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Unified Engineering
Fall 2004

Problem Set #12
Solutions

UNIFIED ENGINEERING

Problem Set #12 -- SOLUTIONS

M14 The following answers, as asked for in the problem statement, include a brief sentence on the functional requirement that needs to be met for each of the given cases. This includes the loads (e.g. tension, compression, shear, impact, cyclic, thermal, electrical) and five material properties that are most relevant to meeting this requirement. Note that the items listed are just ***some*** of the possible requirements, loads, and properties. (**NOTE:** Problem set answers will vary according to what the individual student indicates are the relevant loads and properties.)

(a) Components of a space truss: Must provide load-carrying capacity for loads that a space truss undergoes.

Types of loads:
1. Impact (docking)
2. Thermal (solar)
3. & 4. Tension and Compression (depending on design)
5. Cyclic

Material properties:
1. Thermal - High
2. Density - Low
3. Modulus - High
4. Joining - Medium
5. Longevity - High

(b) Components of a truss of a radio tower: Must provide load-carrying capacity for loads that a radio tower undergoes.

Types of loads:
1. & 2. Tension and Compression (depending on design -- mainly compression due to gravity)
3. Assembly
4. Environmental (Thermal, Aerodynamic)

Material properties:
1. Corrosion - High
2. Modulus - High
3. Strength - Medium
4. Fabrication & Joining - High
5. Price - Low

(c) Kitchen countertop: Must provide an "aesthetic" work surface for a kitchen.

Types of loads:
1. Impact
2. Compression
3. Thermal

Material properties:
1. Price - Low
2. Availability - High
3. Hardness - Medium
4. Appearance - High
5. Finishing - High

(d) Electrical wires strung on telephone poles: Must provide a conductive path for electricity transfer

Types of loads:
1. Tension (gravity, wind-whipping)
2. Electrical
3. Thermal

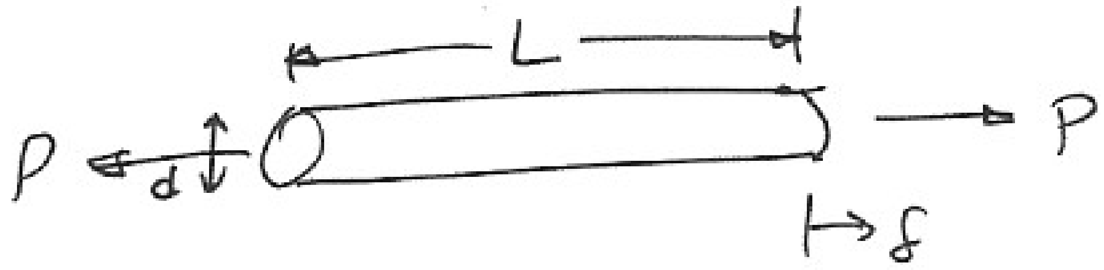
Material properties:
1. Electrical - High
2. Corrosion - High
3. Strength - High
4. Modulus - High
5. Price - Low

(e) Reentry shield on the space shuttle: Must insulate the shuttle structure and its passengers from the extreme heat of reentry.

Types of loads:
1. Thermal
2. Cyclic
3. Impact

Material properties:
1. Thermal - High
2. Density - Low
3. Oxidation Resistance - High
4. Hardness - Medium
5. Strength - Medium

M15. Bar of constant length with solid circular cross-section carrying constant load:



δ: bar deformation

(a) List the constants: P, L

List items to be considered for minimization etc.: mass/weight (m)
deformation (δ)
cost (c)

List key equations

stress-strain: $\sigma = E\epsilon$ (1)

strain-displacement: $\epsilon = \delta/L$ (2)

stress-load: $\sigma = P/A$ (3)

area-diameter: $A = \pi(d/2)^2$ (4)

mass (weight)-density: $m = \rho A L$ (5)

List other variables, parameters:

d = diameter
ρ = density
A = Area

} can be affected

Finally: $\text{Cost} = \frac{\text{cost}}{\text{weight}}(\text{weight})$ (6)

The figures of merit are based on the overall items to be considered and expressing these in terms of geometrical and material parameters/properties

→ First consider deformation.

