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Unified Engineering
Fall 2004

Problem Set #3 Solutions

UNIFIED ENGINEERING

Problem Set #3 - SOLUTIONS

3 (M).1 (a) To draw the free body diagram, replace the person with the load and the supports with the reaction forces.

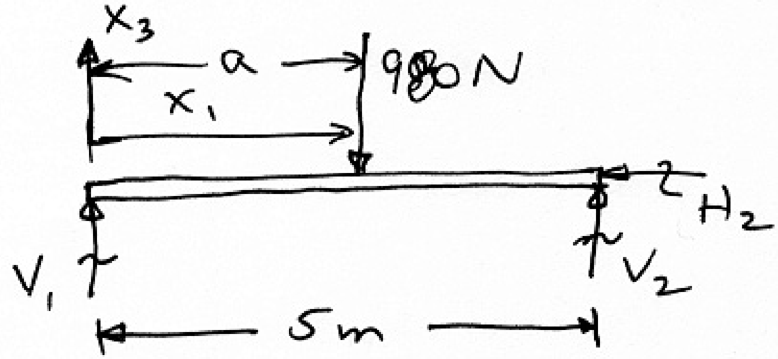
Person has a mass of 100 kg.

$$W = mg$$

and $g = 9.8\ m/s^2$

$$\Rightarrow W = (100 kg)(9.8\ m/s^2) = 980\ N$$

So:



where: roller has a vertical reaction
pin has a vertical and horizontal reaction
person at some location a, in the x_1-direction.

(b) Find the reactions by applying the equations of equilibrium:

$$\sum F_1 = 0 \quad \xrightarrow{+} \Rightarrow -H_2 = 0$$

$$\sum F_3 = 0 \quad +\uparrow \Rightarrow V_1 - 980N + V_2 = 0$$

$$\sum M = 0 \quad \circlearrowleft + \Rightarrow -(980N)\,a + V_2\,(5m) = 0$$

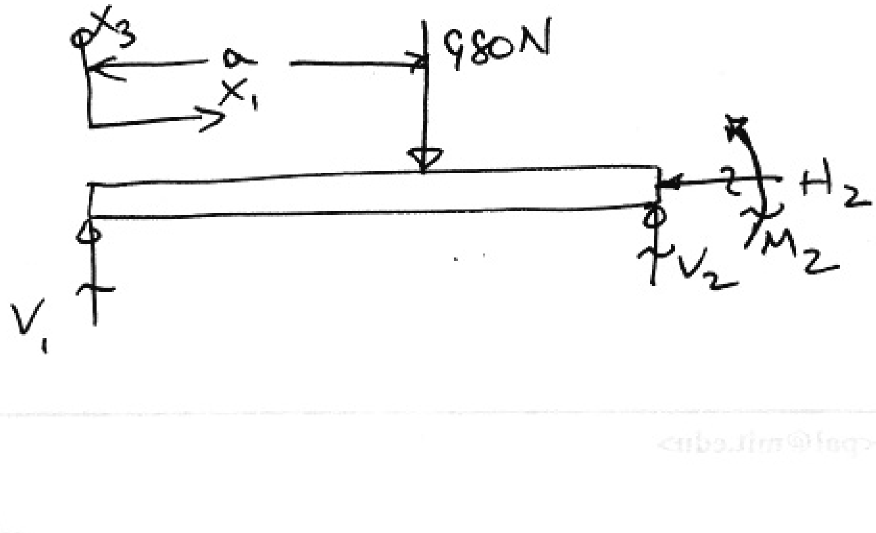
(about $x_1 = 0$)

Third equation yields: $V_2 = 196\,a \left(\frac{N}{m}\right)$

Use in second equation: $V_1 = +980N - 196\,a\left(\frac{N}{m}\right)$

Summarizing:
$$\boxed{\begin{aligned} V_1 &= 980N - 196a\left(\tfrac{N}{m}\right) \\ V_2 &= 196a\left(\tfrac{N}{m}\right) \\ H_2 &= 0 \end{aligned}}$$

(c) If a clamp supports replaces a pin, a moment reaction is added. Thus the Free Body Diagram becomes:



Again go to the three equilibrium equations:

$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -H_2 = 0$

$\Sigma F_3 = 0 +\uparrow \Rightarrow V_1 - 980N + V_2 = 0$

$\Sigma M = 0 \circlearrowleft + \Rightarrow -(980N)a + V_2(5m) + M_2 = 0$
(about $x_1 = 0$)

There are four reactions (unknowns) and three equations (degrees of freedom), so this situation is statically indeterminate. We thus cannot determine all the reactions. But, we can use the equations to say:

$$H_2 = 0$$
$$V_1 + V_2 = 980\ N$$
$$M_2 = (980N)a - V_2(5m)$$

3(M). 2 (a) In order to draw the free body diagram, it is necessary to convert the masses into loads due to gravity:

$$\underset{[N]}{\text{Force}} = \underset{[kg]}{\text{mass}} \times 9.8\ m/s^2$$

For the concentrated mass due to the cable transmitting the force from the hanging of that mass:

$$\text{Force} = 300kg \times 9.8\ m/s^2 = 2940N$$

An equation is needed for the distribution of sand and the associated ~~mass~~ and resulting force. Using the left side of the beam as $x_1 = 0$, one can write:

$$\text{mass/unit length} = f(x_1) = d\left(1 - \frac{x_1}{10\,m}\right)$$

where d is the maximum amount (at $x_1 = 0$) and it goes to zero at $x_1 = 10\,m$

(NOTE: See solution to Problem Set # 2, Problem 2(M).3 for details as to how to derive this)

