Unified Quiz 6F

December 15, 2003

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don't just write equations.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Calculators and a 2-sided sheet of paper are allowed
- Box your final answers.

Exam Scoring

#1 (30 %)	
#2 (20%)	
#3 (50 %)	
Total	

1. (30 %)
A circular cylinder of radius R has an irrotational incompressible circular-streamline flow about it. The fluid's tangential velocity on the surface $V_\theta(R)$ is known.

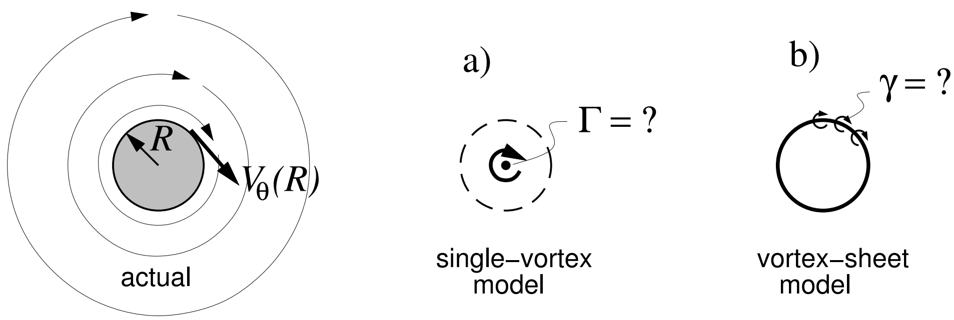
a) In terms of vorticity and conservation of mass, explain why this flow can be represented by a single vortex of strength Γ at the center of the cylinder. Determine the strength Γ required to correctly represent this flow.

b) An alternative representation is sought using a vortex sheet placed on the cylinder surface as shown. Determine the sheet strength γ required to correctly represent the flow.

c) Using the tangential-velocity jump relation for a vortex sheet

$$\Delta V_s = \gamma$$

and circulation arguments, determine the (fictitious) flow inside the cylinder for the surface-sheet model. Be sure to show your reasoning.



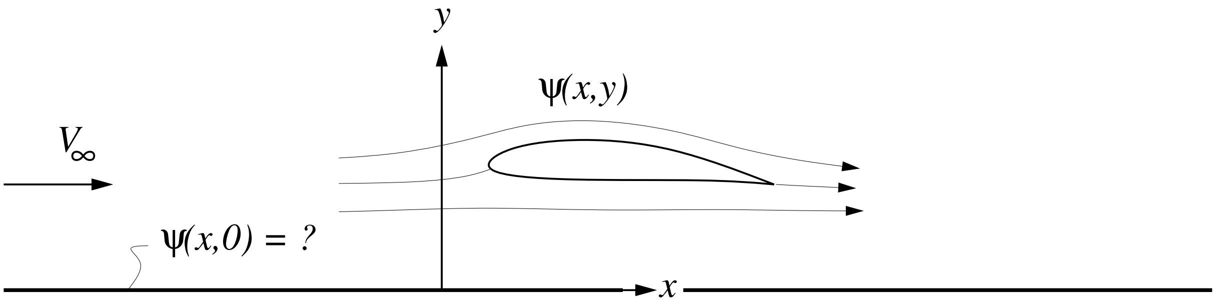
Unified Fluids Quiz 2 (Q6F)
December 15, 2003

Name________________

Problem #1 (continued)

