9:00 - 10:00
35-225

MIT ID# (last four digits) SOLUTIONS

Unified Quiz 1M

October 8, 2004

- Put your name on each page of the exam.
- Read all questions carefully.
- Do all work on that question on the page(s) provided. Use back of the page(s) if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the actual answers.*
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Be sure to show the appropriate units. Intermediate answers and final answers are not correct without the units.
- Report significant digits only.
- Box your final answers.
- ***Calculators and hardwritten "crib sheets" are allowed.***

NOTE: This exam contains material both from "Unified Concepts" (U) and "Materials and Structures" (M).

EXAM SCORING

#1U (25%)	
#2M (25%)	
#3M (25%)	
#4M (25%)	
FINAL SCORE	

Unified Quiz Materials and Structures 1 (Q1M)
October 8, 2004

MIT ID#
(last four digits) SOLUTIONS

PROBLEM #1U (25%)

A set of three forces acts in the x_1-x_2 plane. The force vectors and the (x_1, x_2) points at which they act are as follows:

$\underline{F}_A = (-10\text{ N})\,\underline{i}_2$	acts at (5m, 5m)
$\underline{F}_B = (-5\text{ N})\,\underline{i}_1$	acts at (5m, 0m)
$\underline{F}_C = (3\text{ N})\,\underline{i}_1 - (4\text{ N})\,\underline{i}_2$	acts at (5m, 0m)

(a) Determine the force system acting at the origin that is equipollent to this force system.

The total force and moment with regard to the origin must be determined in finding the equipollent force system acting at the origin.

Force: Must have same magnitude and direction as effect of three forces

Find $\sum \underline{F}_i = \underline{F}_A + \underline{F}_B + \underline{F}_C$

$= (-10N)\underline{i}_2 + (-5N)\underline{i}_1 + (3N)\underline{i}_1 - (4N)\underline{i}_2$

$\uparrow + \xrightarrow{+} \Rightarrow \boxed{\underline{F}_{(0,0)} = (-2N)\,\underline{i}_1 - (14N)\,\underline{i}_2}$

Moment: Must include pure moment equal to sum of moments caused by three ~~of~~ ~~forces~~ acting about origin since moving equipollent force to origin counters this:

$\underline{F}_A$ acts at a distance 5m from (0,0) and causes a counterclockwise moment

$\underline{F}_B$ acts through origin and causes no moment

$\underline{F}_C$ -- break into x_1 and x_2 components. $\underline{i}_1$-component acts through (0,0) and causes no moment. $\underline{i}_2$-component acts at a distance 5m from (0,0) and causes a counterclockwise moment

$\circlearrowleft + M_{(0,0)} = \sum M_i = (10N)(5m) + (-4N)(5m) = -70N\cdot m$

$\boxed{\circlearrowleft + M_{(0,0)} = -70\,N\cdot m}$ acts about $\underline{i}_3$

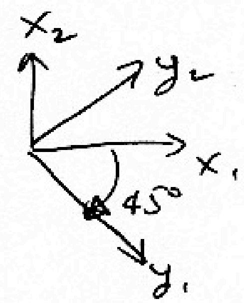
MIT ID# (last four digits) SOLUTIONS

PROBLEM #1U (continued)

(b) Express this equipollent system in a new set of coordinate axes y_1-y_2 that have been rotated - 45° about the x_3 axis.

The moment acts about the x_3-axis. The rotation occurs about this, so the moment is not affected.

Look at $\underline{F}$:



Convention is +CCW
So -45° rotation is clockwise

Using direction cosines, we know:

$$\tilde{\underline{F}}_n = \sum_{m=1}^{2} l_{\tilde{n}m} F_m$$

(Note: summation is to 2 since there are no components in x_3)

in rotated system

where:

$$l_{\tilde{n}m} = \cos(\tilde{x}_n \text{ to } x_m)$$

$\tilde{1} = y_1$, $\tilde{2} = y_2$

So: $l_{\tilde{1}1} = \cos(+45°) = \sqrt{2}/2$

$l_{\tilde{1}2} = \cos(+45° + 90°) = -\sqrt{2}/2$

$l_{\tilde{2}1} = \cos(-45°) = \sqrt{2}/2$

$l_{\tilde{2}2} = \cos(+45°) = \sqrt{2}/2$

So: $\tilde{F}_1 = l_{\tilde{1}1} F_1 + l_{\tilde{1}2} F_2 = \sqrt{2}/2\,(-2N) - \sqrt{2}/2\,(-14N)$
$= 6\sqrt{2}\,N$

$\tilde{F}_2 = l_{\tilde{2}1} F_1 + l_{\tilde{2}2} F_2 = \sqrt{2}/2\,(-2N) + \sqrt{2}/2\,(-14N)$
$= -8\sqrt{2}\,N$.

