Unified Quiz 3S
November 5, 2004

ID number Solution

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Two 8 1/2" x 11" sheets (both sides) of notes allowed.

Calculators are not needed, and may not be used.

No books allowed.

- Put the last 4 digits of your ID on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the two pages provided.
- Show intermediate results.
- Explain your work --- don't just write equations. Any problem (except multiple choice) without an explanation can receive no better than a "B" grade.
- Partial credit will be given, but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

Exam Scoring

#1 (12.5%)	
#2 (12.5%)	
#3 (25%)	
#4 (25%)	
#5 (25%)	
Total	

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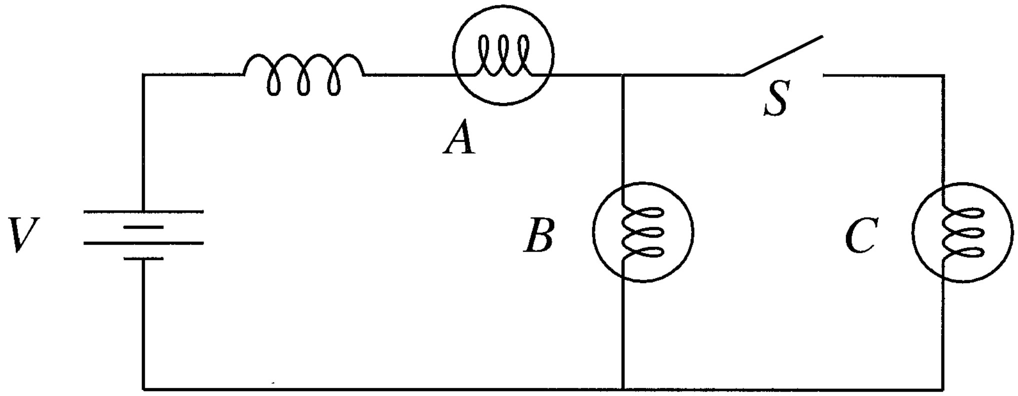
PROBLEM #1 (12.5%)



Consider the circuit above with three identical light bulbs, an inductor, and a switch. Initially, the switch is open, and has been open for a long time. When the switch is closed, what happens to the intensity of each bulb?

1. Immediately after the switch is closed, the intensity of bulb *A*

   increases decreases stays the same

2. Immediately after the switch is closed, the intensity of bulb *B*

   increases decreases stays the same

3. After the switch has been closed for a long time, the intensity of bulb *A* is

   greater than less than the same as

   the intensity of bulb *A* before the switch is closed.

4. After the switch has been closed for a long time, the intensity of bulb *B* is

   greater than less than the same as

   the intensity of bulb *B* before the switch is closed.

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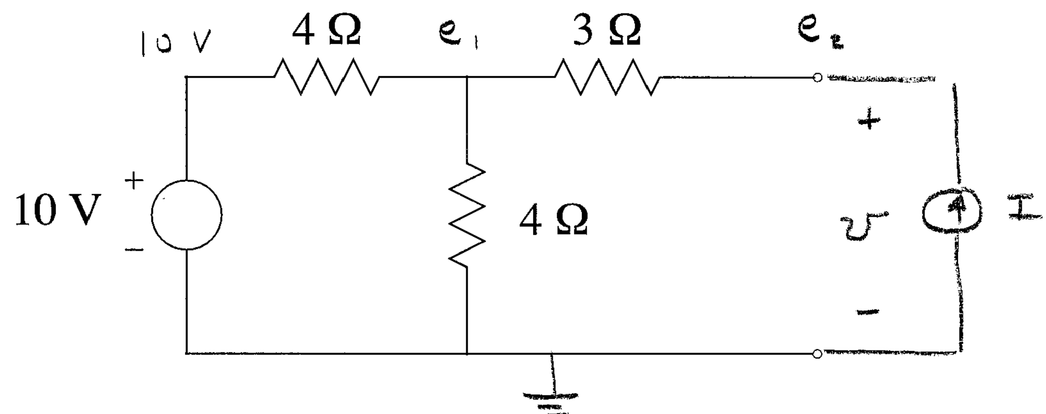
PROBLEM #2 (12.5%)

Find the Thevinin equivalent for the circuit below:



Solve by one of two methods:

Method 1: Add test current, and find v as function of I

To find v, find node voltages using node method. There are two unknown nodes, e_1 and e_2. Apply KCL at each node:

$$e_1: \left(\frac{1}{4}+\frac{1}{4}+\frac{1}{3}\right)e_1 - \frac{1}{3}e_2 - \frac{1}{4}\cdot 10 = 0$$

$$e_2: \quad -\frac{1}{3}e_1 + \frac{1}{3}e_2 - I = 0$$

Therefore,

$$\begin{bmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} = \begin{Bmatrix} 5/2 \\ I \end{Bmatrix}$$

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PROBLEM #2 (continued)

Solve for e_2 using elimination or Cramer's rule:

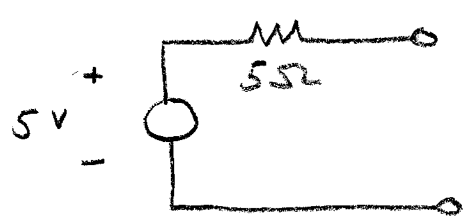
$$e_2 = \frac{\begin{vmatrix} 5/6 & 5/2 \\ -1/3 & I \end{vmatrix}}{\begin{vmatrix} 5/6 & -1/3 \\ -1/3 & 1/3 \end{vmatrix}} = \frac{\frac{5}{6} I + \frac{5}{6}}{\frac{5}{18} - \frac{1}{9}}$$

$$= \frac{(5/6)(I+1)}{1/6} = 5I + 5$$

But $v = e_2$. So $v = 5I + 5 = R_T I + V_T$

$$\Longrightarrow \boxed{\begin{matrix} V_T = 5\ \text{V} \\ R_T = 5\,\Omega \end{matrix}}$$

The Thevinin circuit is then



Method 2: Find open-circuit voltage and impedance at terminals.

When the terminals are open $(I = 0)$, $e_2 = e_1$, since the current through the 3Ω resistor must be zero. Since that current is zero, the two 4Ω resistors form a voltage divider, and

PROBLEM #2 (continued)

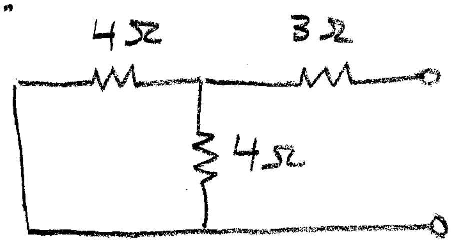
$$e_1 = \frac{4}{4+4} \cdot 10 = 5\text{ V}$$

therefore,

$$V_T = V_{OC} = e_2 = e_1 = 5\text{ V}$$

$$\boxed{V_T = 5\text{ V}}$$

Next, find output impedance by setting sources to 0:



The equivolent impedance is

$$R_T = 3\Omega + 4\Omega \| 4\Omega$$

$$= 3 + \frac{4 \cdot 4}{4+4} = 5\Omega$$

$$\boxed{R_T = 5\Omega}$$

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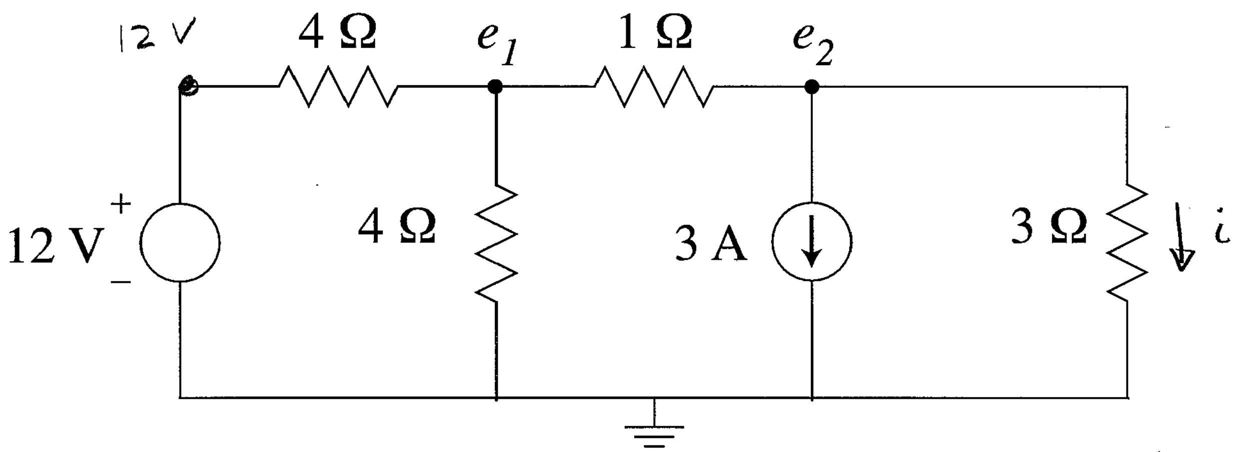
PROBLEM #3 (25%)



For the circuit above, calculate:

(a) The node potentials e_1 and e_2.

(b) The current in the 3 Ω resistor. Make sure that you specify the direction of the current.