- From (2): $\delta = \epsilon L$
  - use (1) to give $\epsilon = \sigma / E$
    and thus: $\delta = \frac{\sigma L}{E}$
  - Now use (3) in this:
    $$\delta = \frac{PL}{EA}$$
  - finally use (4) to get this in terms of load, length, diameter, and modulus:
    $$\delta = \frac{PL}{E\pi(d/2)^2}$$
    $$\Rightarrow \boxed{\delta = \frac{4PL}{\pi E d^2}}$$
    First figure of Merit (*1)

→ Now consider mass/weight

- from (5): weight $= \rho A L$
  ρ ↑ weight density

- use (4) to get this in terms of density, load, length, and diameter:

$$\text{weight} = \rho\pi\left(\frac{d}{2}\right)^2 L$$

$$\Rightarrow \boxed{\text{weight} = \frac{\pi}{4}\rho d^2 L}$$

second Figure of Merit (*2)

→ finally consider cost:

- from (6):

$$\text{cost} = c\,(\text{weight})$$

c ← cost/weight

- using the second figure of merit gets this in terms of key parameters:

$$\boxed{\text{Cost} = \frac{\pi}{4} c\rho d^2 L}$$

third Figure of Merit (*3)

(b) We have three equations that allow us to explore the possibilities in terms of the key items (deformation, ~~mass~~ (weight), cost)

However, minimizing any one consideration separately is generally insufficient to consider. For example, one can decrease deformation by continuing to increase bar diameter. The key is to consider the ability of any choice for other fixed considerations. This leads to

Considering Tradeoffs!

If one considers the ability to provide a specified minimum deformation (call it δ_0) and first consider mass (weight). The first and second figures of merit can be combined for this consideration

from (*1): $\delta_0 = \frac{4PL}{\pi E d^2}$

$$\Rightarrow d^2 = \frac{4PL}{\pi E \delta_0}$$

Place this in (*2)

$$\text{weight} = \frac{\pi}{4} \rho \left(\frac{4PL}{\pi E \delta_0}\right) L$$

$$\Rightarrow \text{weight} = \frac{\rho P L^2}{E \delta_0}$$

Here, $\frac{PL^2}{\delta_0}$ is a constant, so we assess the possibilities via the factor ρ/E

Material	ρ/E $\left[\frac{lb}{in^3} / \frac{10^6 lb}{in^2}\right] = \left[\frac{1}{10^6 in}\right]$
Steel	0.0094
Aluminum	0.0097
Titanium	0.0091
Wood	0.0122
Carbon fiber composite	0.0022
$\boxed{\text{Silicon Carbide}}$	0.0018 ← best choice to minimize mass/weight (for given deformation)

Now consider cost by using (*3)

$$\Rightarrow \quad \text{Cost} = \frac{\pi}{4} c\rho \left(\frac{4PL}{\pi E\delta_0}\right) L$$

giving ... $$\text{Cost} = \frac{cPL^2\rho}{E\delta_0}$$

Here, $\frac{PL^2}{\delta_0}$ is again a constant, so we assess the possibilities via the factor $\frac{c\rho}{E}$

Material	$\frac{c\rho}{E}$ $\left[\frac{\$}{10^6 \text{ in-lb}}\right]$
Steel	0.0097
Aluminum	0.0510
Titanium	0.1775
Wood	0.0091
Carbon fiber Composite	0.1450
Silicon Carbide	0.2178

← best choice to minimize cost (for given displacement)

Finally think about minimizing deformation for a fixed cross-section. Using (*1):

$$\delta = \frac{4PL}{\pi E d^2}$$

Here $\frac{4PL}{\pi d^2}$ is given, so we assess the possibilities via the factor $1/E$

Material	1/E $\left[\frac{in^2}{10^6 lb}\right]$
Steel	0.033
Aluminum	0.096
Titanium	0.056
Wood	0.552
Carbon Fiber Composite	0.041
Silicon Carbide	0.017

– best choice to minimize deformation (for given area)

