Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

Unified Engineering
Fall 2004

Problem Set #9
Solutions

T11 SOLUTIONS BY WAITZ

3 EQNS:
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$\frac{T_T}{T} = 1+\frac{\gamma-1}{2}M^2$$
$$\frac{P_T}{P} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}$$

a) $\frac{T_T}{T} = 1+0.2(0.8)^2 = 1.128 \Longrightarrow \boxed{T_T = 245K \quad T=T_{atm}=217}$

$\frac{P_T}{P} = 1.524 \quad \boxed{P_T = 34.4kP_a,\ P=P_{atm}=22.6kP_a}$

b) T_T & P_T ARE SAME AS ABOVE (INLET IS ADIABATIC & Q-S SO STAG. QUANTITIES ARE CONSTANT & WE HAVEN'T CHANGED REFERENCE FRAMES)

$T = \frac{T_T}{1+\frac{\gamma-1}{2}M^2} \quad \boxed{T_T = 245K,\ T=233K}$

$P = \frac{P_T}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}} \quad \boxed{P_T = 34.4kP_a,\ P=29kP_a}$

c) $P_T = 40(34.4) = 1376kP_a$

$P = \frac{P_T}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1}}, \quad M=0.03$

$\boxed{\begin{array}{l} P_T = 1376kP_a \\ P = 1375kP_a \end{array}}$

$\frac{T_{T_a}}{T_{T_*}} = (40)^{0.4/1.4} = 2.87$

$T_a = T_{T_a}/1+\frac{\gamma-1}{2}M^2$

$\boxed{\begin{array}{l} T_{T_a} = 703K \\ T_a = 703K \end{array}}$

d) $\frac{T_T}{T} = 1+\frac{\gamma-1}{2}M^2$, $M=3$ $\quad T_{atm} = 217K$

$\frac{P_T}{P} = \left[1+\frac{\gamma-1}{2}M^2\right]^{\gamma/\gamma-1}$ $\quad p_{atm} = 7.5kPa$

$$\boxed{\begin{array}{ll} T_T = 607.6\,K & T = T_{atm} = 217K \\ P_T = 275.5\,kPa & p = p_{atm} = 7.5kPa \end{array}}$$

e) IF $T_{T_1} = 607.6$ & $T_{T_2} = 703K$

THEN $\frac{T_{T_2}}{T_{T_1}} = 1.16$ & $\boxed{\frac{P_{T_2}}{P_{T_1}} = (1.16)^{\gamma/\gamma-1} = 1.67}$

THIS SHOWS THAT AS $M\uparrow$ THE RAM PRESSURIZATION (DUE TO RAMMING AIR IN THE INLET) CAN BE QUITE SIGNIFICANT. INDEED, FOR FLIGHT AT VERY HIGH SPEEDS (> M=3) COMPRESSORS AREN'T REQUIRED. THE ENGINES ARE CALLED <u>RAMJETS</u>. NOTE THAT THEY STILL NEED SOMETHING TO GET THEM FROM M=0 TO M=3. IN SOME CASES A ROCKET IS USED (CRUISE MISSILES FOR EXAMPLE), IN OTHER CASES THE ENGINE IS CAPABLE OF BEING RE-CONFIGURED (BYPASSING FLOW AROUND THE COMPRESSOR) IN FLIGHT.

T12 SOLUTIONS BY WAITZ

a) $s - s_0 = c_p \ln\left(\frac{T}{T_0}\right) - R \ln\left(\frac{p}{p_0}\right)$

i) ALL WEIGHTS REMOVED INSTANTANEOUSLY

$p_0 = 405.3\,kPa \longrightarrow p = 101.325\,kPa$
$T_0 = 300K \longrightarrow T = 235.4\,K$

$\Delta s = 1003.5 \ln\left(\frac{235.4}{300}\right) - 287 \ln\left(\frac{101325}{405300}\right)$

$\boxed{\Delta s = 154.5\ J/kg\text{-}K}$

ii) $p_0 = 405.3\,kPa \longrightarrow p = 101.3\,kPa$
$T_0 = 300K \longrightarrow T = 221K$

$\boxed{\Delta s = 91.3\ J/kg\text{-}K}$

NOTE: DON'T HAVE TO DO 2 steps JUST INITIAL & FINAL SINCE S IS A PROPERTY ∴ FUNCT. OF STATE (NOT PATH!)

iii) $\boxed{\Delta s = 0}$ (PLUG IT IN AND CHECK IF YOU LIKE)

