UNIFIED ENGINEERING

Problem Set #12 -- SOLUTIONS

M14  The following answers, as asked for in the problem statement, include a brief sentence on the functional requirement that needs to be met for each of the given cases. This includes the loads (e.g. tension, compression, shear, impact, cyclic, thermal, electrical) and five material properties that are most relevant to meeting this requirement. Note that the items listed are just some of the possible requirements, loads, and properties. (NOTE: Problem set answers will vary according to what the individual student indicates are the relevant loads and properties.)

(a) **Components of a space truss:** Must provide load-carrying capacity for loads that a space truss undergoes.

Types of loads:  
1. Impact (docking)  
2. Thermal (solar)  
3. & 4. Tension and Compression (depending on design)  
5. Cyclic

Material properties:  
1. Thermal - High  
2. Density - Low  
3. Modulus - High  
4. Joining - Medium  
5. Longevity - High

(b) **Components of a truss of a radio tower:** Must provide load-carrying capacity for loads that a radio tower undergoes.

Types of loads:  
1. & 2. Tension and Compression (depending on design -- mainly compression due to gravity)  
3. Assembly  
4. Environmental (Thermal, Aerodynamic)

Material properties:  
1. Corrosion - High  
2. Modulus - High  
3. Strength - Medium  
4. Fabrication & Joining - High  
5. Price - Low

(c) **Kitchen countertop:** Must provide an “aesthetic” work surface for a kitchen.
Types of loads: 1. Impact  
2. Compression  
3. Thermal  

Material properties: 1. Price - Low  
2. Availability - High  
3. Hardness - Medium  
4. Appearance - High  
5. Finishing - High  

(d) **Electrical wires strung on telephone poles**: Must provide a conductive path for electricity transfer  

Types of loads: 1. Tension (gravity, wind-whipping)  
2. Electrical  
3. Thermal  

Material properties: 1. Electrical - High  
2. Corrosion - High  
3. Strength - High  
4. Modulus - High  
5. Price - Low  

(e) **Reentry shield on the space shuttle**: Must insulate the shuttle structure and its passengers from the extreme heat of reentry.  

Types of loads: 1. Thermal  
2. Cyclic  
3. Impact  

Material properties: 1. Thermal - High  
2. Density - Low  
3. Oxidation Resistance - High  
4. Hardness - Medium  
5. Strength - Medium
M15. Bar of constant length with solid circular cross-section carrying constant load:

\[ P \]

\[ \rightarrow \]

\[ f \]

\( f \): bar deformation

(a) List the constants: \( P, L \)

List items to be considered for minimization, etc.:

- mass/weight (M)
- deformation (f)
- cost (c)

List key equations:

1. stress-strain: \( \sigma = E \varepsilon \)  
2. strain-displacement: \( \varepsilon = \frac{f}{L} \)
3. stress-load: \( \sigma = \frac{P}{A} \)
4. area-diameter: \( A = \pi \left( \frac{d}{2} \right)^2 \)
5. mass-density: \( m = \rho A L \)

List other variables, parameters:

\( d \): diameter 
\( \rho \): density 
\( A \): Area
Finally: \[ \text{Cost} = \frac{\text{Cost}}{\text{weight (weight)}} \] (6)

The figures of merit are based on the overall items to be considered and expressing these in terms of geometrical and material parameters/properties.

→ First consider the formation.

- From (2): \[ \delta = EL \]
  - Use (1) to give \[ E = \frac{P}{L} \]
  - From: \[ \delta = \frac{PL}{E} \]

- Now use (3) in this:
  \[ \delta = \frac{PL}{EA} \]

- Finally use (4) to put this in terms of load, length, diameter and modulus:
  \[ \delta = \frac{PL}{E/\pi (d/2)^2} \]
  \[ \Rightarrow \delta = \frac{4PL}{\pi E d^2} \]  
  \[ \text{Figure of Merit} \]  
  \[ \times 1 \]

→ Now consider mass/weight

- From (5): weight = \( \rho AL \)
  - weight density
Use (4) to put this in terms of

\[ \text{Weight} = \rho \pi \left( \frac{d}{2} \right)^2 L \]

\[ \Rightarrow \text{Weight} = \frac{\pi}{4} \rho d^2 L \]

\[ \text{Figure of Merit} \]

\[ \text{Second} \]

Finally consider cost:

\[ \text{from (6):} \]

\[ \text{Cost} = C \left( \text{Weight} \right) \]

\[ \frac{\text{Cost}}{\text{Weight}} \]

Using the second figure of merit sets this in

terms of key parameters:

\[ \text{Cost} = \frac{\pi}{4} C \rho d^2 L \]

\[ \text{Figure of Merit} \]

\[ \text{Third} \]

(6) We have three equations that allow us to

explore the possibilities in terms of the key

items (information, weight, cost).

