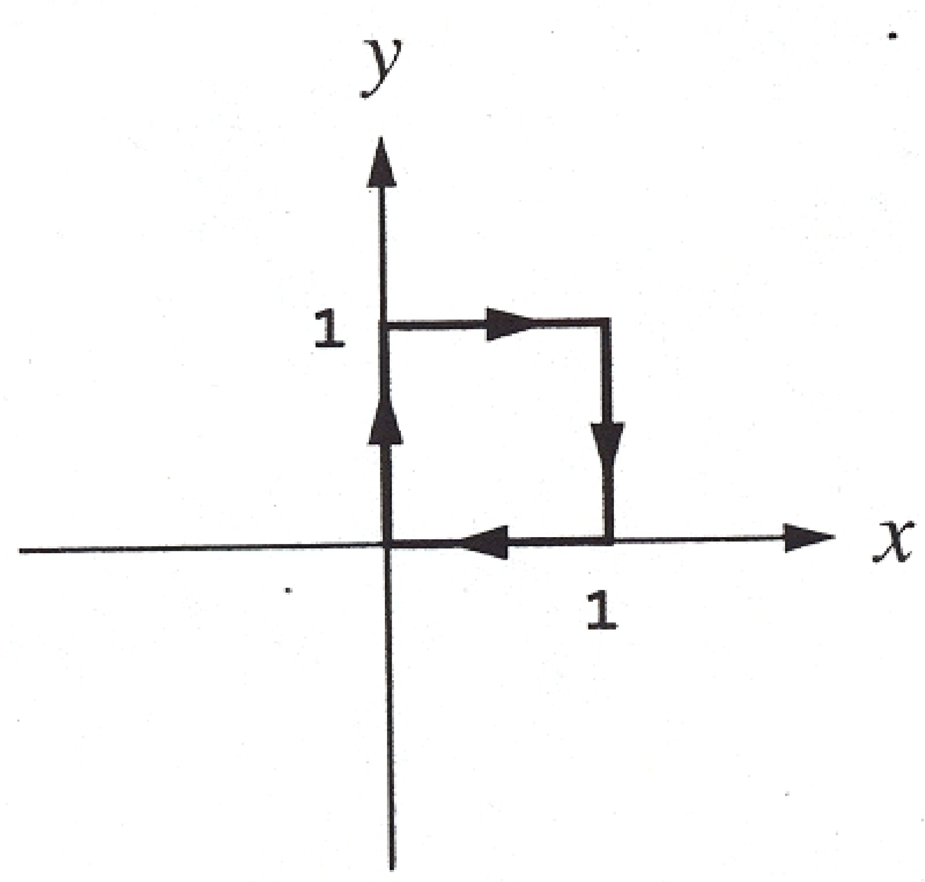
1. (35 %)
Consider the following three low-speed flows (ρ = constant).

$$
\begin{aligned}
1: &\quad \phi = x^2 + y^2 \\
2: &\quad \psi = x^2 + y^2 \\
3: &\quad \psi = \exp(x^2 + y^2)
\end{aligned}
$$

a) Which flows are irrotational?

b) Which flows conserve mass?

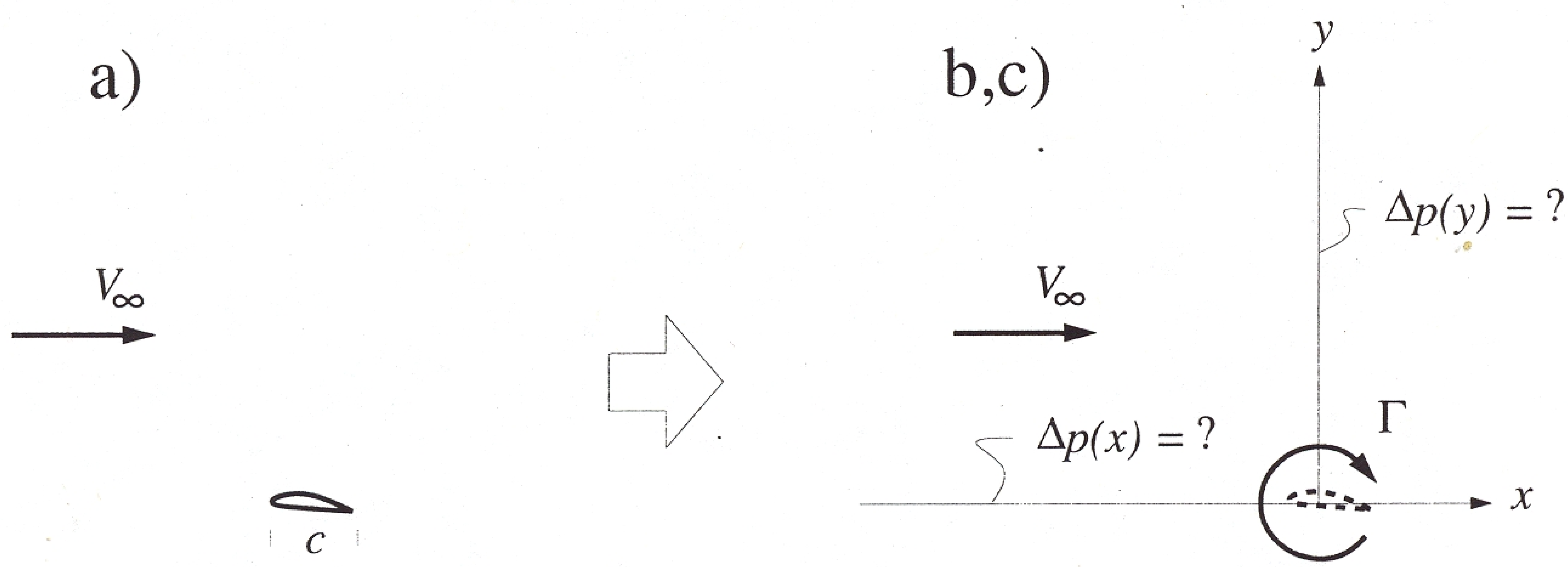
c) For flow 2 above, determine the circulation about the unit-square circuit pictured below.



2. (30 %)
a) An airfoil of chord c is operating at a lift coefficient $c_\ell = 0.5$, in a freestream velocity V_∞. Determine the airfoil's circulation Γ.

b) Far away, the effect of the airfoil on the flow must be nearly the same as that of a point vortex with the same circulation Γ. Using superposition, estimate the u, v velocities and the corresponding overpressure $\Delta p(y) = p - p_\infty$ directly above the airfoil. Also find $\Delta p(x)$ directly in front of the airfoil.

c) A pitot probe is needed to measure p_∞ to within 0.5% of the freestream's dynamic pressure $\frac{1}{2}\rho V_\infty^2$. Determine the probe's minimum y-distance above the airfoil, and also the minimum x-distance in front of the airfoil.



3. (35 %)
The potential flow about a non-lifting 2-D airfoil is represented by some number of source panels with strengths λ_i, $i = 1 \dots N$. The length of each panel is ℓ_i.

a) Determine the total strength Λ of all the panels in terms of the individual λ_i strengths.

b) From physical considerations, what must Λ be equal to?

c) Another airfoil with some nonzero circulation Γ in the direction shown is now added to the flow below the symmetric airfoil, and all the panel strengths are recomputed to correctly represent the flow again.
Compared to its value in case a,b), how must λ_a of the frontmost panel change as a result of the addition (more positive or more negative)? How must λ_b of the rearmost panel change?

d) After the lifting airfoil is added, what happens to the total Λ of the top airfoil?