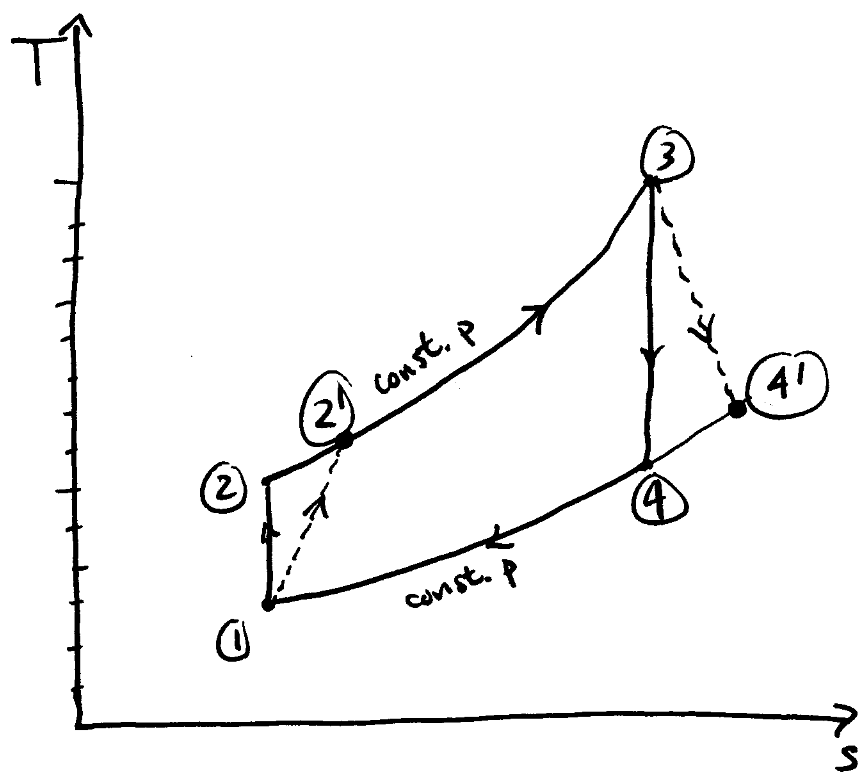
SO WHAT DOES THIS TELL US?

AS DEGREE OF IRREVERSIBILITY (FREE EXPANSION IN THIS CASE) INCREASES, WORK ↓ AND Δs ↑

ENTROPY IS A MEASURE OF THE LOST OPPORTUNITY TO DO WORK.

b) $P_1 = 100\,kPa,\ T_1 = 300K$ — $\Delta s_{1-2} = 0$
$P_2 = 1300\,kPa,\ T_2 = 624K$ — $\Delta s_{2-3} = 810.9\ J/kg\text{-}K$
$P_3 = 1300\,kPa,\ T_3 = 1400K$ — $\Delta s_{3-4} = 0$
$P_4 = 100\,kPa,\ T_4 = 673K$ — $\Delta s_{4-1} = -810.9\ J/kg\text{-}K$

b) & c)



DASHED LINES = IRREV. PROCESSES FOR COMP. & TURB.