However, minimizing any one consideration

separately is generally insufficient to consider. For

example, one can increase information by

continuing to increase bar diameter. The key

is to consider the ability of any choice to

other fixed considerations. This leads to
considering Tradeoffs!

If one considers the ability to provide a specified minimum deformation (call it $f_0$) and test consider mass (weight). The first and second figures of merit can be combined for this consideration

from (*1): \[ f_0 = \frac{4PL}{\pi D^2} \]
\[ \Rightarrow D^2 = \frac{4PL}{\pi E f_0} \]

Place this in (*2)

\[ \text{Weight} = \frac{\pi}{8} \rho \left( \frac{4PL}{\pi E f_0} \right) L \]
\[ \Rightarrow \text{Weight} = \frac{\rho PL^2}{E f_0} \]

Here, $\frac{PL^2}{f_0}$ is a constant so we express the possibilities via the factor $\frac{\rho}{E}$

<table>
<thead>
<tr>
<th>Material</th>
<th>$\frac{\rho}{E}$ [10(^6) N/m(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.0094</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.0097</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.0091</td>
</tr>
<tr>
<td>Wood</td>
<td>0.0122</td>
</tr>
<tr>
<td>Carbonfiber Composites</td>
<td>0.0022</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>0.0018</td>
</tr>
</tbody>
</table>

\[ \text{best choice to minimize (for given mass/weight proportion): Silicon Carbide} \]
Now consider cost by using (*)3

\[ Cost = \frac{1}{4} CP \left( \frac{4PL}{\pi E_S} \right) L \]

Simplifying...

\[ Cost = \frac{CPL^2}{E_S} \]

Here, \( \frac{PL^2}{E_S} \) is again a constant, so we assess the possibilities via the factor \( \frac{C}{E} \).

<table>
<thead>
<tr>
<th>Material</th>
<th>( \frac{C}{E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.0099</td>
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<tr>
<td>Aluminum</td>
<td>0.0510</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.1779</td>
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<tr>
<td>Wood</td>
<td>0.0091</td>
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<tr>
<td>Carbon Fiber Composite</td>
<td>0.1450</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>0.2178</td>
</tr>
</tbody>
</table>

Wood is the best choice to minimize cost (for given constraint).

Finally, think about minimizing deformation for a fixed cross-section. Using (*)1:

\[ f = \frac{4PL}{\pi D^2} \]

Here, \( \frac{4PL}{\pi D^2} \) is given, so we assess the possibilities via the factor \( \frac{1}{E} \).
<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon$ [in^2/in^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.032</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.096</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.056</td>
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<tr>
<td>Wood</td>
<td>0.552</td>
</tr>
<tr>
<td>Carbon Fiber Composite</td>
<td>0.041</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Silicon Carbide is the best choice to minimize deformation (for given area).

One again, there is no fixed answer without further clarification of the objectives. It all depends on the decisions with regard to the needs.
a) For mass conservation, must have \( \nabla \cdot \vec{V} = 0 \), or \( \nabla \cdot \vec{\phi} = \nabla^2 \phi = 0 \)

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 1 - 1 = 0 \quad \checkmark
\]

b) \begin{align*}
\frac{\partial \phi}{\partial x} &= x \\
\frac{\partial \phi}{\partial y} &= -y
\end{align*}

c) \begin{align*}
\frac{\partial y}{\partial y} &= x & \Rightarrow & y = xy + f_1(x) \\
\frac{\partial x}{\partial x} &= -y & \Rightarrow & x = xy + f_2(y)
\end{align*}

\( y(x, y) = xy \)

d) \[
\dot{m} = \int \rho \vec{V} \cdot \hat{n} \, dA = \int (\rho u \, dy - \rho v \, dx) = \int \rho \left( \frac{\partial y}{\partial y} \, dy + \frac{\partial x}{\partial x} \, dx \right) = \int_0^1 \rho \, d\gamma
\]

\[
\dot{m} = \rho \left( y(1, 1) - y(0, 0) \right) = \rho (1 - 1 - 0.1) = \rho = 1
\]

since \( \rho = 1 \)
a) Parallel flow: \( \frac{\partial P}{\partial n} = 0 \) since fluid doesn’t accelerate.

