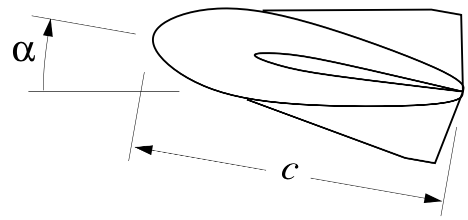
1. (30 %) A proposed winged blimp design flies at some angle of attack α, and uses both aerodynamic lift and aerostatic lift (buoyancy) to generate its total lift force L. The blimp has a given shape, but its length c is as yet undecided.



a) In addition to the given α and c, list all the remaining physical parameters which significantly influence L.

$$g(L, \alpha, c, \ldots) = 0$$

b) Determine a set of nondimensional parameters (or Pi products) which describe this situation.

c) Identify the nondimensional parameter which determines whether or not the aerodynamic force is significant compared to the buoyancy force.

Question 1. Solution

a)

Parameter	Units
L	ml/t^2
α	—
c	l
V_∞	l/t
μ_∞	m/tl
ρ_∞	m/l^3
g	m/t^2
a_∞	l/t (not likely to be important for slow blimp)

minimum set for full credit: L, α, c, V_∞, μ_∞, ρ_∞, g

b) $N = 8$ $K = 3$ $\rightarrow$ $N-K = 5$ Π groups

$$\Pi_1 = \frac{L}{\frac{1}{2}\rho V_\infty^2 c^2} \equiv C_L$$

$$\Pi_2 = \alpha \equiv \alpha$$

$$\Pi_3 = \frac{\rho_\infty V_\infty c}{\mu_\infty} \equiv Re$$

$$\Pi_4 = \frac{V_\infty^2}{gc} \equiv Fr^2 \quad \text{(Froude number)}^2$$

$$\Pi_5 = \frac{V_\infty}{a_\infty} = M_\infty$$

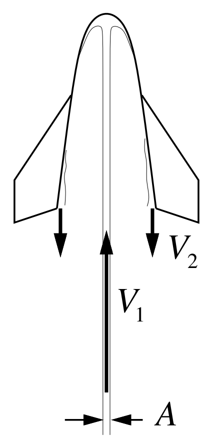
$$C_L = C_L(\alpha, Re, Fr, M_\infty)$$

, M_∞ not likely to be important

c) $$\frac{\text{aerodynamic lift}}{\text{aerostatic lift}} \sim \frac{\rho V_\infty^2 c^2}{\rho g c^3} \sim \frac{V^2}{gc} \equiv Fr^2$$

The (Froude number)^2 indicates the relative magnitude of aerodynamic lift & buoyancy lift

2. (40 %) A toy rocket traveling at steady speed is propelled by a thin water jet with velocity V_1 and cross-sectional area A directed ino the rocket's open bottom end. The water then pours out of the bottom at speed V_2. These velocities are as seen by an observer moving alongside the rocket.



a) Draw a suitable control volume for analyzing this flow situation. Determine the mass and momentum flows for your chosen control volume.

b) What is the vertical thrust force imparted by the water? You may neglect the effect of gravity on the water velocities.

