Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Fall 2004

Problem Set #8

Due Date: Tuesday, November 2, 2004 at 5pm

	Time Spent (minutes)
T7	
T8	
T9	
T10	
S13	
S14	
S15	
Study Time	

Name: ______________

UNIFIED ENGINEERING Fall 2004

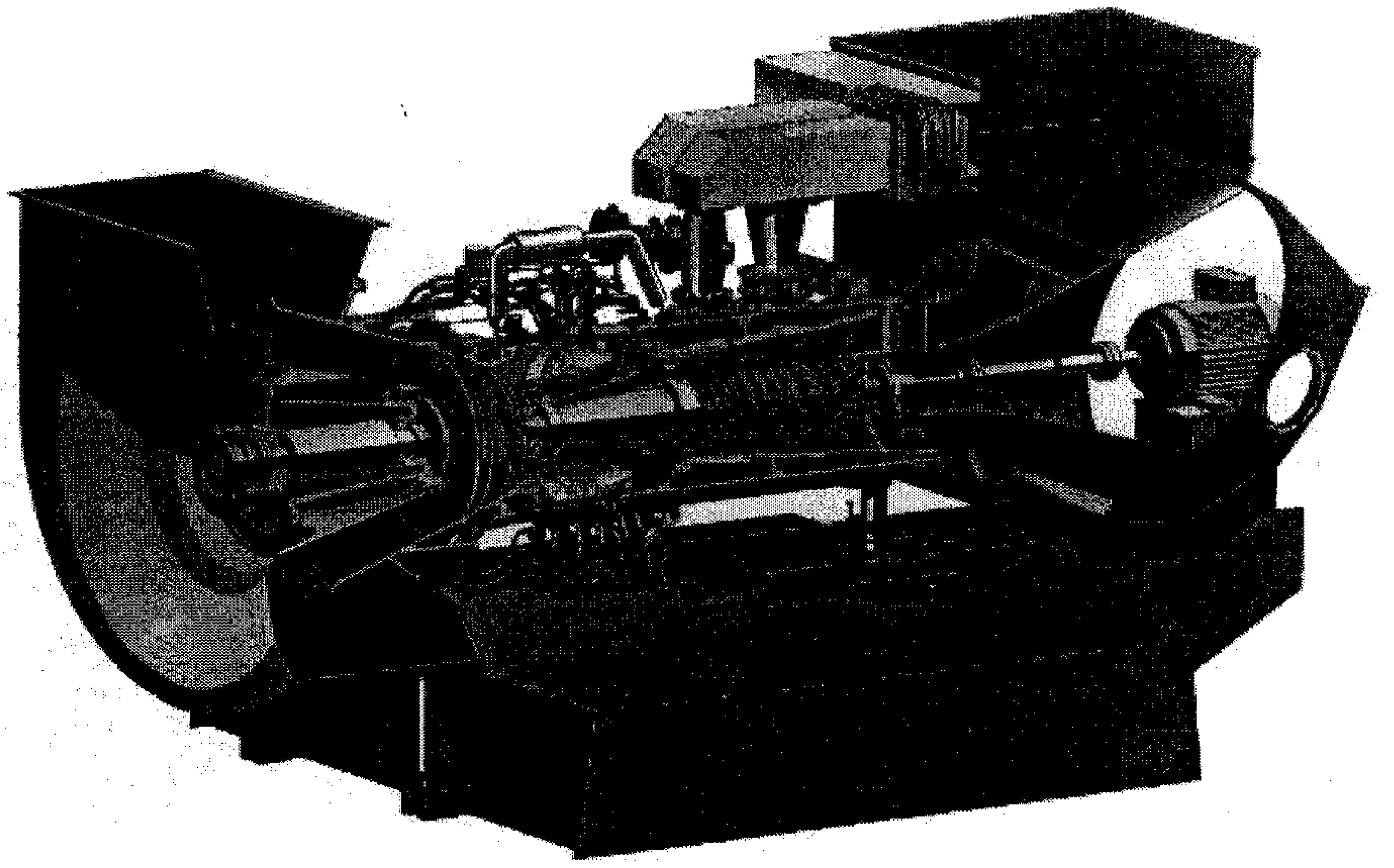
Ian A. Waitz

Problem T7 (Unified Thermodynamics)