Since \( \frac{\partial P}{\partial n} \approx \frac{P_{\text{atm}} - P_{\text{test}}}{\Delta n} = 0 \)

\[ \Rightarrow P_{\text{atm}} - P_{\text{test}} = 0 \]

b) \( P_i = P_{\text{test}} \) since \( P_0 = \text{const along streamline} \)

\[ P_i + \frac{1}{2} \rho V_i^2 = P_{\text{test}} + \frac{1}{2} \rho V_{\text{test}}^2 \]

But by mass conservation,

\[ \rho V_i A_i = \rho V_{\text{test}} A_{\text{test}} \]

\[ \Rightarrow V_i = V_{\text{test}} \frac{A_{\text{test}}}{A_i} = \frac{1}{36} V_{\text{test}} \]

So \( P_i - P_{\text{test}} = \frac{1}{2} \rho \left( V_{\text{test}}^2 - V_i^2 \right) = \frac{1}{2} \rho V_{\text{test}}^2 \left( 1 - \frac{1}{36^2} \right) \approx \frac{1}{2} \rho V_{\text{test}}^2 \)

Since \( P_{\text{test}} = P_{\text{atm}} \):

\[ P_i - P_{\text{atm}} = \frac{1}{2} \rho V_{\text{test}}^2 = \frac{1}{2} \cdot 1.2 \text{ kg/m}^3 \cdot 45^2 \text{ m/s} \]

\[ P_i - P_{\text{atm}} = 1218 \text{ Pa} \]

c) \( F = (P_i - P_{\text{atm}}) \cdot A_{\text{wall}} = 1218 \text{ Pa} \cdot (6 \text{ ft} \times 15 \text{ ft}) \cdot \frac{1}{(3.27 \text{ ft/m})^2} \approx 11022 \text{ lb} \)

So \( F \approx 1.0226 \times 10^4 \text{ N} \)
F14  Solution  Fall '04

a) \[ U = \frac{\partial y}{\partial y} = Cy \rightarrow y = \frac{1}{2} Cy^2 + f_1(x) \] 
\[ V = -\frac{\partial y}{\partial x} = 0 \rightarrow y = \text{const} + f_2(y) \]  
\[ f_1(x) = \text{const} = 0 \text{ is ok} \] 
\[ f_2(y) = \frac{1}{2} Cy^2 \]

\[ y(x, y) = \frac{1}{2} Cy^2 \]

b) For given velocities, \( \nabla \cdot \vec{V} = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} = -Cy \neq 0 \) rotational flow
\[ \therefore \phi \text{ does not exist} \]
Can also try \[ U = \frac{\partial \phi}{\partial x} = Cy \rightarrow \phi = Cxy + f_1(y) \] \[ V = \frac{\partial \phi}{\partial y} = 0 \rightarrow \phi = \text{const} + f_2(y) \] incompatible, cannot match \( f_2(y) \) with \( Cxy \)

C) Check mass conservation:
\[ m_1 = \int_0^h \rho dy = \int_0^h \rho Cy dy = \rho \frac{1}{2} Ch^2 \]
\[ m_2 = \int_{\frac{1}{2}}^{h} \rho dy = \int_{\frac{1}{2}}^{h} \rho 2Cy dy = \rho \frac{1}{4} Ch^2 \]
\[ m_1 \neq m_2 \text{ mass not conserved} \]

Can also compare \( y \) on top wall: \[ y_1(h) = \rho \frac{1}{2} Ch^2 \]
\[ y_2(h/2) = \rho \frac{1}{4} Ch^2 \]
Evaluate \( \zeta = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \)
At 1: \[ \zeta = 0 - C = -C \]
At 2: \[ \zeta = 0 - 2C = -2C \]
\( \zeta \) changes on any streamline between 1 and 2
\[ \frac{D\zeta}{Dt} \neq 0 \text{ Helmholtz doesn't hold} \]