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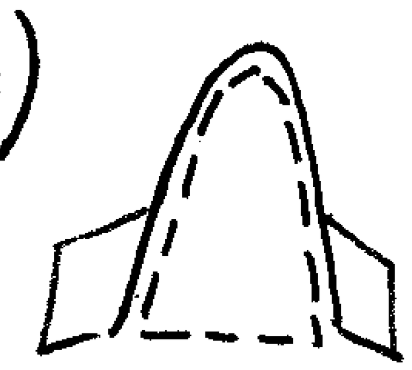
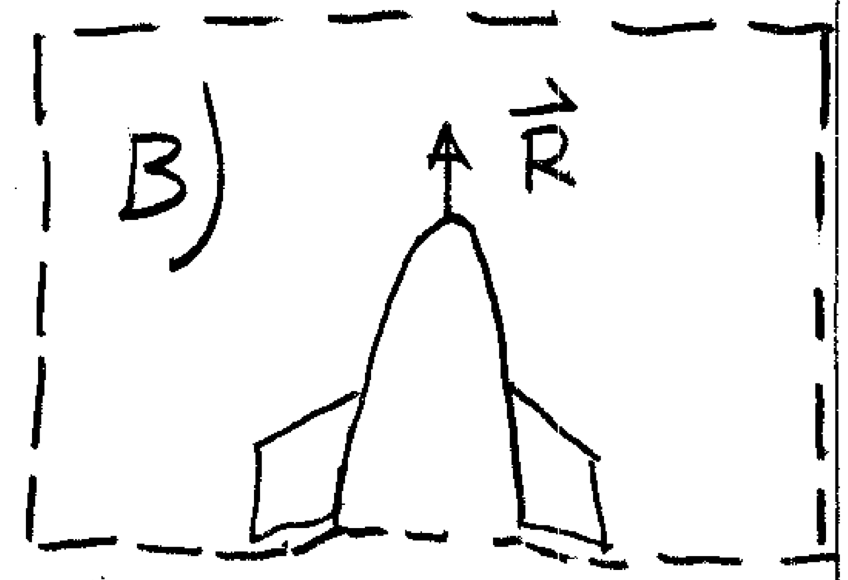
Question 2 Solution

a) Two usable control volumes: A)



$$\oint \rho (\vec{V}\cdot\hat{n})\vec{V}\,dA + \oint p\hat{n}\,dA + \vec{F}_{viscous} = 0$$

B)



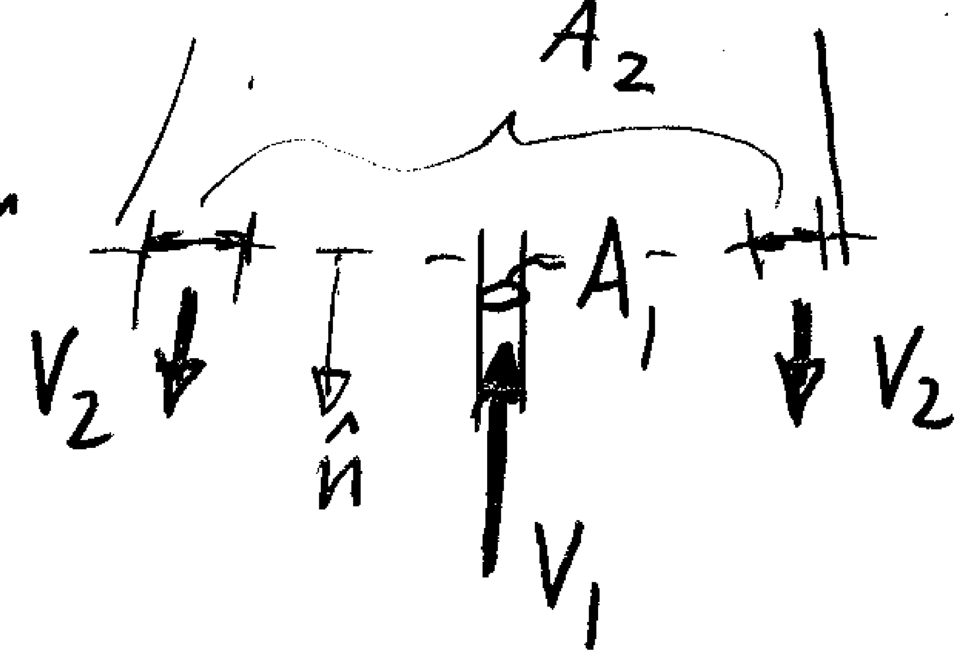
$$\oint \rho (V\cdot\hat{n})\vec{V}\,dA + \oint p\hat{n}\,dA + \vec{R} = 0$$