We know: $$\int_0^{10m} d\left(1 - \frac{x_1}{10\,m}\right) dx_1 = 1000\,kg$$

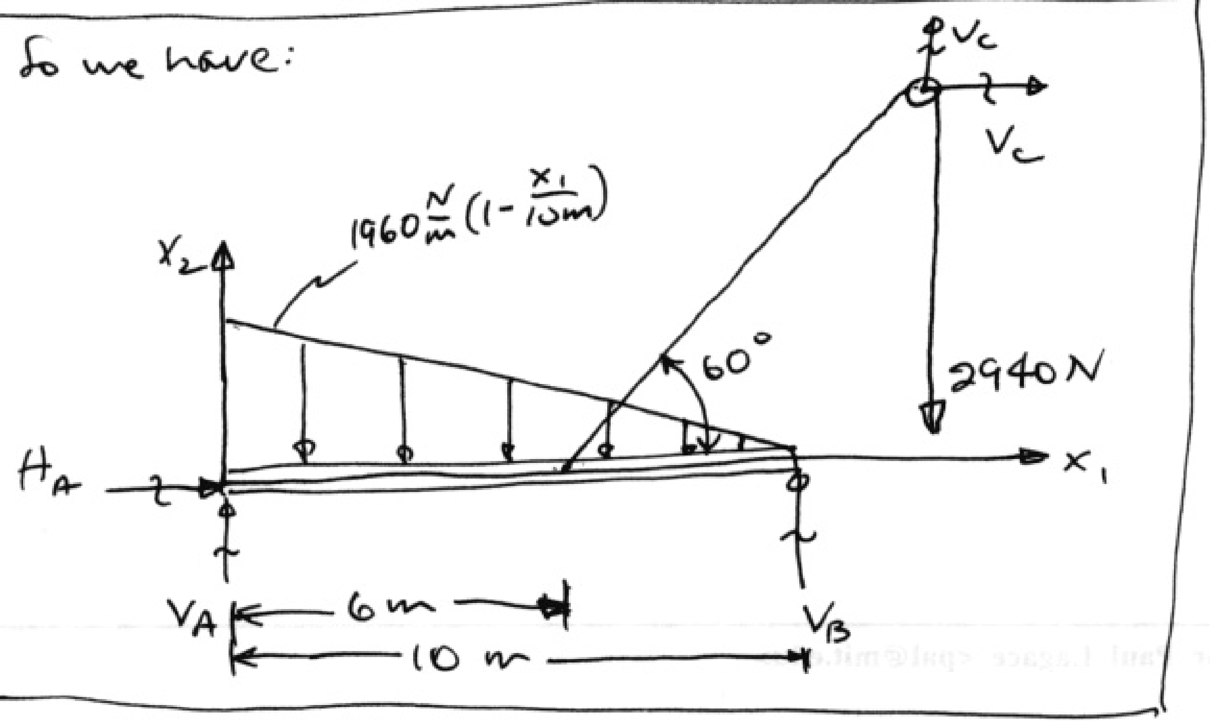
$$\Rightarrow d\left(x_1 - \frac{x_1^2}{20\,m}\right)\Big]_0^{10m} = 1000\,kg$$

$$d(10\,m - 5\,m) = 1000\,kg \Rightarrow d = 200\ kg/m$$

So: $\text{Force} = 200\ kg/m \times 9.8\ m/s^2 = 1960\ N/m$

$\Rightarrow$ dirt gives force of $1960\ N/m\left(1 - \frac{x_1}{10\,m}\right)$

So we have:



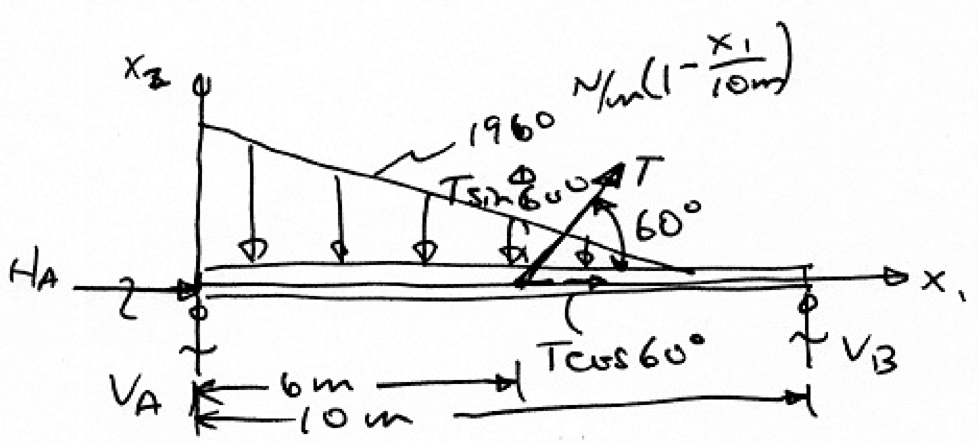
where: A is the pin support at $x_1 = 0$ with horizontal and vertical reactions

B is the roller support at $x_1 = 10$ m with vertical reaction

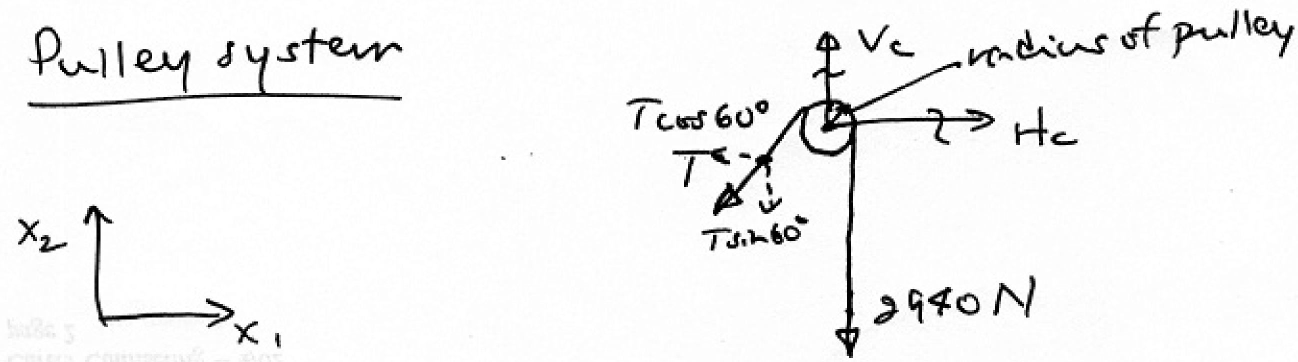
C is the pin support for the pulley with vertical and horizontal reactions