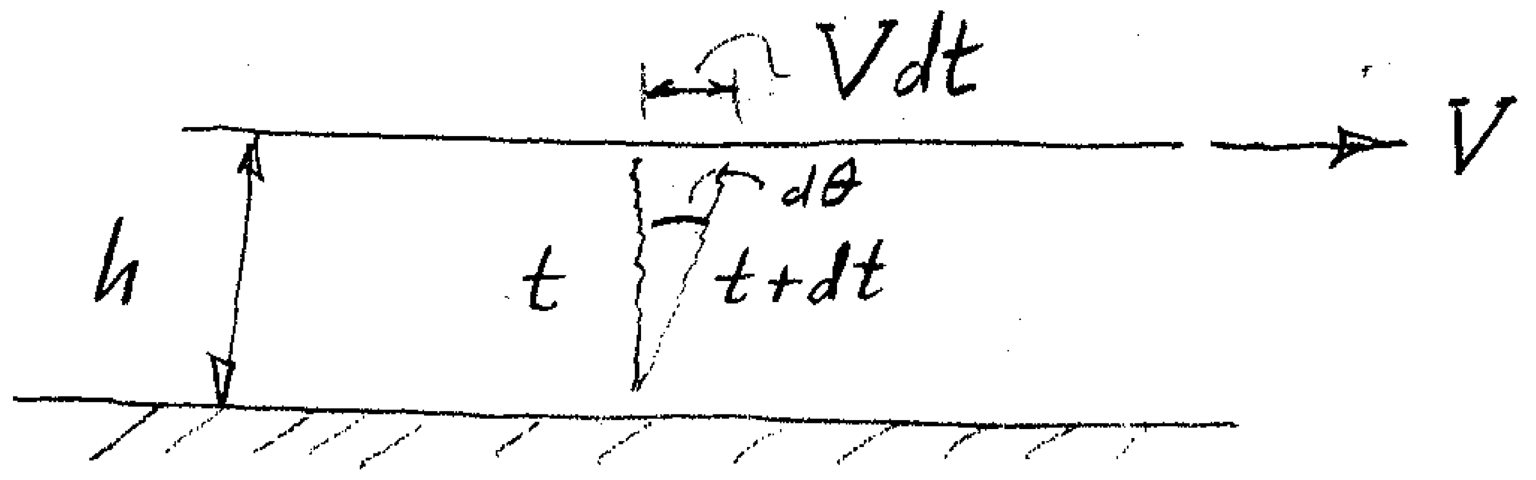
T13 Solution by Waitz

Energy can take many different forms (kinetic energy, internal energy, potential energy, chemical energy). Energy can be exchanged from one form to another (e.g. using fuel to get power out of an engine, or using an electric stove to heat water, or dropping a rock from your hand to the ground) but the only way to change the total energy (the sum of all the various forms of energy) of a system is to add or remove heat or for the system to do work or have work done on it. If heat is added to, or if work is done on a system its energy increases. This is a statement of the First Law of Thermodynamics. It is a very good law but it allows many processes that are not observed in our world.

If I touch a hot and a cold brick together I get two medium temperature bricks. The First Law of Thermodynamics also allows that the opposite can happen. It allows that two medium temperature bricks in contact with each other can spontaneously produce a hot brick and a cold brick, as long as the total energy of the two bricks together hasn't changed. This is not observed in our world, as are many other "reverse" processes: the spontaneous un-mixing of two gases in a container (the reverse of perfume spreading across a room), low pressure air spontaneously collecting itself into a small high pressure volume (the reverse of a balloon breaking), a rock collecting heat from the ground and spontaneously converting it to kinetic and then potential energy to jump into your hand (the reverse of dropping a rock from your hand). Of course, it is possible to reverse all of these processes but only by using work (they don't spontaneously happen). So when we say something is irreversible, we mean that it can not be reversed *without the application of work from the surroundings*.

All real processes are to some degree irreversible. Things that cause processes to be irreversible are unrestrained expansion, friction, molecular diffusion, heat transfer across a finite temperature difference, etc. The Second Law of Thermodynamics provides the tool for sorting out which processes allowed by the First Law are observed and which are not. For processes that are observed in our world, a thermodynamic property called entropy will increase (if calculated and summed for both the system and the surroundings). The Second Law also allows us to characterize the "degree of irreversibility" by measuring how much the entropy of the system and surroundings increases as a result of a process. Some processes have more irreversibility (e.g. more friction) than others. The more irreversibilities in a device, the less efficient it is. Reversible processes are useful idealizations that let us understand the best that can be achieved (we know that a pendulum without friction will run forever, but how efficient could an engine be without friction?). For example, if the power plant at MIT were ideal (no friction, no heat transfer across finite temperature differences, no free expansions, etc.), then all the processes that make it work could run equally well forward and reverse without application of additional work from the surroundings. In this ideal case, application of the First Law of Thermodynamics can be used to show that for each unit of energy that goes into the power plant as fuel, about half that unit of energy comes out as useful work. However, for the real MIT power plant , with all its irreversibilities, each unit of fuel energy only results in about 1/4 of a unit of useful work. Quite a significant impact indeed.

a) Vertical line of fluid, at times t and $t+dt$



Given: $h = 1\,mm = 0.001\,m$

$V = 1\,m/s$

From geometry: $d\theta = \frac{V\,dt}{h}$ (small angle approximation)

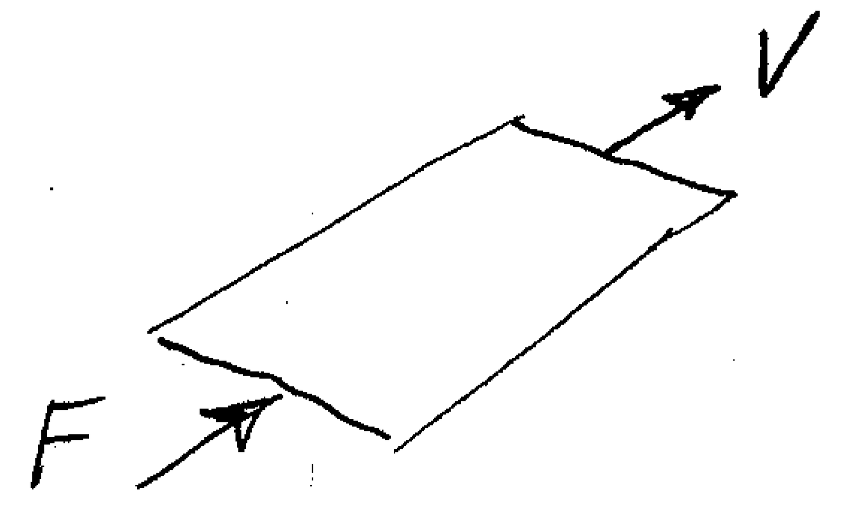
or $\frac{d\theta}{dt} = \dot{\theta} = \frac{V}{h} = \frac{1\,m/s}{0.001\,m} = \boxed{1000\ rad/s}$