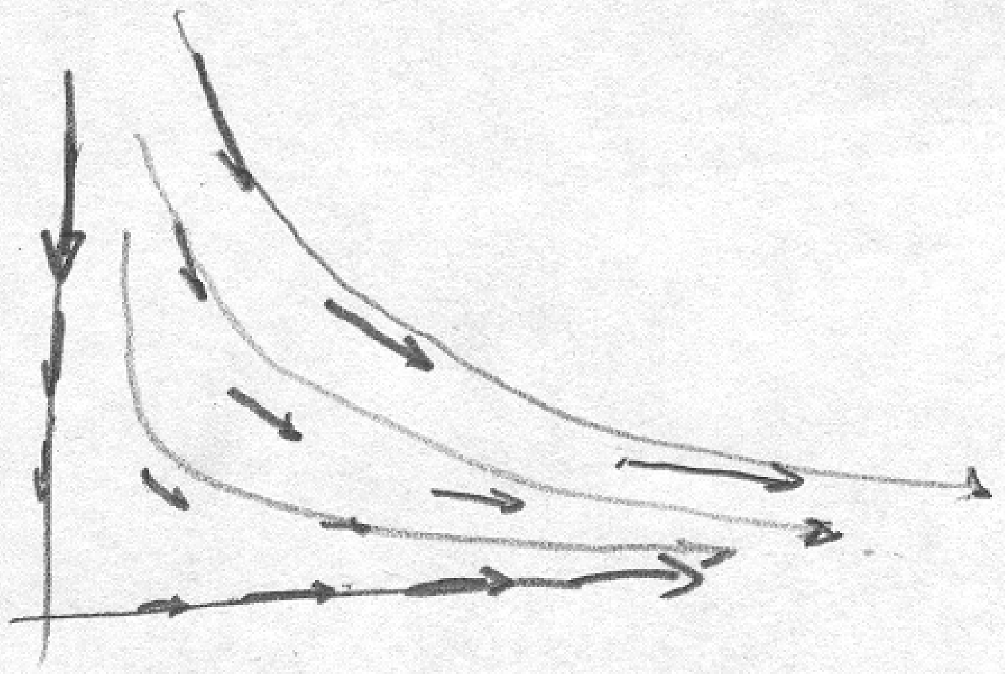
One again, there is no final answer without further clarification of the objectives. It all depends on the decisions with regard to tradeoffs.

F12 Solution Fall '04

a) For mass conservation, must have $\nabla \cdot \vec{V} = 0$, or $\nabla \cdot \nabla \phi = \nabla^2 \phi = 0$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 1 - 1 = 0} \ \checkmark$$

b)
$$\boxed{\begin{aligned} u &= \frac{\partial \phi}{\partial x} = x \\ v &= \frac{\partial \phi}{\partial y} = -y \end{aligned}}$$