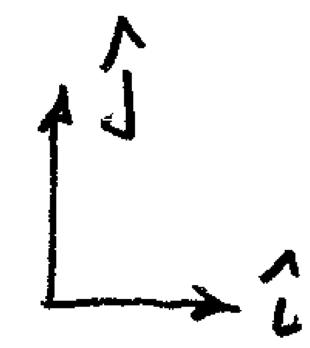
Exit plane:

$\hat{n} = -\hat{j}$, $A_1 = A$ given

$\vec{V}_1 = V_1\hat{j}$

$\vec{V}_2 = -V_2\hat{j}$





$$\text{Mass flow} = \oint \rho \vec{V}\cdot\hat{n}\,dA = \rho(V_1\hat{j}\cdot(-\hat{j}))A + \rho(-V_2\hat{j}\cdot(-\hat{j}))A_2$$

$$\boxed{\text{Mass flow} = \rho\left[-V_1 A + V_2 A_2\right]}$$

← Same for both C.V.'s

$$\overrightarrow{\text{Momentum flow}} = \oint \rho(\vec{V}\cdot\hat{n})\vec{V}\,dA = \rho(-V_1)A\,V_1\hat{j} + \rho V_2 A_2(-V_2\hat{j})$$

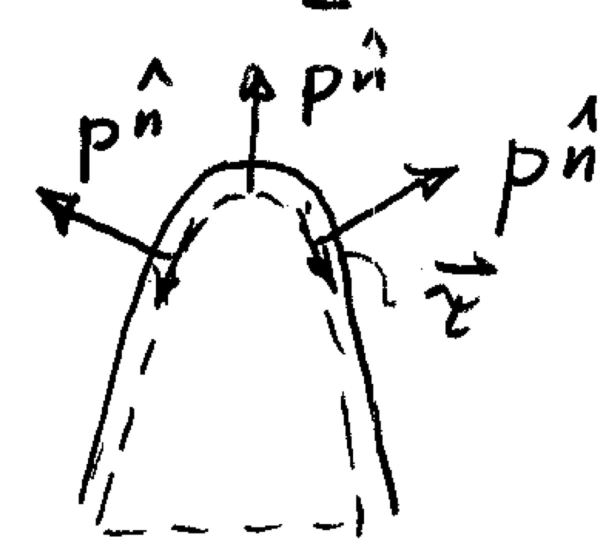
$$\boxed{\overrightarrow{\text{Momentum flow}} = \rho\left[(-V_1^2 A - V_2^2 A_2)\hat{j} + 0\,\hat{i}\right]}$$

b) Since situation is steady, there's no accumulation of water in rocket:

$$\rightarrow \text{mass flow} = 0 \rightarrow -V_1 A + V_2 A_2 = 0 \rightarrow A_2 = A\frac{V_1}{V_2}$$

Therefore $\overrightarrow{\text{momentum flow}} = -\rho A\left[V_1^2 + V_1 V_2\right]\hat{j} + 0\,\hat{i}$

For C.V. A): $\oint p\hat{n}\,dA + \vec{F}_{viscous} = \text{force on rocket} (= \vec{R})$



$$\oint \rho \vec{V}\cdot\hat{n}\,\vec{V}\,dA + \underbrace{\oint p\hat{n}\,dA + \vec{F}_{viscous}}_{\vec{R}} = 0$$

$$\rightarrow \boxed{\vec{R} = -\oint \rho\vec{V}\cdot\hat{n}\,\vec{V}\,dA = \rho A V_1(V_1 + V_2)\hat{j}}$$

For C.V. B): $\oint p\hat{n}\,dA + \vec{F}_{viscous} = 0$

$$\oint \rho\vec{V}\cdot\hat{n}\,\vec{V}\,dA + \vec{R} = 0$$

$$\rightarrow \boxed{\vec{R} = -\oint \rho\vec{V}\cdot\hat{n}\,\vec{V}\,dA = \rho A V_1(V_1 + V_2)\hat{j}}$$