$$\boxed{\tau = \mu\dot{\theta} = 1.78\times10^{-5}\,kg/m\cdot s \cdot 1000\,rad/s = 1.78\times10^{-2}\,Pa}$$

b) For steady motion, must have $F = F_{shear}$

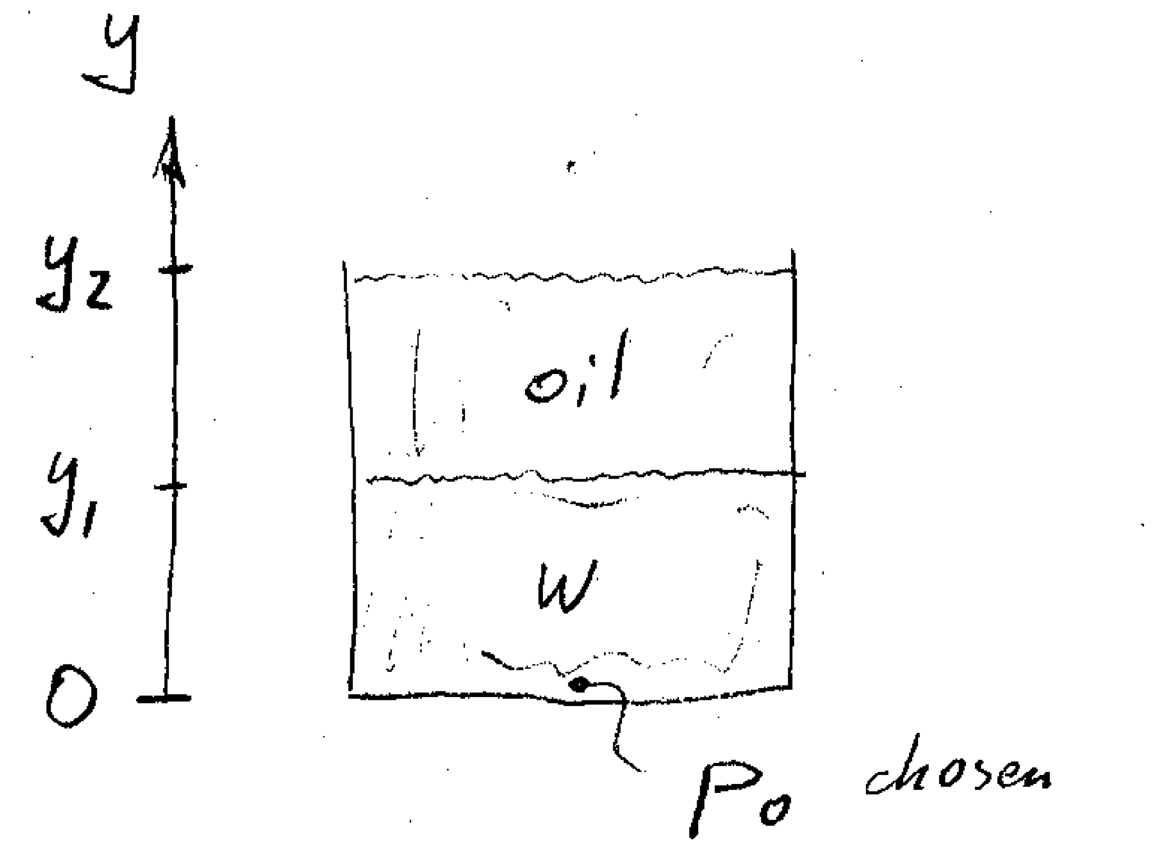
$F = \tau \cdot A$, $A = 8.5\,in \cdot 11\,in \cdot \frac{1}{(39.37\,in/m)^2} = 0.0603\,m^2$