MIT's gas turbine power plant operates on a Brayton cycle. The current operating conditions are shown at http://cogen.mit.edu/unified/. Assume the cycle is ideal and the gas behaves as an ideal gas with constant specific heats $c_p = 1.0035$kJ/kg-K, and $c_v = 0.7165$kJ/kg-K. The maximum temperature is 1400K and the minimum is 300K. The minimum pressure is 100kPa. The compressor pressure ratio is 13.

a. Calculate the pressure and temperature at each point of the cycle.
b. Calculate the ideal efficiency of the cycle.
c. How does the efficiency you calculated compare with the actual efficiency of the plant. [You can determine this from the data on the web page. If you get a full listing of variables, the fuel flow is listed under "total gas energy flow" and the net power is listed under "active load". If you use the buttons next to the picture, you need to add up the main and primary fuel flows to get the total gas energy flow. Note that 1 BTU is 1055 Joules, and 1 KBTU/s is 10^{00} BTU/s.] Why is the actual efficiency different from that you calculated in part b)?
d. A generalized schematic of a heat engine is shown at the end of Chapter 5. Sketch a similar representation for a refrigerator.
e. Could you use a Brayton cycle for a refrigerator? If no, why not? If yes, what would be required?

25 MW Gas Turbine GT10



ABB

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Problem T8 (Unified Thermodynamics)

A steady stream of high pressure air flows through a part of an aerospace power system. The velocity at the inlet is 200 m/s, the pressure is 2 x 106 N/m2 and the temperature is 1000K. In between the inlet and the outlet are various components that can extract or inject heat and work into the flowing fluid. Approximate the air as an ideal gas with constant specific heat s, R=287 J/kg-K, c_p=1003.5 J/kg-K, and c_v = 716.5 J/kg-K. (MO# 5)

Suppose the outlet flow is at a pressure of 105 N/m2, a velocity of 1000 m/s, and a temperature of 600K, and the rate of power out of the device to turn a shaft is 100 x 103 J/kg per unit mass flow rate.

a) What, if any, quantities are the same at the inlet and the outlet?

b) What is the change in stagnation enthalpy from inlet to outlet?

c) What is the rate of heat transfer between the inlet and the outlet?

d) If there were <u>no heat transfer</u>, if the process through the components were quasi-static, and if the pressure at the outlet were maintained at the same value, what would now be the temperature and velocity at the outlet?

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Problem T9 (Unified Thermodynamics)

Consider a Brayton cycle for an ideal gas. Assume all processes are quasi-static.

a. Derive expressions for the shaft work, flow work, work, and heat transfer for each leg of the cycle in terms of the temperatures T_1, T_2, T_3, T_4, c_v, c_p, and R.
b. Using these expressions, show that the net work for the cycle is equal to the net heat for the cycle.
c. Show that the net flow work for the cycle is zero and that the net shaft work is equal to the net work for the cycle.
d. Describe in a sentence or two what flow work is.
e. For the ABB gas turbine (problem T7) calculate the work, shaft work, and flow work for the turbine. What fraction of the shaft work is used to drive the compressor?

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Problem T10 (Unified Thermodynamics)

The compressor in a turbojet engine has a pressure ratio (ratio of exit stagnation pressure to inlet stagnation pressure) of 12. The ratio of combustor exit stagnation temperature to compressor inlet stagnation temperature is 6. The atmospheric temperature is 220 K and the pressure is 50kPa

and the jet is flying at a speed of 250m/s. Assume that the flow through the inlet, the compressor and the turbine can all be assumed adiabatic and quasi-static, with no friction or other losses, and the air can be treated as a perfect gas with constant specific heats (R=287 J/kg-K, c_p=1003.5 J/kg-K, c_v=716.5 J/kg-K).

a) What is the stagnation temperature at the compressor inlet?

b) What is the ratio of the stagnation pressure at the inlet to the atmospheric pressure?