It is useful in this problem to split this into two "subsystems" each with their own Free Body Diagram. Why? We have two structures: the beam and the cable. We "split" this by replacing the beam/cable connection by a force representing the tension in the cable, T. Thus:

Beam/bar system



Pulley system



(b) For it to be possible to determine the reactions, the system must be statically determinate (or we need constitutive relations). In this case, that means each of these subsystems must be statically determinate

→ <u>Beam/bar system:</u>

\# of dof = 3

- lateral in x_1
- lateral in x_2
- rotation in $x_1 - x_2$ plane

\# of reactions = 3 (H_A, V_A, H_B)

(# of dof) = (# of reactions) ⇒ Statically Determinate

→ <u>Pulley system</u>

\# of dof = 2

- lateral in x_1
- lateral in x_2

(<u>Note</u>: A cable is not rigid, so it cannot rotate as a rigid body!)

\# of reactions = 2 (H_c, V_c)

(# of dof) = (# of reactions) ⇒ Statically Determinate

What about T?

This is a transmission across the two systems so there will be an "equation" as well as "across the cable ...".

→ Beam/bar system

Take equilibrium:

$$\Sigma F_1 = 0 \quad \overset{+}{\rightarrow} \Rightarrow H_A + 0.5T = 0 \qquad (1)$$

$$\Sigma F_2 = 0 \quad \uparrow + \Rightarrow V_A - \int_0^{10m} 1960\tfrac{N}{m}\left(1 - \tfrac{x_1}{10m}\right)dx_1 + 0.866T + V_B = 0 \qquad (2)$$

$$\Rightarrow V_A + V_B + 0.866T - 9800\,N = 0 \qquad (2)$$

$$\Sigma M = 0 \; \circlearrowleft + \Rightarrow 0.866T(6m) + V_B(10m) - \int_0^{10m} 1960\tfrac{N}{m}\left(1 - \tfrac{x_1}{10m}\right)x_1\,dx_1 = 0 \qquad (3)$$

(about $x_1 = 0$)

$$\Rightarrow 5.196T[m] + V_B(10m) - 1960\tfrac{N}{m}\left(\tfrac{x_1^2}{2} - \tfrac{x_1^3}{30m}\right)\Big]_0^{10m} = 0$$

$$5.196T[m] + V_B(10m) - 1960\tfrac{N}{m}(16.7m^2) = 0$$

$$\Rightarrow 5.196T + V_B(10) - 32{,}732\,N = 0 \qquad (3)$$

→ Pulley system

$$\Sigma F_1 = 0 \quad \overset{+}{\rightarrow} \Rightarrow H_c - 0.5T = 0 \qquad (4)$$

$$\Sigma F_2 = 0 \; +\uparrow \Rightarrow V_c - 2940\,N - 0.866T = 0 \qquad (5)$$

$$\Sigma M = 0 \; \circlearrowleft + \Rightarrow Tr - 2940\,N(r) = 0 \qquad (6)$$

(about pulley pin) where: r = pulley radius

Rearranging and summarizing the equations:

$$H_A = -0.5T \qquad (1)$$

$$V_A + V_B = 9800\,N - 0.866T \qquad (2)$$

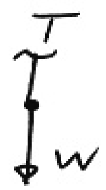
$$V_B = 32{,}732\,N - 5.196\,T \qquad (3)$$

$$H_C = 0.5T \qquad (4)$$

$$V_c = 2940\,N + 0.866T \qquad (5)$$

$$T = 2940\,N \qquad (6)\ *$$

(*Note: Can also get this by noting that a cable transmits axial force, so the tension in the cable must be equal to the weight hanging at its end. Can also do this by cutting the cable near the end:



$$\Rightarrow T = W$$)

Using (6) and knowing T, this can be used in all the other equations to get the reactions:

$$H_A = -1470\,N$$
$$V_B = 1746\,N$$
$$V_A = 5508\,N$$
$$V_c = 5486\,N$$
$$H_c = 5486\,N$$

(c) If one wants $V_B = 0$, all the equilibrium equations basically stay the same except (3) becomes:

$$0 = V_B = 32{,}732\,N - 5.196\,T$$

This gives $T = 6300\ N$

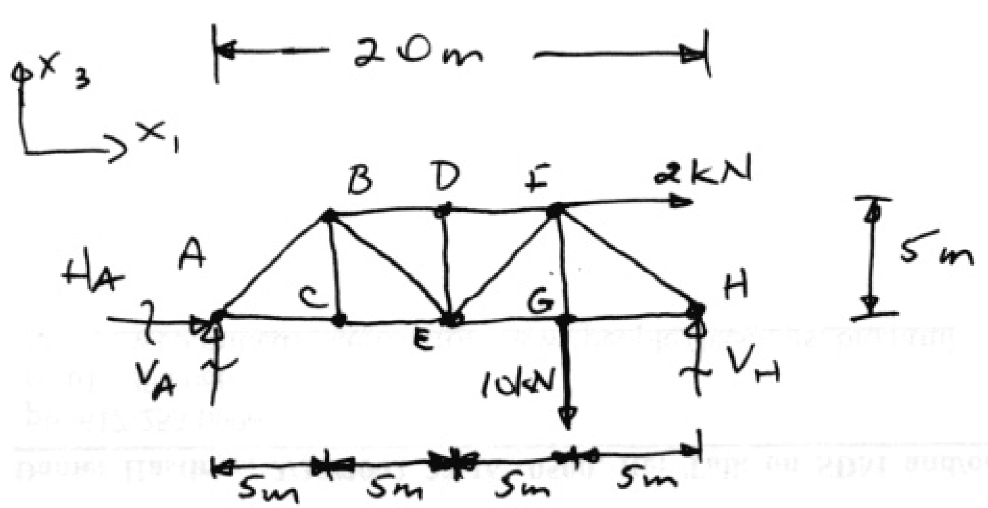
Recalling that: $T = \text{mass} \times 9.8\ m/s^2$

gives: $\boxed{m = 643\,kg}$ Yes, it can be done

Note: The reaction is still there, it is just zero. Things just balance at this point.