so $\boxed{F = 1.78\times10^{-2}\,Pa \cdot 0.0603\,m^2 = 1.074\times10^{-3}\,N}$ in direction of V



a) For either fluid, $\frac{dp}{dy} = -\rho g$

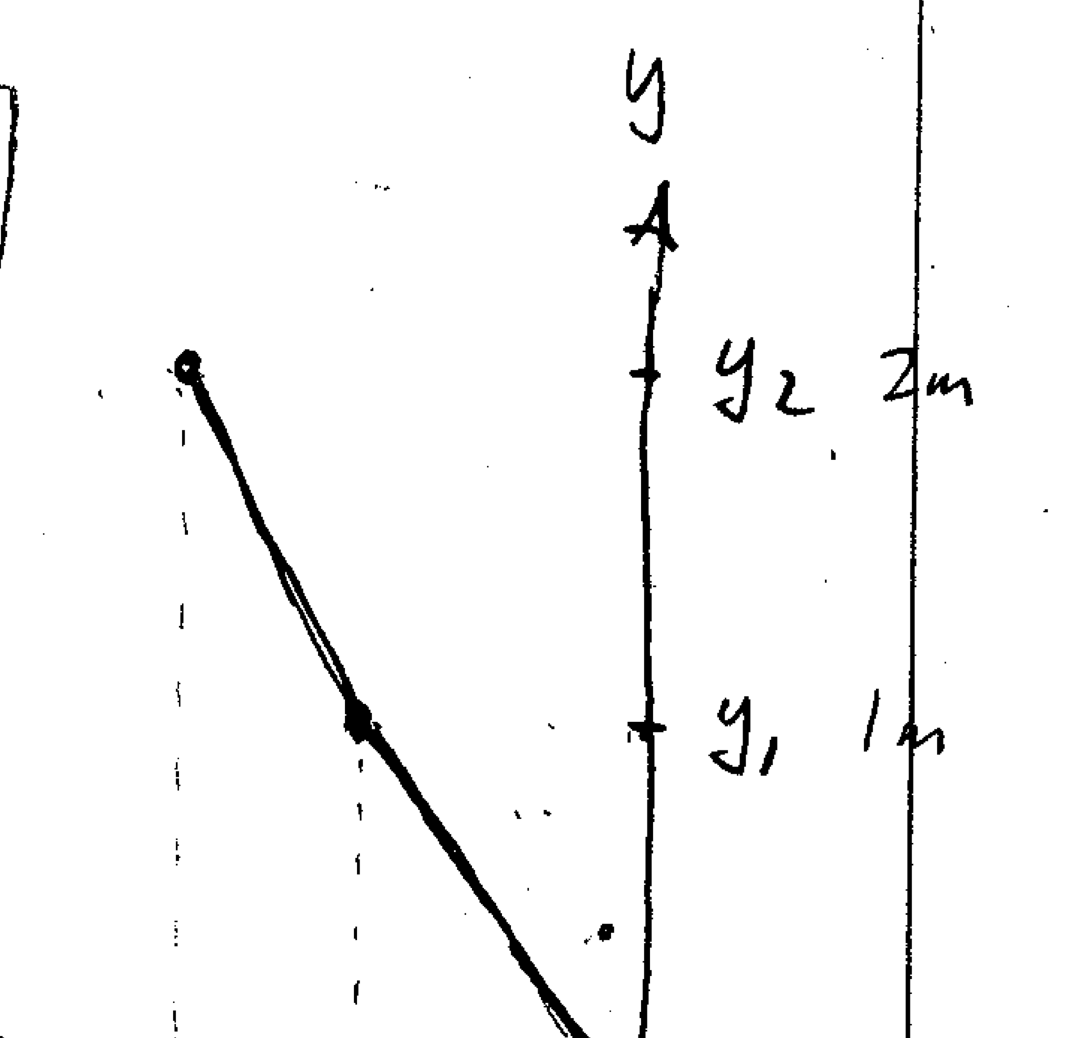
or $p = -\rho g y + \cancel{c}$



For water portion: $\rho_w = 1000\ kg/m^3$

$p(y=0) = \cancel{c} = p_0$

$\rightarrow \boxed{p_w - p_0 = -\rho_w g y \quad for \quad 0 < y < y_1}$



For oil portion: $\rho_{oil} = 850\ kg/m^3$

$p_{oil} = -\rho_{oil} g y + \cancel{c}$