c) $u = \frac{\partial \psi}{\partial y} = x \rightarrow \psi = xy + f_1(x)$

$v = -\frac{\partial \psi}{\partial x} = -y \rightarrow \psi = xy + f_2(y)$

comparing, we see that $f_1 = 0, f_2 = 0$

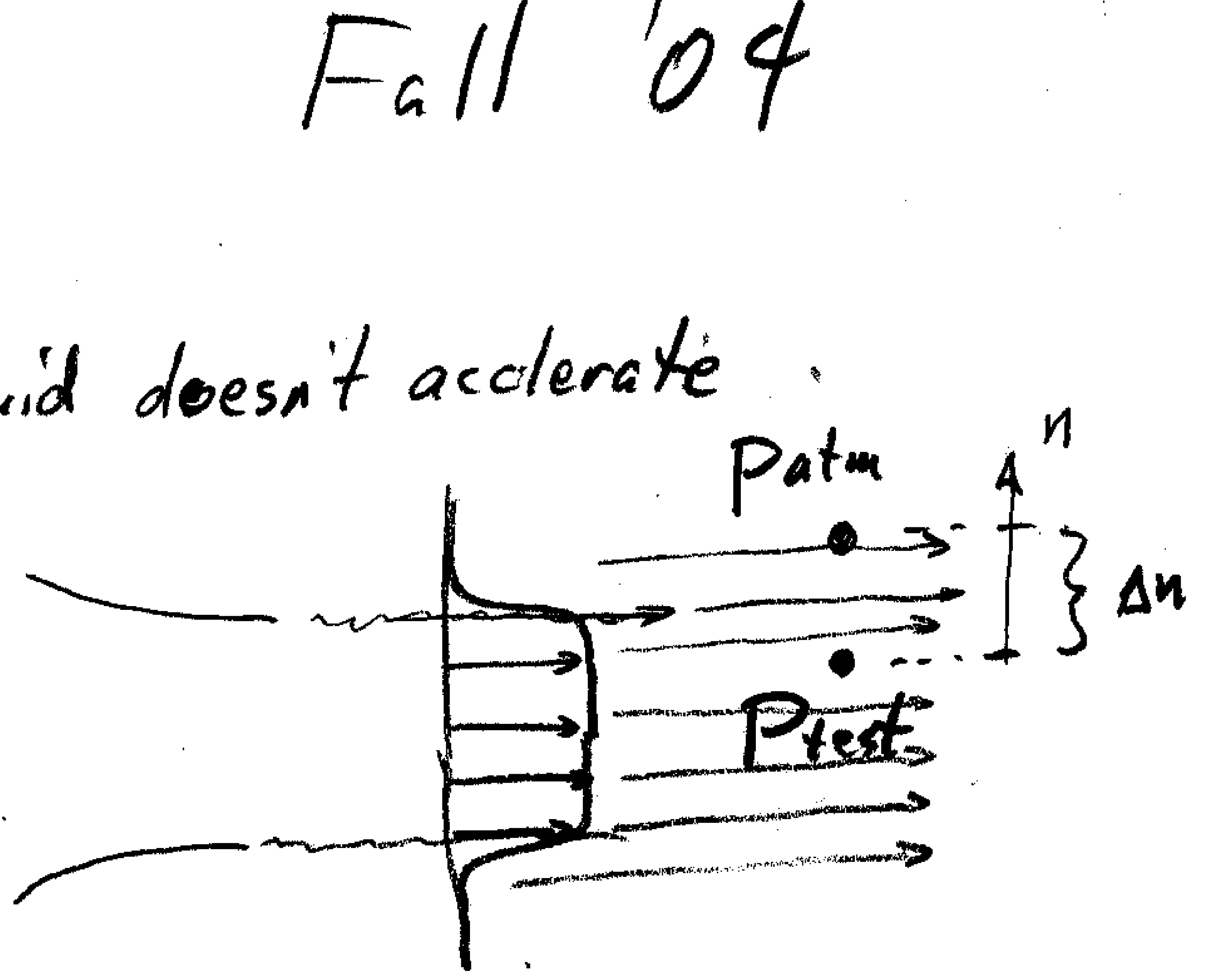
$$\boxed{\psi(x,y) = xy}$$

d) $\dot{m} = \int \rho \vec{V} \cdot \hat{n} \, dA = \int [\rho u \, dy - \rho v \, dx] = \int \rho \left(\frac{\partial \psi}{\partial y} dy + \frac{\partial \psi}{\partial x} dx \right) = \int_{0,0}^{1,1} \rho \, d\psi$

$$\boxed{\dot{m} = \rho(\psi(1,1) - \psi(0,0)) = \rho(1 \cdot 1 - 0 \cdot 0) = \boxed{\rho = 1}}$$ since $\rho = 1$

F13 Solution Fall '04

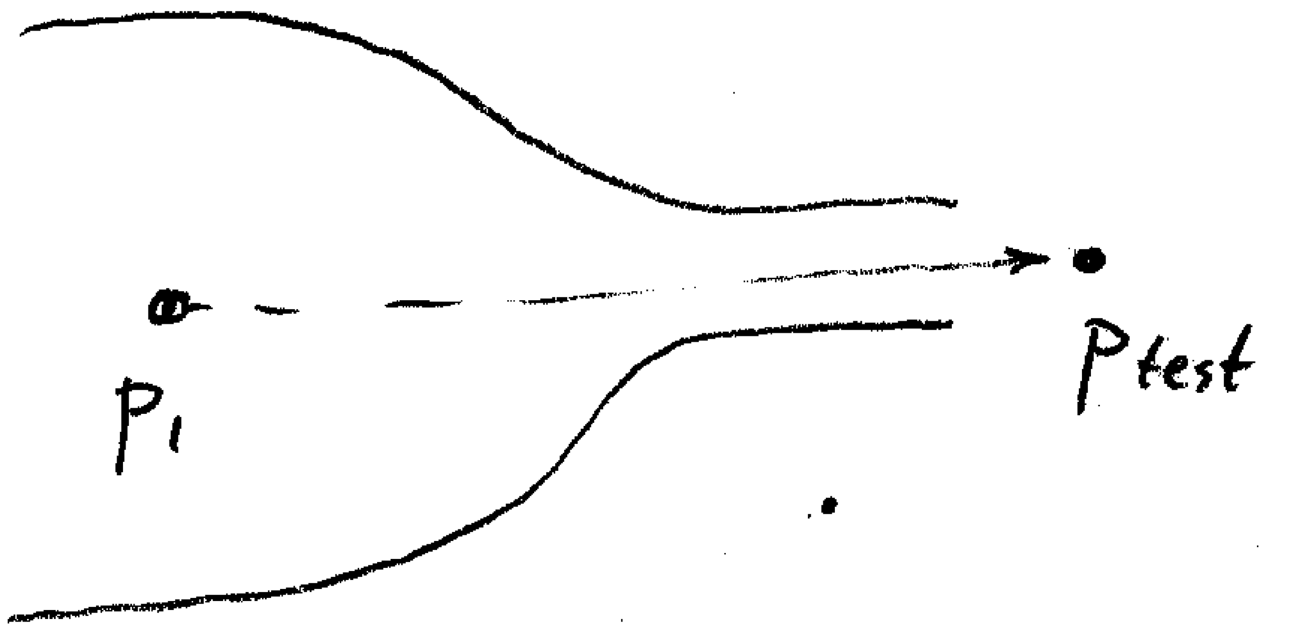
a) Parallel flow: $\frac{\partial p}{\partial n} = 0$ since fluid doesn't accelerate