3 (M). 3 (a)

Draw the Free Body Diagram:



(b) Determine the reaction forces by applying the equilibrium equations:

$$\Sigma F_1 = 0 \ \overset{+}{\rightarrow} \ \Rightarrow \ H_A + 2000\,N = 0$$

$$\Rightarrow H_A = -2000\,N$$

$$\Sigma F_3 = 0 \ +\!\uparrow \ \Rightarrow \ V_A + V_H - 10{,}000\,N = 0$$

$$\Sigma M = 0 \ \circlearrowleft^{+} \ \Rightarrow \ V_H(20\,m) - 10{,}000\,N(15\,m)$$

(about point A)

$$-2{,}000\,N(5\,m) = 0$$

$$\Rightarrow V_H = 8000\,N$$

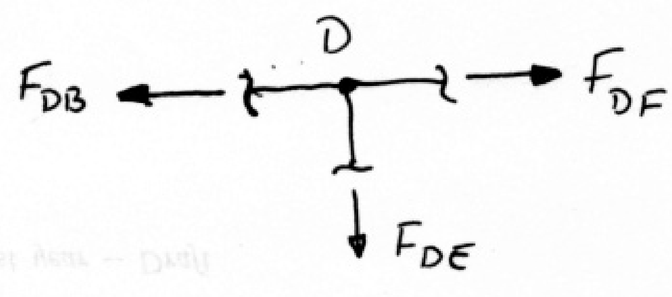
using this in the second equation

$$\Rightarrow V_A + 8000N - 10{,}000N = 0 \Rightarrow V_A = 2000\,N$$

Summarizing:

$$\boxed{\begin{aligned} V_A &= 2000\,N \\ H_A &= -2000\,N \\ V_H &= 8000\,N \end{aligned}}$$

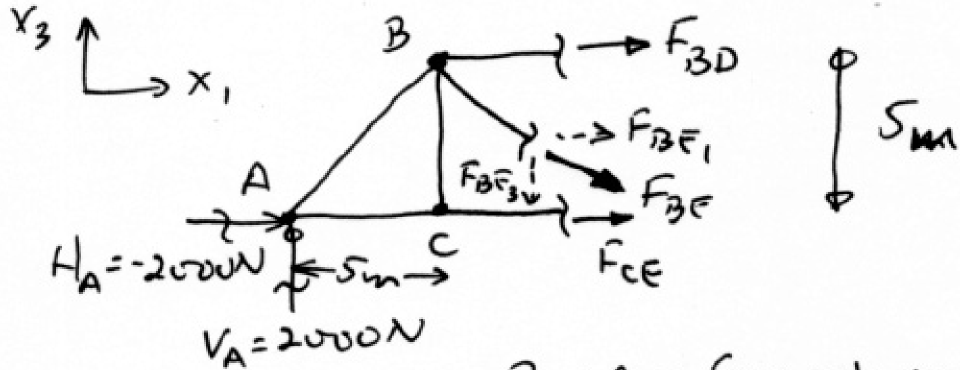
(c) <u>YES!</u> Consider the point D and equilibrium there from the forces from the three bars (BD, DE, DF) that connect there:



F_{DE} is the only bar that can carry vertical (x_3-direction) load. Since there must be equilibrium at each joint, then $\Sigma F_3 = 0$ tells us:

$$\boxed{F_{DE} = 0}$$

(d) Multiple sections can be chosen to be cut in order to determine the load in bar CE by the method of sections. A common choice is to cut through the section. Thus:



Resolve F_{BE} into components along x_1 and x_3. The geometry gives an angle of 45° and:

$$F_{BE_1} = F_{BE} \cos\theta = 0.707\, F_{BE}$$
$$F_{BE_3} = F_{BE} \sin\theta = 0.707\, F_{BE}$$

Now apply the equations of equilibrium:

$$\Sigma F_1 = 0 \ \overset{+}{\rightarrow} \Rightarrow -2000\,N + F_{CE} + F_{BE_1} + F_{BD} = 0$$

$$\Sigma F_3 = 0 \ +\uparrow \Rightarrow 2000\,N - F_{BE_3} = 0$$

$$\Rightarrow F_{BE_3} = 0.707\, F_{BE} = 2000\,N$$

$$\Rightarrow \quad F_{BF} = 2829\,N$$

$$\underset{(\text{about } A)}{\sum M = 0} \quad \circlearrowleft + \;\Rightarrow\; -F_{BD}(5m) \underbrace{- F_{BF_1}(5m) - F_{BF_3}(5m)}_{\substack{\text{or express as } F_{BF} \\ \text{acting over distance of bar} \\ AB = \sqrt{2(5m)^2} = \sqrt{2}(5m) \text{ SAME!}}} = 0$$

use the result of $F_{BF} = 2829\,N$ in the last equation:

$$-F_{BD} - 2829\,N(\sqrt{2}) = 0$$

$$\Rightarrow F_{BD} = -4000\,N$$

and use these results in the first equation:

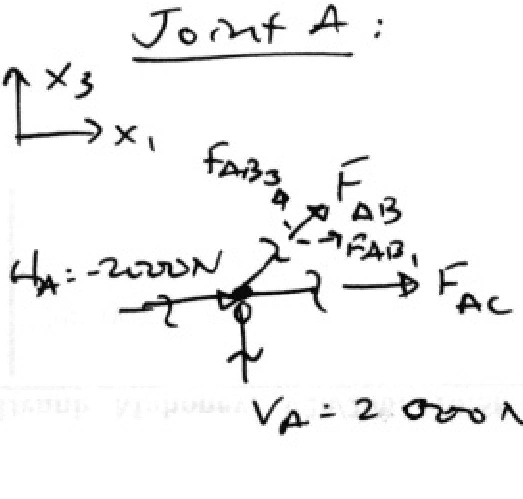
$$-2000N + F_{CE} + 2000N - 4000N = 0$$

$$\Rightarrow \boxed{F_{CE} = 4000\,N}$$

in $+x_1$ direction (a tensile force)