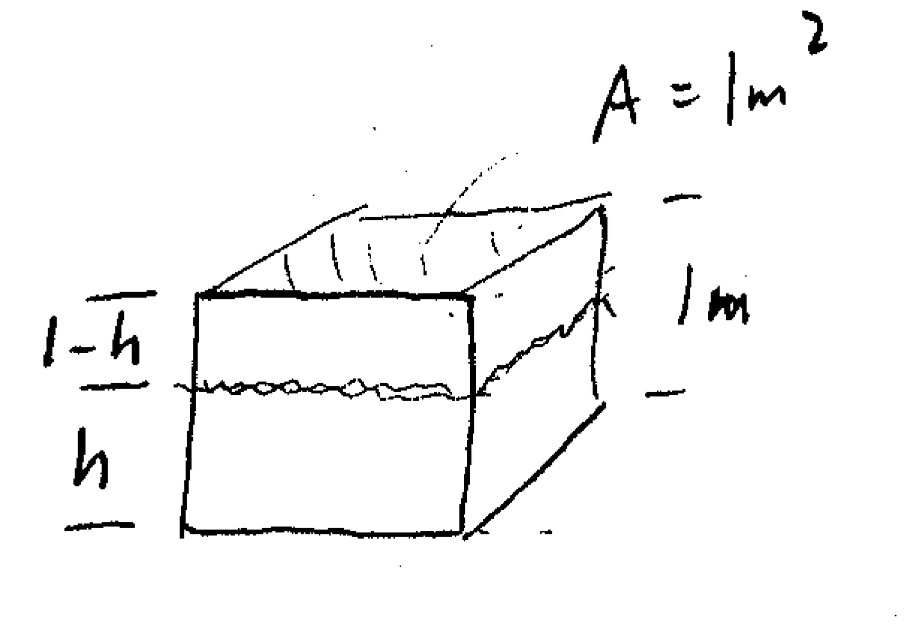
At $y = y_1$ must have $p_w = p_{oil}$

$p_0 - \rho_w g y_1 = -\rho_{oil} g y_1 + \cancel{c}$

or $\cancel{c} = p_0 + (\rho_{oil} - \rho_w) g y_1$

$\boxed{p_{oil} - p_0 = (\rho_{oil} - \rho_w) g y_1 - \rho_{oil} g y \quad for \quad y_1 < y < y_2}$

Numerically: $p(y_1) - p_0 = -9800\ Pa$
$p(y_2) - p_0 = -18130\ Pa$



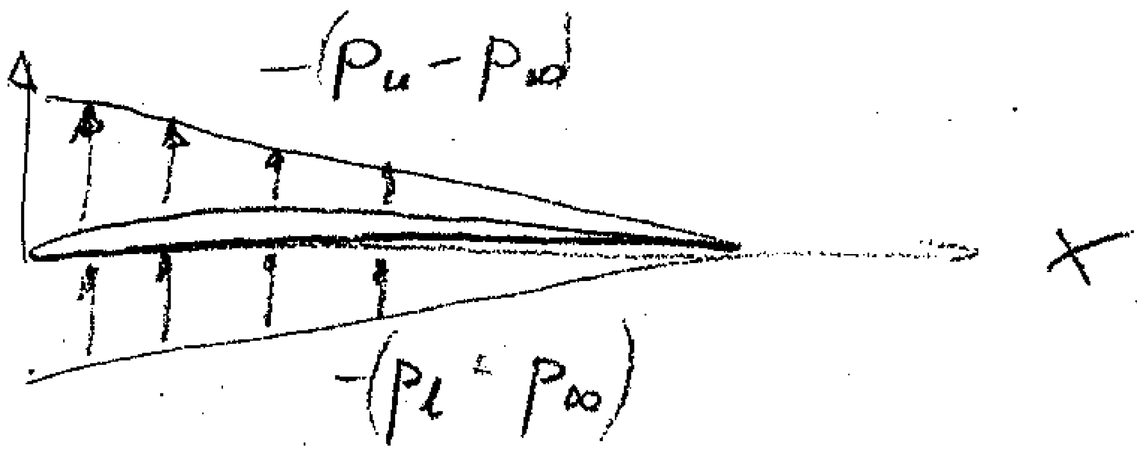
b) F_{buoy} = displaced weight
$= \rho_{oil} g A h + \rho_w g A (1-h)$