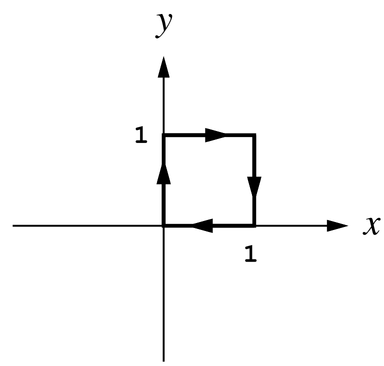
same result

3. (30 %) A 2-D velocity field is given by

$$u(x,y) \;=\; x \qquad , \qquad v(x,y) \;=\; -y$$

a) Determine and sketch the streamline pattern.

b) Determine the circulation around the unit-square curve shown (Note: This is curve is <u>not</u> a streamline of this flow)



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Question 3 Solution

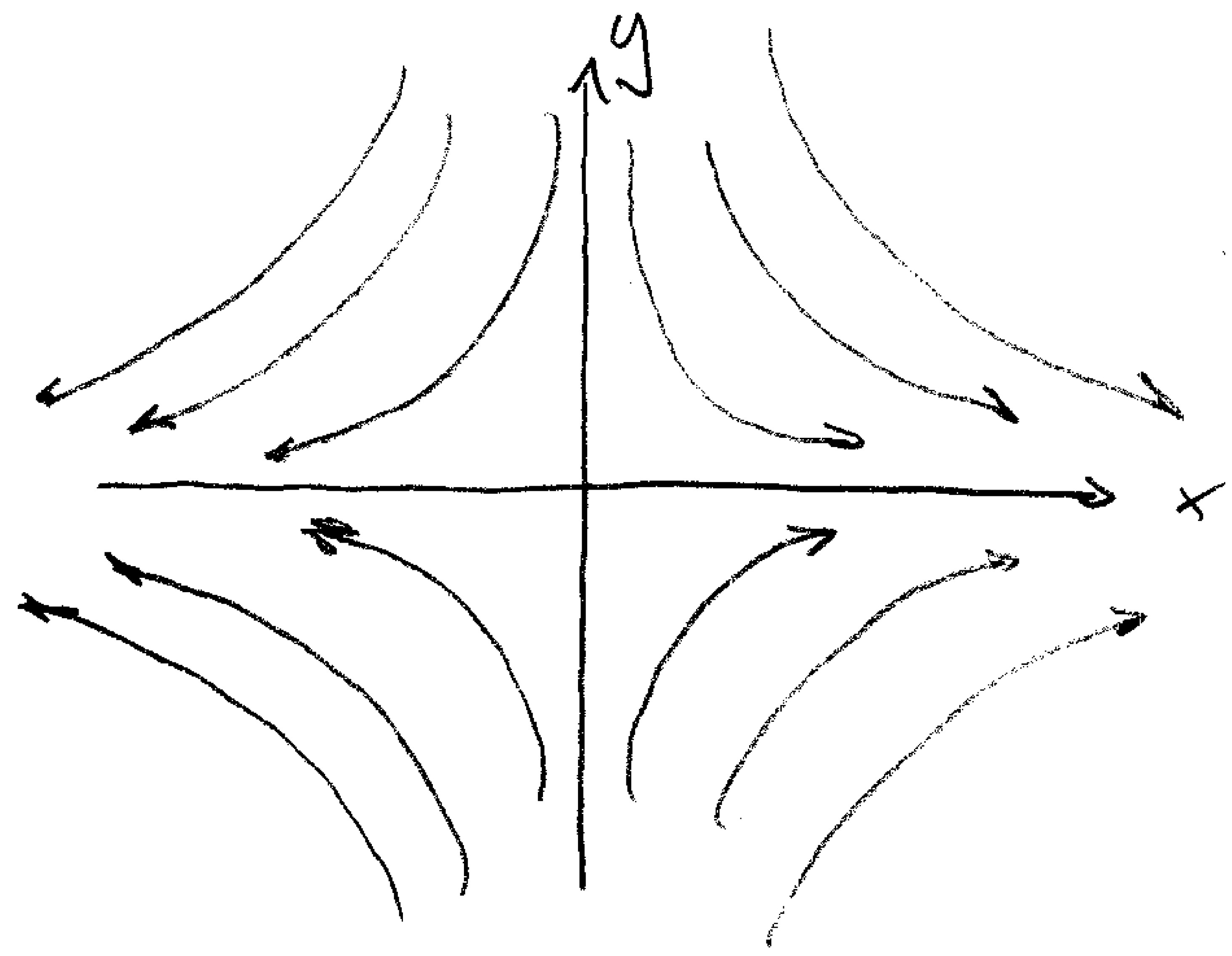
a) $\frac{dy}{dx} = \frac{v}{u} = \frac{-y}{x}$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\ln y = -\ln x + C$$