(e) To check the load in bar CF by the method of joints. Choose a joint at a support. Here, A is chosen:

<u>Joint A</u>:



then apply the two equations of equilibrium

Note that loads in diagonal bars can/must be resolved into components in the x_1 and x_3 directions. In all cases, the bars are at angles of 45° so:

$$x_1 \text{ component} = \cos\theta\,(\text{load}) = \frac{\sqrt{2}}{2}\,\text{load} = 0.707\,\text{load}$$

$$x_3 \text{ component} : \sin\theta\,(\text{load}) : \frac{\sqrt{2}}{2}\,\text{load} = 0.707\,\text{load}$$

For the method of joints, there are two equations of equilibrium (only force, moment cannot be used)

Here:

$$\sum F_1 = 0 \;\overset{+}{\rightarrow}\; \Rightarrow -2000\text{N} + F_{AC} + 0.707\,F_{AB} = 0$$

$$\sum F_3 = 0 \;+\!\uparrow\; \Rightarrow 2000\text{N} + 0.707\,F_{AB} = 0$$

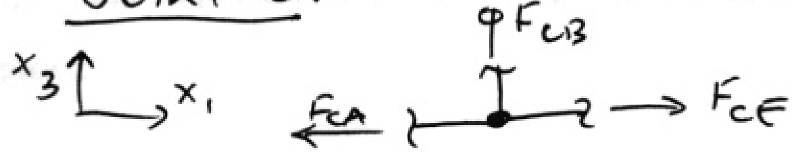
$$\Rightarrow F_{AB} = -2829\text{ N}$$

using in the first:

$$F_{AC} = 4000\text{ N}$$

Proceed to ...

<u>Joint C</u>:



$$\sum F_1 = 0 \;\overset{+}{\rightarrow}\; \Rightarrow F_{CE} - F_{CA} = 0$$

with $F_{CA} = 4000$ N

$$\Rightarrow \boxed{F_{CE} = 4000\text{ N}}$$ checks ✓

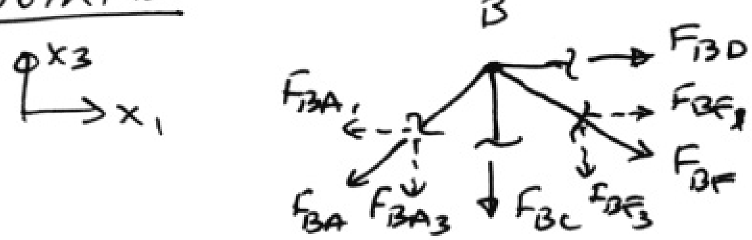
in the $+x_1$ direction

(f) Continue first with the Method of Joints:

Finalize with Joint C:

$\Sigma F_3 = 0 \; +\uparrow \;\Rightarrow\; F_{CB} = 0$

Now Joint B



$\Sigma F_1 = 0 \; \xrightarrow{+} \;\Rightarrow\; -0.707 F_{BA} + F_{BD} + 0.707 F_{BE} = 0$

$\Sigma F_3 = 0 \; +\uparrow \;\Rightarrow\; -0.707 F_{BA} - F_{BC} - 0.707 F_{BE} = 0$

using previous results

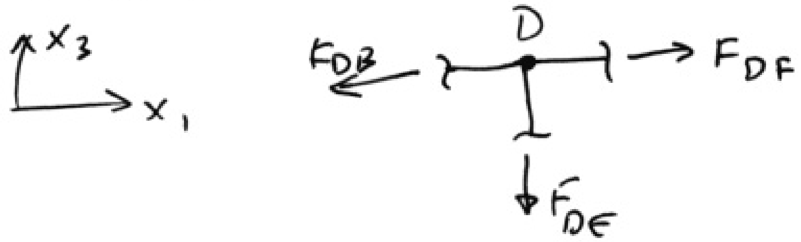
$F_{BE} = -F_{BA} = 2829\text{ N}$

and into ΣF_1 equation:

$2000\text{ N} + F_{BD} + 2000\text{ N} = 0$

$\Rightarrow F_{BD} = -4000\text{ N}$

Now Joint D

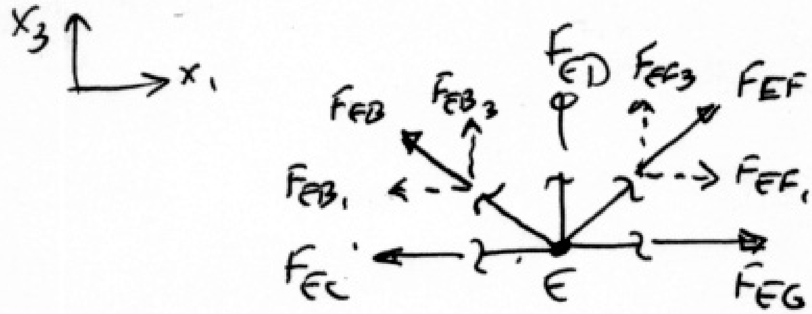


$\Sigma F_3 = 0 \; +\uparrow \;\Rightarrow\; -F_{DE} = 0$

$\Sigma F_1 = 0 \; \xrightarrow{+} \;\Rightarrow\; -F_{DB} + F_{DF} = 0$

$\Rightarrow F_{DF} = F_{DB} = -4000\text{ N}$

Next Joint E



$\Sigma F_1 = 0 \Rightarrow -F_{EC} + F_{EG} - 0.707 F_{EB} + 0.707 F_{EF} = 0$

$\Rightarrow -4000N + F_{EG} - 2000N + 0.707 F_{EF} = 0$

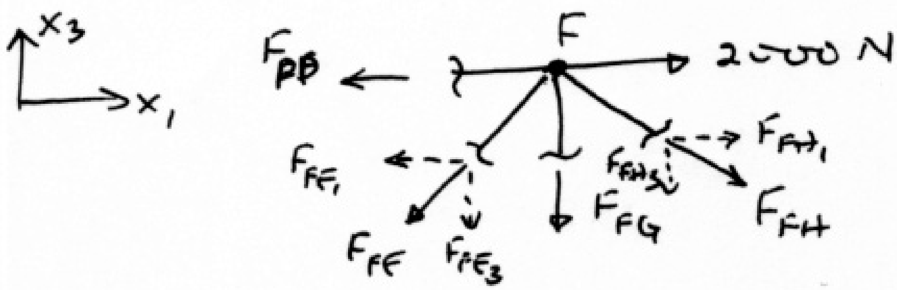
$\Sigma F_3 = 0 \Rightarrow 0.707 F_{EB} + F_{ED} + 0.707 F_{EF} = 0$

$\Rightarrow -F_{EB} = F_{EF} = -2829\ N$

using in ΣF_1 gives:

$F_{EG} = 8000\ N$

Next Joint F



$\Sigma F_1 = 0 \Rightarrow -0.707 F_{FE} - F_{FD} + 2000N + 0.707 F_{FH} = 0$

$\Rightarrow -0.707 F_{FE} + 0.707 F_{FH} = -6000\ N$

$\Sigma F_3 = 0 \Rightarrow -0.707 F_{FE} - F_{FG} - 0.707 F_{FH} = 0$

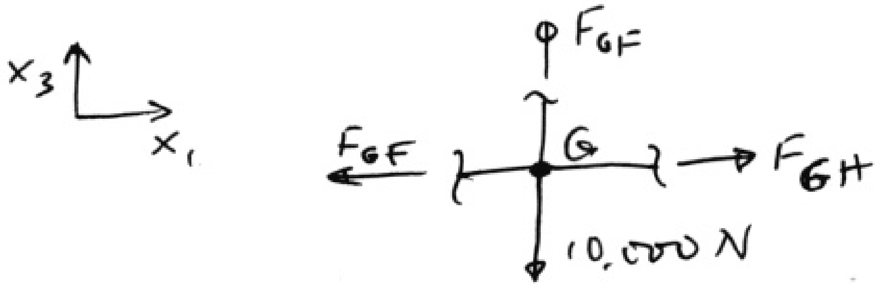
The $\Sigma F_1 = 0$ gives: $0.707 F_{FH} = -8000\text{ N}$

$$\Rightarrow F_{FH} = -11,315\text{ N}$$

using in ΣF_3 gives:

$$F_{FG} = 10,000\text{ N}$$

and to <u>Joint G</u>



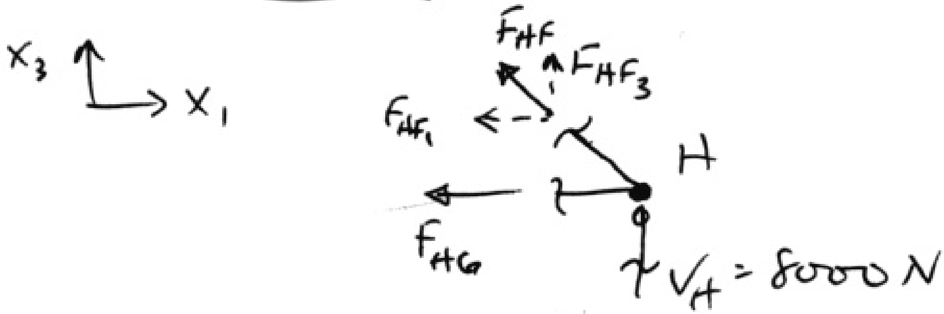
$$\Sigma F_1 = 0 \ \overset{+}{\rightarrow} \ \Rightarrow -F_{GE} + F_{GH} = 0$$

$$\Rightarrow F_{GH} = F_{GE} = 8000\text{ N}$$

$$\Sigma F_3 = 0 \ +\uparrow \ \Rightarrow \ F_{GF} - 10,000\text{ N} = 0$$ ✓ checks

(<u>NOTE</u>: with method of joints can check results)

Check at <u>Joint H</u>:

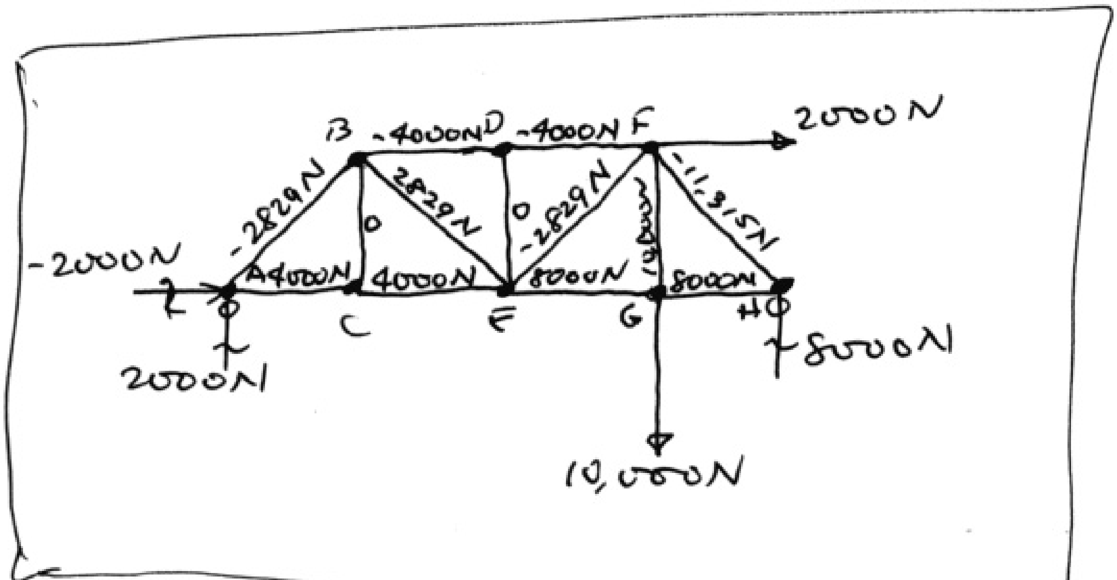


$$\Sigma F_1 = 0 \ \overset{+}{\rightarrow} \ \Rightarrow -0.707 F_{HF} - F_{HG} = 0$$ ✓ checks