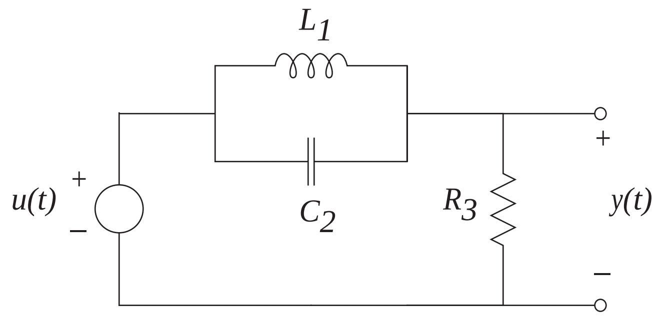
c) What is the stagnation temperature ratio across the turbine?

d) What is the stagnation pressure ratio across the turbine?

Problem S13 (Signals and Systems)

Note: for this problem and Problem S14, please do not use bibles or previous solutions.

Consider the RLC circuit below:



This circuit is a *notch filter*, meaning that the output $y(t)$ is almost the same as the input $u(t)$, except that the circuit "filters out" frequencies in a narrow range, determined by the component values. For example, this circuit might be used to filter out 60 Hz noise caused by electrical wiring from the input to an audio system, to prevent 60 Hz "hum."

For this circuit, find a state-space description of the system, in the form

$$\begin{aligned}\frac{d\underline{x}(t)}{dt} &= A\underline{x}(t) + Bu(t)\\ y(t) &= C\underline{x}(t) + Du(t)\end{aligned}$$

No component values are given, so just find the matrices A, B, C, and D in symbolic form.

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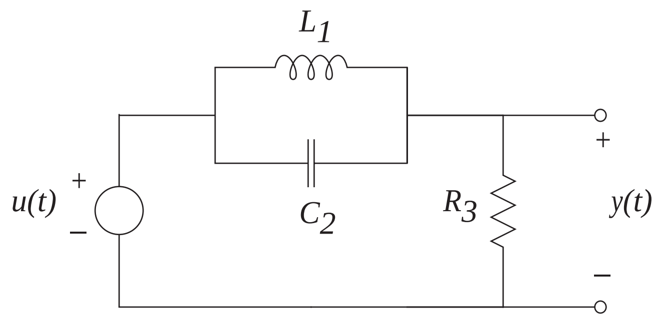
Problem S14 (Signals and Systems)

Note: for this problem and Problem S13, please do not use bibles or previous solutions.

Consider the RLC circuit of Problem S13, shown below:



1. Find the transfer function, $G(s)$, of the system, using

$$G(s) = C(sI - A)^{-1}B + D \tag{1}$$

2. Find the transfer function using impedance methods. Show that your result agrees with the result in part (1).

3. For component values

$$L_1 = 1 \text{ H}, \quad C_2 = 0.25 \text{ F}, \quad R_3 = 10\ \Omega$$

plot the magnitude of the transfer function $G(j\omega)$ vs. ω. Explain why the filter is called a notch filter.

Note: You may find it useful to use Matlab or a spreadsheet to calculate values of the transfer function, since there is a fair amount of complex arithmetic.

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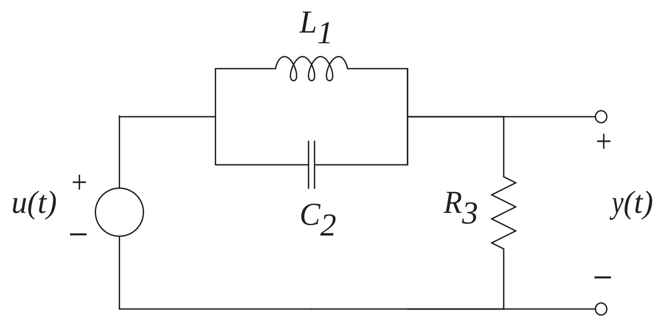
Problem S15 (Signals and Systems)

Consider the RLC circuit below:



In Problems S13 and S14, you found the state and measurement equations for this circuit, and transfer function. For component values

$$L_1 = 1 \text{ H}, \quad C_2 = 0.25 \text{ F}, \quad R_3 = 10\ \Omega$$

find the response of the circuit, $y(t)$, for the following input signals:

1. $u(t) = \cos t$
2. $u(t) = \cos 2t$
3. $u(t) = \cos 4t$

You need find only the particular solution, that is, the steady-state sinusoidal response. You do not need to find the homogenous solutions, which are exponentially decaying sinusoids.