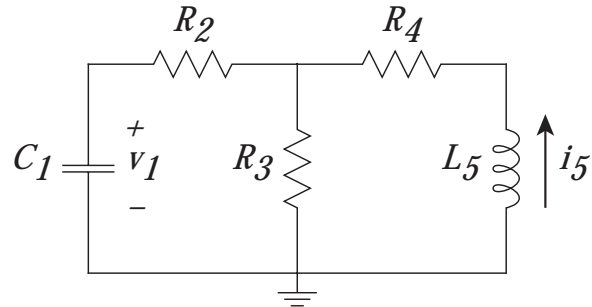
$$C_1 = 0.25 \text{ F}, \quad R_2 = 4 \, \Omega, \quad R_3 = 4 \, \Omega, \quad R_4 = 1 \, \Omega, \quad L_5 = 1 \text{ H}$$

The initial conditions on the capacitor and inductor are

$$v_1(0) = 2 \text{ V}, \quad i_5(0) = 1 \text{ A}$$

Find $v_1(t)$ and $i_5(t)$, using the methods discussed in Lecture. Note: If you wish, you may use the loop method instead of the node method.

Problem S12 (Signals and Systems)



Consider the circuit of Problem S11 above, with

$$C_1 = 0.25 \text{ F}, \quad R_2 = 4 \text{ } \Omega, \quad R_3 = 4 \text{ } \Omega, \quad R_4 = 1 \text{ } \Omega, \quad L_5 = 1 \text{ H}$$

Find the state-space equations that describe the evolution of the circuit, in the form

$$\frac{d}{dt} \underline{x}(t) = A \underline{x}(t)$$

where

$$\underline{x}(t) = \begin{bmatrix} v_1(t) \\ i_5(t) \end{bmatrix}$$

Confirm that the eigenvalues of the matrix A are the same as the characteristic values found in Problem S11.