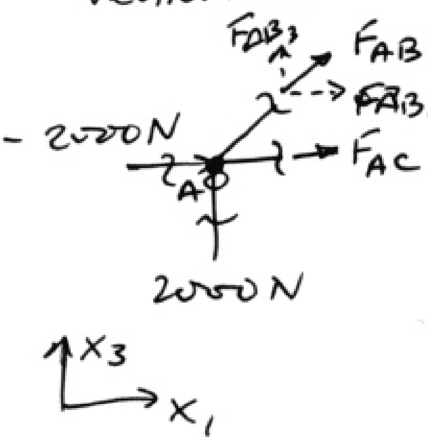
$$\Sigma F_3 = 0 \ +\uparrow \ \Rightarrow \ 0.707 F_{HF} + 8000\text{ N} = 0$$ ✓ checks

(17)

Summarize by drawing the truss and placing the bar load above each bar with (+) tension; and (−) compression



If do by method of sections, go through each section....



$$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow F_{AC} + F_{AB_1} - 2000\text{N} = 0$$

$$\Sigma F_3 = 0 \; +\uparrow \Rightarrow 0.707 F_{AB} + 2000\text{ N} = 0$$

$$\Rightarrow F_{AB} = -2829\text{N} \checkmark$$

and this results in

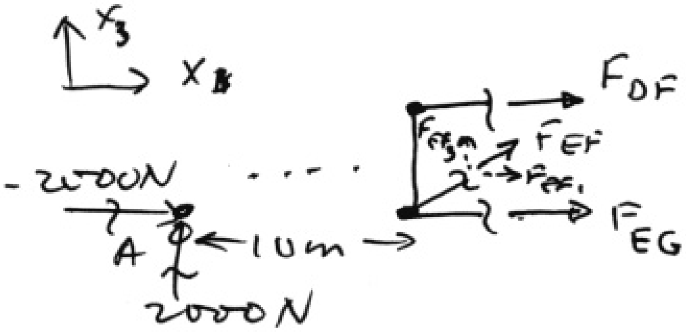
$$F_{AC} = 4000\text{N} \checkmark$$

($\checkmark$ = same as before)

We already had done the next section and found F_{BE}, F_{CF} and F_{DE} (see section (d))

Furthermore in section (c) we found $F_{DF} = 0$
We can use the same argument to show $F_{CB} = 0$.

Move to the next "bay/section" (DEFG)



$$\sum F_1 = 0 \xrightarrow{+} \Rightarrow -2000\,N + F_{DF} + 0.707 F_{EF} + F_{EG} = 0$$

$$\sum F_3 = 0 + \uparrow \Rightarrow 2000\,N + 0.707 F_{EF} = 0$$

$$\Rightarrow F_{EF} = -2829\,N \checkmark$$

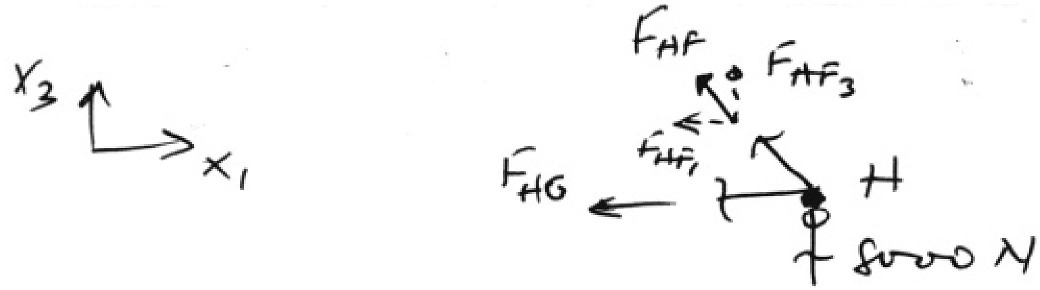
$$\sum M_A = 0 \circlearrowleft + \Rightarrow F_{DF}(5m) + 0.707 F_{EF}(10m) = 0$$

$$\Rightarrow F_{DF} = -4000\,N \checkmark$$

use in $\sum F_1 = 0$

$$\Rightarrow F_{EG} = 8000\,N \checkmark$$

for F_{FH} and F_{GH} cut around H:



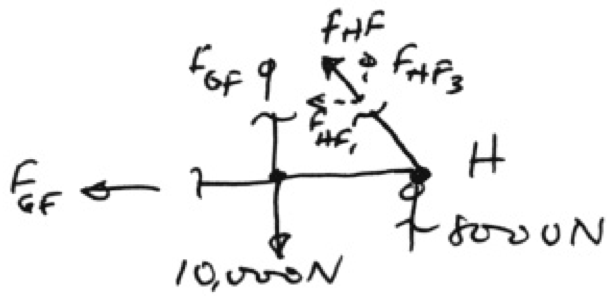
$\Sigma F_1 = 0 \xrightarrow{+} \Rightarrow -F_{HG} - 0.707 F_{HF} = 0$

$\Sigma F_3 = 0 +\uparrow \Rightarrow 8000N + 0.707 F_{HF} = 0$

$\Rightarrow F_{HF} = -11,315 N$ ✓

using in $\Sigma F_1 \Rightarrow$ $F_{HG} = 8000 N$ ✓

finally cut to get F_{FG}



$\Sigma F_3 : 0 \;+\uparrow \Rightarrow F_{GF} + 0.707 F_{HF} + 8000 N - 10,000 N = 0$

$\Rightarrow F_{GF} = 10,000 N$ ✓

All the same as before
(results in same diagram)

C&P PSET 3 Solutions

1. **12 points**

a. Convert 27_{10} from decimal into binary and hexadecimal notation

The conversion from decimal to binary is carried out using the following algorithm:

- Divide the value by 2 and record the remainder
- As long as the quotient obtained is not 0, continue to divide the newest quotient by 2 and record the remainder
- Now that a quotient of 0 has been obtained, the binary representation of the original value consists of the remainders listed from right to left in the order they were recorded

(i) 27_{10} is converted into binary as shown below:

Number	Remainder
27	1
13	1
6	0
3	1
1	

$$\mathbf{27_{10} = 11011_2}$$

Similarly, the conversion from decimal to hexadecimal is carried out by dividing by 16.