since $\frac{\partial p}{\partial n} \approx \frac{P_{atm} - P_{test}}{\Delta n} \approx 0$

$\rightarrow \boxed{P_{atm} - P_{test} \approx 0}$

b) $p_{o_1} = p_{o_{test}}$ since $p_o = \text{const}$ along streamline



$p_1 + \frac{1}{2}\rho V_1^2 = p_{test} + \frac{1}{2}\rho V_{test}^2$

but by mass conservation,

$\rho V_1 A_1 = \rho V_{test} A_{test}$

$\rightarrow V_1 = V_{test} \frac{A_{test}}{A_1} = \frac{1}{36} V_{test}$

so $p_1 - p_{test} = \frac{1}{2}\rho\left(V_{test}^2 - V_1^2\right) = \frac{1}{2}\rho V_{test}^2\left(1 - \overbrace{\frac{1}{36^2}}^{\text{negligible}}\right) \approx \frac{1}{2}\rho V_{test}^2$

since $p_{test} = p_{atm}$: $p_1 - p_{atm} \approx \frac{1}{2}\rho V_{test}^2 = \frac{1}{2}\cdot 1.2\,\text{kg/m}^3 \cdot 45^2\,\text{m/s}$

$\boxed{p_1 - p_{atm} = 1215\ \text{Pa}}$

c) $F = (p_1 - p_{atm}) \cdot A_{wall} = 1215\ \text{Pa}\cdot(6\ \text{ft} \times 15\ \text{ft})\frac{1}{(3.27\ \text{ft/m})^2} \approx 1.0226^{4}$

So $\boxed{F \approx 1.0226\,{}^{4}\ \text{N}}$

F14 Solution Fall '04

a) $u = \frac{\partial\psi}{\partial y} = Cy \rightarrow \psi = \frac{1}{2}Cy^2 + f_1(x)$
$v = -\frac{\partial\psi}{\partial x} = 0 \rightarrow \psi = \text{const} + f_2(y)$ } compare