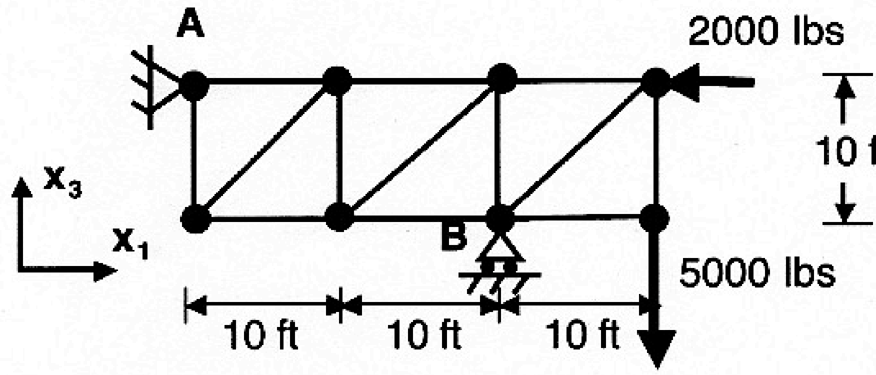
$$\Rightarrow \boxed{\begin{aligned} \tilde{\underline{F}} &= 6\sqrt{2}\,N\,\tilde{i}_1 - 8\sqrt{2}\,N\,\tilde{i}_2 \\ \circlearrowleft^{+} M &= -70\ N\cdot m \end{aligned}}$$

Unified Quiz Materials and Structures 1 (Q1M)
October 8, 2004

MIT ID#
(last four digits) SOLUTIONS

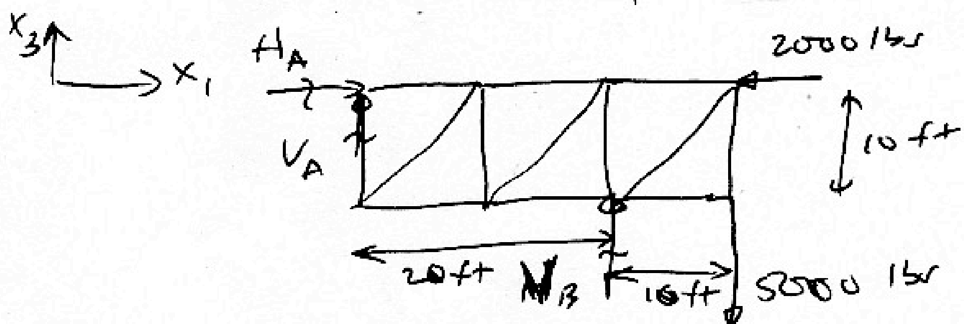
PROBLEM #2M (25%)

The 30-foot long by 10-foot high truss structure, depicted in the figure below, is supported by a pin at its upper left joint, A, and by a roller at its two-thirds length point, B. It is subjected to a tip load of 5000 pounds and by a load of 2000 pounds at its upper right joint.



Determine the reaction forces at support points A and B ***or*** indicate all information available to determine the reaction forces and the additional information that is needed to fully determine these forces.

First draw the Free Body Diagram:



Have 3 reactions and 3 degrees of freedom, so this is statically determinate and can determine reactions

Use equations of equilibrium:

$\Sigma F_1 = 0 \xrightarrow{+} H_A - 2000\,lbs = 0$

$\Rightarrow H_A = 2000\,lbs$

MIT ID# (last four digits) SOLUTIONS

PROBLEM #2M (continued)

$\sum F_3 = 0$ $\uparrow +$ $V_A + V_B - 5000 \text{ lbs} = 0$

$\sum M = 0$ $\circlearrowright +$ $V_B(20\text{ft}) - 5000(30\text{ft}) = 0$

(take about A) $\Rightarrow V_B = 7500 \text{ lbs}$

using $\sum F_3 = 0$ equation:

$$V_A + 7500 \text{ lbs} - 5000 \text{ lbs} = 0$$
$$\Rightarrow V_A = -2500 \text{ lbs}$$

Summarizing:

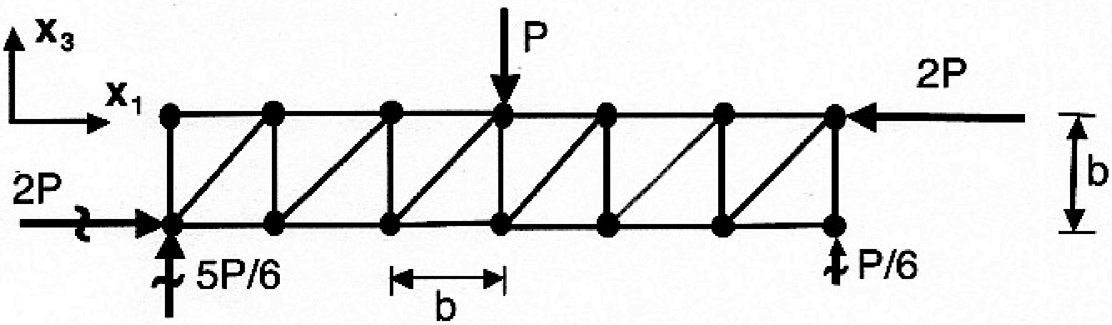
$$\boxed{\begin{aligned} H_A &= 2000 \text{ lbs} \\ V_A &= -2500 \text{ lbs} \\ V_B &= 7500 \text{ lbs} \end{aligned}}$$

Unified Quiz Materials and Structures 1 (Q1M)
October 8, 2004

MIT ID# (last four digits) SOLUTIONS

PROBLEM #3M (25%)

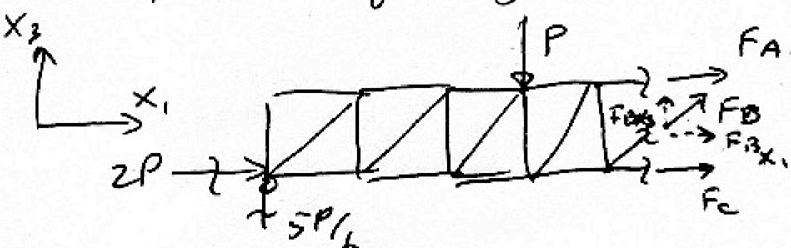
The free body diagram for a six-bay truss, with square bays of length and height b, is shown in the figure below.



Determine the load in the diagonal bar member of the fifth bay (highlighted in the figure) or indicate the additional information needed in order to determine this load.

This can be determined directly using the Method of Sections (NOTE: Method of Joints can be used, but requires significantly more work)

Cut forces through bay with bar of interest:



This gives 3 unknowns (3 bar loads) and have 3 equations of equilibrium $\Rightarrow$ Can be determined

First resolve F_B into x_1 and x_3 components.
Bays are square so B-bar is at angle of 45°

$$\Rightarrow \quad F_{B_{x_1}} = F_B \cos 45^\circ = \frac{\sqrt{2}}{2} F_B$$

$$F_{B_{x_3}} = F_B \sin 45^\circ = \frac{\sqrt{2}}{2} F_B$$

MIT ID#
(last four digits) SOLUTIONS

PROBLEM #3M (continued)

Now use equations of equilibrium:

$$\Sigma F_{x_1} = 0 \quad \xrightarrow{+} \quad 2P + F_{B_{x_1}} + F_C + F_A = 0$$

$$\Sigma F_{x_3} = 0 \quad P + \frac{5P}{6} - P + F_{B_{x_3}} = 0$$

$$\Rightarrow \frac{\sqrt{2}}{2} F_B = \frac{P}{6}$$

$$\Rightarrow F_B = \frac{P}{3\sqrt{2}} = \frac{\sqrt{2}P}{6}$$

no further work needed

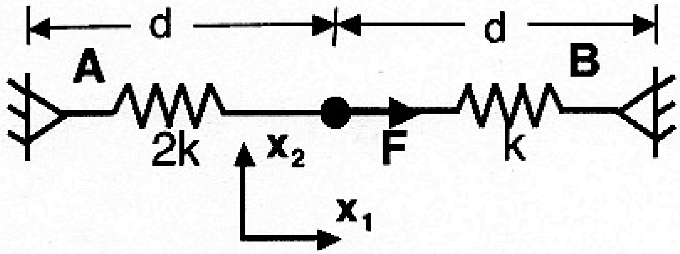
$$\boxed{\text{load in diagonal bar} = \frac{\sqrt{2}P}{6}}$$