(ii) 27_{10} is converted into hexadecimal as shown below:

Number	Remainder
27	1
11	

$$\mathbf{27_{10} = 1B_{16}}$$

b. Convert FA_{16} from hexadecimal into binary and decimal

The conversion from hexadecimal to binary is carried out as follows:

- Convert each hexadecimal digit into the equivalent nibble (group of 4 bits)
- The final binary representation is a composition of the individual nibbles going from left to right of the most significant hexadecimal digit.

(i) FA_{16} can be represented in binary as:

$F_{16} = 1111_2$
$A_{16} = 1010_2$

$$\mathbf{FA_{16} = 11111010_2}$$

The conversion from hexadecimal to decimal is carried out as follows:

- ❖ Convert each hexadecimal digit into the equivalent decimal digit
- ❖ Multiply each equivalent decimal digit by $16^{(position-1)}$

(ii) FA_{16} can be represented in decimal as:

$F_{16} = 15_{10}$
$A_{16} = 10_{10}$

$FA_{16} = 15_{10}*16^{(2-1)} + 10_{10}*16^{(1-1)}$
$= 250_{10}$

$$\mathbf{FA_{16} = 250_{10}}$$

c. Convert 10111001_2 from binary into decimal and hexadecimal

The conversion from binary to decimal is carried out as follows:

- ❖ Multiply each bit value by $2^{(position-1)}$

(i) 10111001_2 can be represented in decimal as:

$10111001_2 = 1*2^7 + 1*2^5+1*2^4+1*2^3+1$
$= 128 + 32 + 16 + 8 + 1$
$= 185_{10}$

$$\mathbf{10111001_2 = 185_{10}}$$

The conversion from binary to hexadecimal is carried out as follows:

- ❖ Start from right to left
- ❖ Break bit patterns into nibbles
- ❖ Add 0's to the front to complete the leftmost nibble
- ❖ Convert the nibbles into hexadecimal symbols

(ii) 10111001_2 can be represented in hexadecimal as:

$10111001_2 = 1011\ 1001$
$= B9_{16}$

$10111001_2 = B9_{16}$

2. **35 points**

a. Write an algorithm to carry out integer subtraction using only addition. Assume that the numbers are stored in num1 and num2, and the operation to be performed is num1-num2.

Precondtions – Legal 8 bit integers are stored in locations num1 and num2

Inputs – None

Outputs – Display the result of the subtraction to the user

Postconditions – Result of the subtraction operation stored in the location sum

Algorithm

1. Read in the number from num1 and store it in sum
2. Compute the 2's complement of num2 as follows:
   a. Compute the 1's complement by computing the negation of num2
   b. Add 1 to the result
3. Add the 2's complement to num2 to sum
4. Display the result of the subtraction to the user
5. Store the result in sum

b. Implement your algorithm as a Pep7 program. Turn in a hard copy of your assembly code and an electronic copy of your code

```
;Program to carry out subtraction using addition
;Programmer : Jane B
;Date Created:September 22,2004
;Date Last Modified: September 27, 2004

       BR     Main; branch to location main

num1: .BYTE       d#1   ; byte to store num1
num2: .BYTE       d#1   ; byte to store num2
sum1: .BYTE       d#1   ; byte to store result of the subtraction
sum2: .BYTE       d#1   ; byte to capture any overflow information

Main: LOADA h#0020, I   ; load h#0020 into accumulator
      STBYTA num1, d    ; store h#20 into location num1

      LOADA h#0030, I   ; load h#0030 into accumulator
      STBYTA num2, d    ; store h#30  into num2

      NOTA              ; negate num2 to find the 1s complement
      ADDA d#1, i       ; find the two's complment by adding 1

      STOREA sum1, d    ; store the result of the negation into sum

      LOADA h#0000, I   ; initialize the accumulator to 0
      LDBYTA num1,d     ; load num1 into accumulator

      ADDA sum1, d      ; add the 2s complement to the accumulator

      STOREA sum1,d     ; store the result of the negation into input

      DECO sum1,d       ; display the result of the negation

      STOP              ; stop the processing

     .END                ; end of the program
```

c. Implement your algorithm as an Ada95 program. Turn in a hard copy of your code listing and an electronic copy of your code.

```
------------------------------------------------------------
--| Program to demonstrate 2s complement using Ada95
--| Programmer: Joe B
--| Date Created: September 20, 2004
--| Date Last Modified : September 21,2004
------------------------------------------------------------


with Ada.Text_Io;

procedure Demo_2_Complement is
   type Byte is mod 256;

   Num1  : Byte := 128;
   Num2  : Byte := 10;

   Sum    : Byte;

begin
   --convert num2 into its 2's complement
   -- find the 1's complment by negating num2
   Sum := not Num2;
   -- find the 2's complement by adding 1 to the 1's complement
   Sum := Sum+1;

   -- compute the subtraction by adding num1 to sum
   Sum := Sum + Num1;

   Ada.Text_Io.Put("The result of the subtraction operation is : ");
   if Sum > 127 then
      --then the result of the subtraction is negative
      Ada.Text_Io.Put("-");
      Ada.Text_Io.Put(Byte'Image(255-Sum+1));
   else
      Ada.Text_Io.Put_Line(Byte'Image(Sum));
   end if;

end Demo_2_Complement;
```