$F_{grav} = -\rho_{poly} g A \cdot 1$

when neutral must have $F_{buoy} + F_{grav} = 0$

or $\rho_{oil} g A h + \rho_w g A (1-h) - \rho_{poly} g A \cdot 1 = 0$

$\rightarrow \boxed{h = \frac{\rho_{poly} - \rho_w}{\rho_{oil} - \rho_w} \cdot 1m = 0.333\ m}$



a) Thin airfoil $\rightarrow$ neglect $\frac{dy}{dx}$

From notes: $L' = \cos\alpha \int_0^c (p_l - p_u)\,dx$

$$= \cos\alpha \int_0^c (p_{l_0} - p_{u_0})\left(1 - \frac{x}{c}\right)dx$$

$$= \cos\alpha\,(p_{l_0} - p_{u_0})\left[x - \frac{1}{2}\frac{x^2}{c}\right|_0^c$$

$$\boxed{L' = \cos\alpha\,(p_{l_0} - p_{u_0})\frac{c}{2} = 622.6 \text{ N/m}}$$

$$M'_{LE} = \int_0^c -(p_l - p_u)\,x\,dx$$

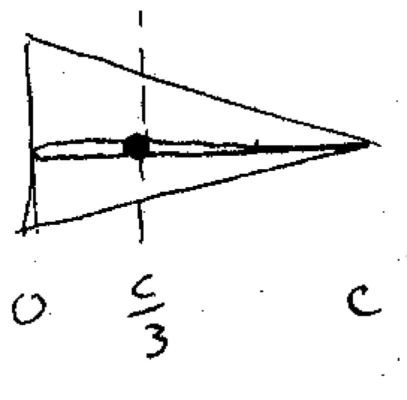
$$= \int_0^c -(p_{l_0} - p_{u_0})\left(x - \frac{x^2}{c}\right)dx$$

$$= -(p_{l_0} - p_{u_0})\left[\frac{1}{2}x^2 - \frac{1}{3}\frac{x^3}{c}\right|_0^c$$

$$M'_{LE} = -(p_{l_0} - p_{u_0})\frac{c^2}{6} = -104.17 \text{ N}$$

$$\boxed{M'_{c/4} = M'_{LE} + L'\frac{c}{4} = -26.3 \text{ N}}$$

$$\boxed{x_{cp} = -M'_{LE}/L' = 0.167 \text{ m} = \frac{c}{3}}$$



b) $L' = \frac{1}{2}\rho V_\infty^2\, c\, c_l$

Given: $c_l = 0.8$, $\rho = 1.225$ kg/m^3 (sea level)

$$\rightarrow \boxed{V_\infty = \sqrt{\frac{2L'}{\rho c c_l}} = 50.4 \text{ m/s}}$$

Given: $D = f(\rho_\infty, V_\infty, c, g)$ $\qquad (*)$

Parameter	Units	
D	$m\ell t^{-2}$	(force)
ρ_∞	$m\ell^{-3}$	(density)
V_∞	ℓt^{-1}	(speed)
c	ℓ	(length)
g	ℓt^{-2}	(accel.)
$N = 5$	$K = 3$	(m, ℓ, t)

$\rightarrow$ Expect $N - K = 2$ Π groups.

By inspection or experience:

$$\Pi_1 = \frac{D}{\rho_\infty V_\infty^2 c^2} \qquad \frac{\cancel{m}\cancel{\ell}\cancel{t^{-2}}}{\cancel{m}\cancel{\ell^{-3}}\cancel{\ell^2}\cancel{t^{-2}}\cancel{\ell^2}} \checkmark \text{ dimensionless}$$

$$\Pi_2 = \frac{V_\infty^2}{cg} \qquad \frac{\cancel{\ell^2}\cancel{t^{-2}}}{\cancel{\ell}\cancel{\ell}\cancel{t^{-2}}} \checkmark \text{ dimensionless}$$

Alternative, more traditional Π groups are

$$\Pi_1 = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^2 c^2} \equiv C_D \quad \text{drag coefficient}$$

$$\Pi_2 = \frac{V_\infty}{\sqrt{cg}} \equiv Fr \quad \text{Froude number}$$

Dimensionless version of $(*)$ is $\Pi_1 = \bar{f}(\Pi_2)$

or $C_D = f(Fr)$

QED.

PAL 10/26/04

(1)

Grading Scheme

(see last page)

UNIFIED ENGINEERING

Problem Set # 9 - SOLUTIONS

9(M)-1 for all cases, recall rules for tensor/indicial notation:

- Latin subscripts take on values 1, 2, 3
- Greek subscripts take on values 1, 2
- when subscript is repeated in one term, it is a "dummy index" and is summed on
- When subscript appears only once on left side of equation in one term, it is a "free index" and represents separate equations

So . . .