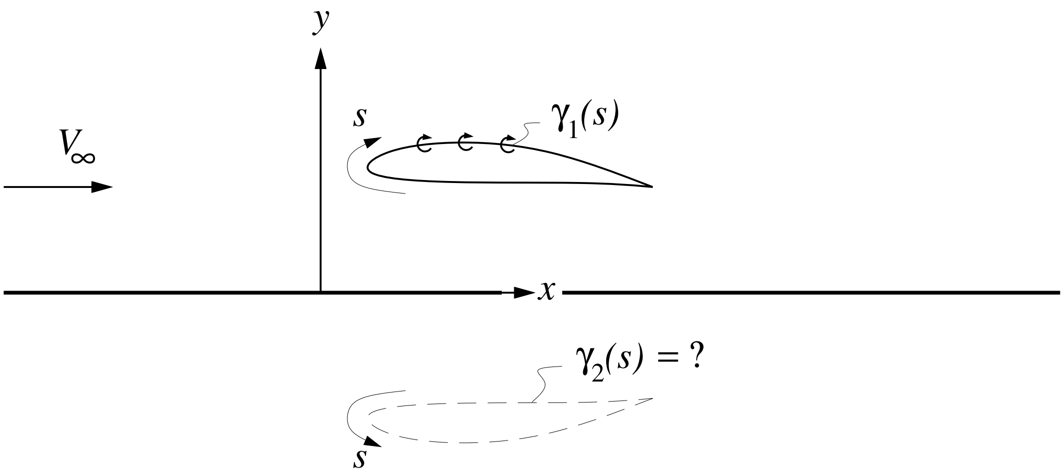
2. (20 %)
A lifting airfoil is positioned over an infinite flat ground plane. The air velocity far away is V_∞. The flowfield is described by a stream function $\psi(x, y)$.

a) How does the stream function vary along the ground plane (i.e. what does $\psi(x, 0)$ look like)?



The stream function $\psi(x, y)$ is to be represented by placing on the airfoil surface a vortex sheet of strength $\gamma_1(s)$. At the same time, a sheet of strength $\gamma_2(s)$ is placed on the fictitious mirror image airfoil of the same shape, as shown.

b) How must $\gamma_2(s)$ be related to $\gamma_1(s)$ so that the flow is correctly represented? Explain.



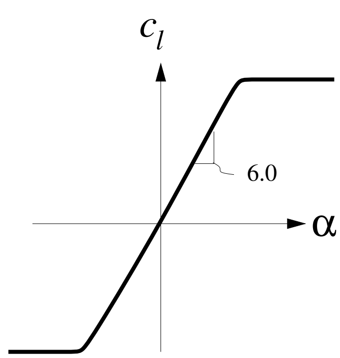
Name________________

Problem #2 (continued)

3. (50 %)
The symmetrical NACA 0015 airfoil has a lift coefficient function approximated by

$$c_\ell(\alpha) = \begin{cases} 1.2 & , \quad \alpha > 0.2 & \text{(positive stall)} \\ 6.0\alpha & , \quad 0.2 > \alpha > -0.2 & \text{(unstalled)} \\ -1.2 & , \quad -0.2 > \alpha & \text{(negative stall)} \end{cases}$$

where α is in radians.

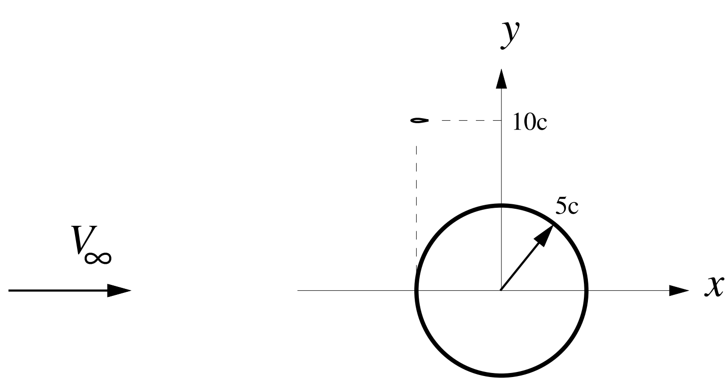


A NACA 0015 airfoil of chord c is held parallel to the x-axis, in a flow with freestream velocity V_∞ along the x-axis. The air density ρ is effectively constant. A large circular cylinder of radius $R = 5c$ is now stuck in the flow, such that the airfoil is at $x, y = (-5c, 10c)$ relative to the cylinder center, as shown.

a) Construct a velocity field $u(x, y)$ and $v(x, y)$ for this configuration, for the purpose of estimating the apparent velocity seen by the little airfoil. Assume the cylinder has inviscid flow about it, and that the airfoil itself has a negligible effect on the overall flow.

b) Evaluate the velocity components u and v at the airfoil location. Is the airfoil stalled?

c) Determine the lift L' on the airfoil.



Unified Fluids Quiz 2 (Q6F)
December 15, 2003

Name_______________

Problem #3 (continued)