Unified Quiz Materials and Structures 1 (Q1M)
October 8, 2004

MIT ID# (last four digits) SOLUTIONS

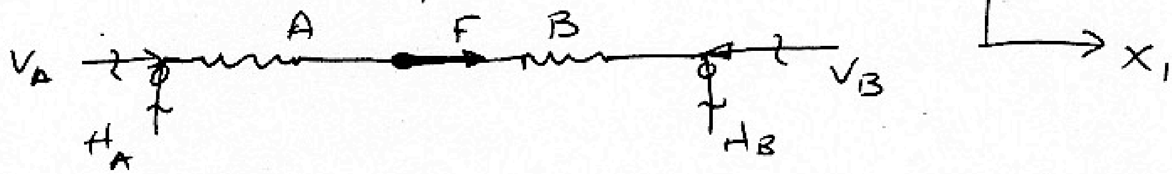
PROBLEM #4M (25%)

A collinear, two-spring system, with each spring of equal original length, d, and pinned at each end, is loaded at the system midpoint by a load, F, acting along the spring line. The left spring, A, has a stiffness of 2k, and the right spring, B, has a stiffness of k. This configuration is depicted below.

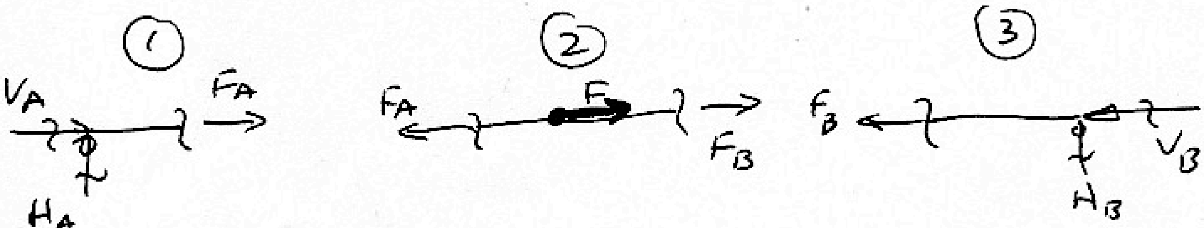


(a) Set up the equations available to determine the reaction loads.

Draw the Free Body Diagram



Cut each spring and consider separate subsystem



F_A and F_B are forces in springs

Do equilibrium for each subsystem:

① $\Sigma F_1 = 0 \xrightarrow{+} V_A + F_A = 0 \Rightarrow V_A = -F_A$ (1)

$\Sigma F_2 = 0 \uparrow + \quad H_A = 0$

② $\Sigma F_1 = 0 \xrightarrow{+} -F_A + F + F_B = 0$ (2)

$\Sigma F_2 = 0$ no info

PROBLEM #4M (continued)

③ $\Sigma F_1 = 0 \overset{+}{\rightarrow} \quad -F_B - V_B = 0 \Rightarrow V_B = -F_B$ (3)

$\Sigma F_2 = 0 \quad \uparrow + \quad H_B = 0$ ✓

Have 3 equations and 4 unknowns (V_B, F_B, V_A, F_A)

$\Rightarrow$ Statically indeterminate.

Use constitutive equations: ($F = kx$)

$F_A = 2k\delta_A$ (4)

$F_B = k\delta_B$ (5)

Now have 5 equations in 6 unknowns (added δ_A, δ_B)

Use compatibility and carefully define deflection:

$\delta_B \longleftarrow \bullet \longrightarrow \delta_A \quad \Rightarrow -\delta_A = \delta_B$ (6)

(extra credit) 6 equations in 6 unknowns $\Rightarrow$ have enough.

(b) Determine the reactions loads or indicate what additional information is needed to do such.

Use (4) and (5) in (6):

$$\Rightarrow -\delta_A = -\frac{F_A}{2k} = +\delta_B = +\frac{F_B}{k}$$

$$\Rightarrow F_A = -2F_B \quad (7)$$

Use in (2):

$$2F_B + F + F_B = 0$$

$$\Rightarrow 3F_B = -F \Rightarrow F_B = -\frac{F}{3}$$

and in (7):

$$F_A = +\frac{2F}{3}$$

use in (1) and (3):

$$\boxed{V_A = -\frac{2F}{3} \qquad V_B = +\frac{F}{3}}$$

from before $\boxed{H_A = 0 \qquad H_B = 0}$