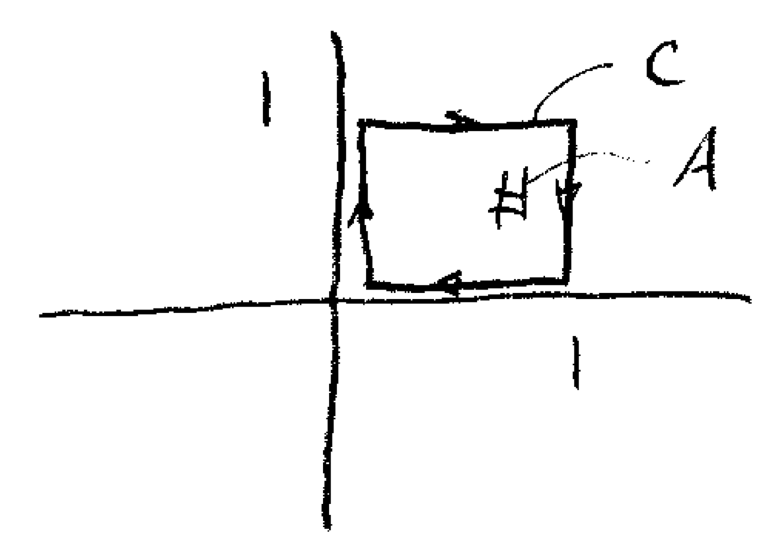
$$y = \frac{C}{x}$$

$$\boxed{xy = C}$$



"corner flow"

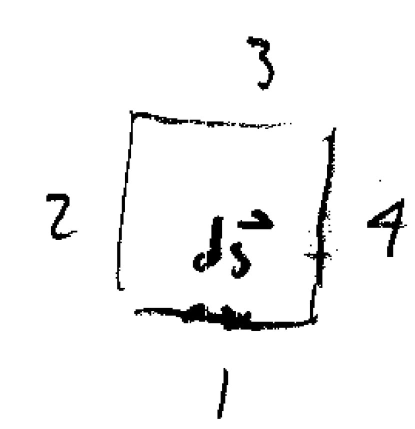
b) $\Gamma = -\oint_C \vec{V} \cdot d\vec{s} = -\iint_A \xi \, dA$



Easiest to note that

$$\xi_z \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 0 = 0$$

$$\rightarrow \quad \Gamma = \iint \xi \, dA = 0$$



Can also evaluate $-\oint \vec{V} \cdot d\vec{s} = \oint_1 + \oint_2 + \oint_3 + \oint_4$

$$= -\tfrac{1}{2} - \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{2} = 0$$

For example, for side 1: $d\vec{s} = \hat{\imath}\, dx$, $\vec{V} = u\hat{\imath} = x\hat{\imath}$

$$\vec{V} \cdot d\vec{s} = x \, dx$$

$$\Gamma_{(\text{side } 1)} = -\int_0^1 x \, dx = -\frac{1}{2} x^2 \Big|_0^1 = -\frac{1}{2}$$

Similarly for sides 2, 3, 4