(a) $C_{mn} = R_{mnjk} \gamma_j z_k$ (for $m = 1, n = 3$)

$\Rightarrow C_{13} = R_{13jk} \gamma_j z_k$

- j and k are dummy indices and are summed on from 1 to 3 (they are latin subscripts)

$\Rightarrow$ same as $C_{13} = \sum_{j=1}^{3} \left(\sum_{k=1}^{3} R_{13jk} \gamma_j z_k \right)$

Thus:

$$\boxed{\begin{aligned} C_{13} = {} & R_{1111}\gamma_1 z_1 + R_{1112}\gamma_1 z_2 + R_{1113}\gamma_1 z_3 \\ & + R_{1121}\gamma_2 z_1 + R_{1122}\gamma_2 z_2 + R_{1123}\gamma_2 z_3 \\ & + R_{1131}\gamma_3 z_1 + R_{1132}\gamma_3 z_2 + R_{1133}\gamma_3 z_3 \end{aligned}}$$

(b) $E = \frac{1}{2}\sigma_{\alpha\beta}\epsilon_{\alpha\beta}$

- α and β are dummy indices and are summed on from 1 to 2 (They are Greek subscripts)

$\Rightarrow$ same as $\epsilon = \frac{1}{2}\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2}\sigma_{\alpha\beta}\epsilon_{\alpha\beta}$

Thus: $\boxed{E = \frac{1}{2}\left\{\sigma_{11}\epsilon_{11} + \sigma_{12}\epsilon_{12} + \sigma_{21}\epsilon_{21} + \sigma_{22}\epsilon_{22}\right\}}$

(c) $H_i = b_{\alpha\beta}P_{\alpha\beta}n_i$

- α and β are dummy indices and are summed on from 1 to 2 (they are Greek subscripts)
- i is a free index and indicates there are 3 equations (3 - Latin subscripts)

$\Rightarrow$ same as $H_i = n_i\sum_{\alpha=1}^{2}\sum_{\beta=1}^{2} b_{\alpha\beta}P_{\alpha\beta}$

Thus:

$i=1$: $H_1 = n_1(b_{11}P_{11} + b_{12}P_{12} + b_{21}P_{21} + b_{22}P_{22})$

$i=2$: $H_2 = n_2(b_{11}P_{11} + b_{12}P_{12} + b_{21}P_{21} + b_{22}P_{22})$

$i=3$: $H_3 = n_3(b_{11}P_{11} + b_{12}P_{12} + b_{21}P_{21} + b_{22}P_{22})$

(d) $\sigma_{31} = l_{3m'}\, l_{1n'}\, \sigma_{m'n'}$

- m' and n' are <u>dummy indices</u> and are summed on from 1 to 3 (they are latin subscripts)

$\Rightarrow$ same as $\sigma_{31} = \sum_{m'=1}^{3} \sum_{n'=1}^{3} l_{3m'}\, l_{1n'}\, \sigma_{m'n'}$.

Thus:

$$\boxed{\begin{aligned}\sigma_{31} = {} & l_{31} l_{11} \sigma'_{11} + l_{31} l_{12} \sigma'_{12} + l_{31} l_{13} \sigma'_{13} \\ & + l_{32} l_{11} \sigma'_{21} + l_{32} l_{12} \sigma'_{22} + l_{32} l_{13} \sigma'_{23} \\ & + l_{33} l_{11} \sigma'_{31} + l_{33} l_{12} \sigma'_{32} + l_{33} l_{13} \sigma'_{33}\end{aligned}}$$

(e) $f_{pq}\left(\dfrac{\partial g_q}{\partial t}\right) + x_p = 0$

- q is a <u>dummy index</u> and is summed on from 1 to 3 (it is a latin subscript)
- p is a <u>free index</u> (repeated only once in a term) and indicates there are 3 equations (3 - latin subscript)

$\Rightarrow$ same as $\sum_{q=1}^{3} f_{pq} \left(\frac{\partial g_q}{\partial t} \right) + x_p = 0$

Thus:

$$\boxed{\begin{array}{ll} p=1: & f_{11} \frac{\partial g_1}{\partial t} + f_{12} \frac{\partial g_2}{\partial t} + f_{13} \frac{\partial g_3}{\partial t} + x_1 = 0 \\ p=2: & f_{21} \frac{\partial g_1}{\partial t} + f_{22} \frac{\partial g_2}{\partial t} + f_{23} \frac{\partial g_3}{\partial t} + x_2 = 0 \\ p=3: & f_{31} \frac{\partial g_1}{\partial t} + f_{32} \frac{\partial g_2}{\partial t} + f_{33} \frac{\partial g_3}{\partial t} + x_3 = 0 \end{array}}$$

For getting the following number of problems correct:

5 → A+
4 → B+
3 → C+
2 → C-
1 → D

Use other + and - for small errors, etc.
Being "correct" means that the basic item is correct (not all details -- use a "minus" as needed)