$f_1(x) = \text{const} = 0$ is OK
$f_2(y) = \frac{1}{2}Cy^2$

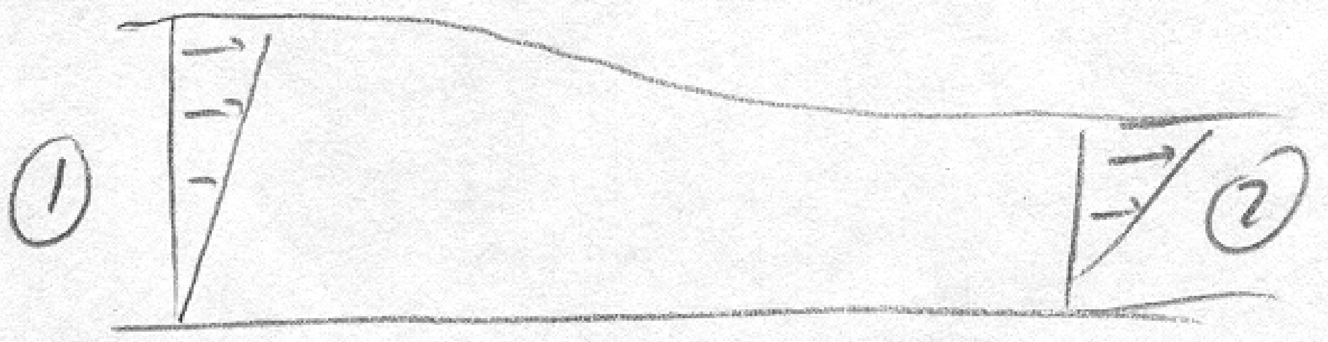
$$\boxed{\psi(x,y) = \frac{1}{2}Cy^2}$$

b) For given velocities, $\nabla\times\vec{V} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -Cy \neq 0$ rotational flow

$\therefore$ $\boxed{\phi \text{ does not exist}}$

Can also try $u = \frac{\partial\phi}{\partial x} = Cy \rightarrow \phi = Cxy + f_1(y)$
$v = \frac{\partial\phi}{\partial y} = 0 \rightarrow \phi = \text{const.} + f_2(x)$ } incompatible, cannot match $f_2(x)$ with Cxy

c) Check mass conservation:



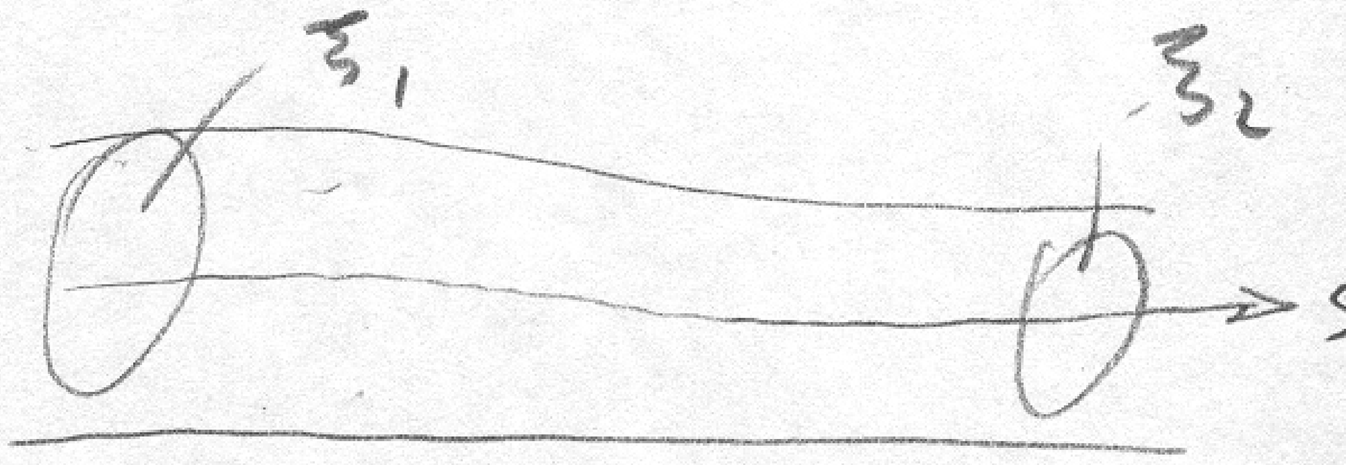
$\dot{m}_1 = \int_0^h \rho u\,dy = \int_0^h \rho Cy\,dy = \rho\frac{1}{2}Ch^2$

$\dot{m}_2 = \int_0^{h/2} \rho u\,dy = \int_0^{h/2} \rho\,2Cy\,dy = \rho\frac{1}{4}Ch^2$

$\dot{m}_1 \neq \dot{m}_2$ mass not conserved.

Can also compare ψ on top wall: $\psi_1(h) = \rho\frac{1}{2}Ch^2$
$\psi_2(h/2) = \rho C\left(\frac{h}{2}\right)^2 = \rho\frac{1}{4}h^2$ different

Evaluate $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$



At ①: $\xi = 0 - C = -C$
At ②: $\xi = 0 - 2C = -2C$

ξ changes on any streamline between ① and ②

so $\frac{D\xi}{Dt} \neq 0$ Helmholtz doesn't hold