Use the node method to solve. Apply KCL at unknown nodes:

$$e_1: \quad \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{1}\right) e_1 - \frac{1}{1} e_2 - \frac{1}{4} \cdot 12 = 0$$

$$e_2: \quad -\frac{1}{1} e_1 + \left(\frac{1}{1} + \frac{1}{3}\right) e_2 + 3 = 0$$

Therefore,

$$\begin{bmatrix} 3/2 & -1 \\ -1 & 4/3 \end{bmatrix} \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} = \begin{Bmatrix} 3 \\ -3 \end{Bmatrix}$$

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PROBLEM #3 (continued)

Solve by elimination or Cramer's rule. Result is

$$\boxed{\begin{aligned} e_1 &= 1\ V \\ e_2 &= -1.5\,V \end{aligned}}$$

The current through the 3Ω resistor is

$$i = \frac{e_2 - 0}{3\,\Omega} = \frac{-1.5\,\Omega}{3\,\Omega} = -\frac{1}{2}\,A$$

So current is

$$\boxed{i = -\frac{1}{2}\,A \Rightarrow \frac{1}{2}\,A \text{ upwards}}$$

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PROBLEM #4 (25%)

A circuit has dynamics described by the state-space equation

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

Find $v_1(t)$ and $i_2(t)$ for the initial conditions

$$v_1(0) = 1 \text{ V}$$
$$i_2(0) = 4 \text{ A}$$

The eigenvalues are the roots of

$$\phi(s) = |sI - A| = 0 = \begin{vmatrix} s+3 & 1 \\ -2 & s \end{vmatrix} = s^2 + 3s + 2$$

$$= (s+1)(s+2)$$

$$\Rightarrow \quad \lambda_1 = -1, \quad \lambda_2 = -2$$

Eigenvectors solve $[\lambda_i I - A]\underline{v}_i = 0$

λ_1: $\quad \lambda_1 I - A = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \Rightarrow \underline{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

λ_2: $\quad \lambda_2 I - A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \Rightarrow \underline{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The general sol'n is the

$$\begin{Bmatrix} v_1(t) \\ i_2(t) \end{Bmatrix} = a\begin{bmatrix} 1 \\ -2 \end{bmatrix}e^{-t} + b\begin{bmatrix} 1 \\ -1 \end{bmatrix}e^{-2t}$$

To match the ICs,

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PROBLEM #4 (continued)

$$a + b = 1$$
$$-2a - b = 4$$

Solve for a & b by elimination or Cramer's rule:

$$a = -5$$
$$b = 6$$

Therefore,

$$v_1(t) = \left(-5e^{-t} + 6e^{-2t}\right) \text{ volts}$$
$$i_2(t) = \left(10e^{-t} - 6e^{-2t}\right) \text{ amp}$$

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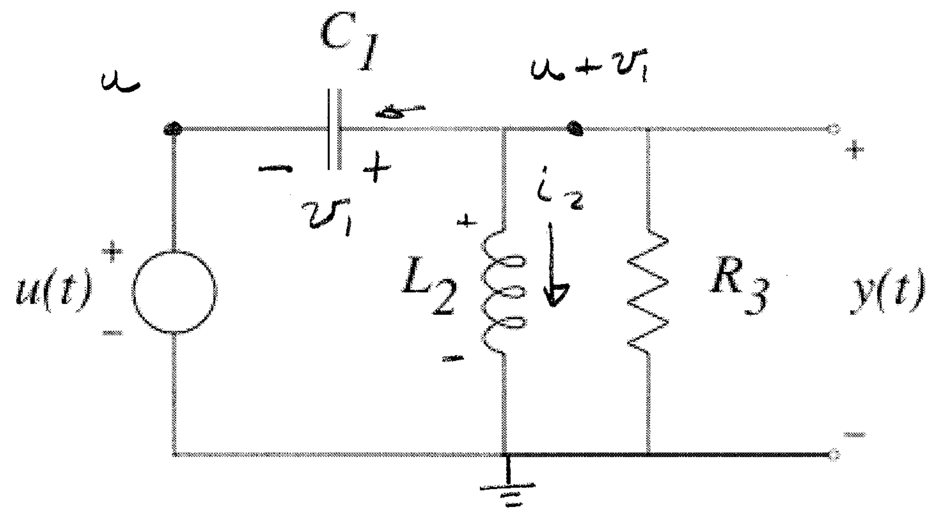
PROBLEM #5 (25%)



Find the differential equations that describe the input-output behavior of the circuit above, in state-space form,

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t)$$
$$y(t) = C\underline{x}(t) + Du(t)$$

That is, you must define the state vector $\underline{x}$, and then derive the matrices A, B, C, and D. The component values are not given, so your answer should be in symbolic form.

States are v_1, i_2. (Note that choice of signs is not unique)

Constitutive laws are

$$\frac{dv_1}{dt} = \frac{1}{C_1} i_1 , \quad \frac{di_2}{dt} = \frac{1}{L_2} v_2$$

So need to find i_1, v_2. Use node method. 1st step is to label known & unknown nodes, as above. There are no unknown nodes.

v_2 is then easy — $v_2 = v_1 + u$.

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PROBLEM #5 (continued)

To find $\dot{i}_1$, use KCL at $u + v_1$:

$$\dot{i}_1 + \dot{i}_2 + \frac{u + v_1}{R_3} = 0$$

$$\Rightarrow \quad \dot{i}_1 = -\frac{v_1}{R_3} - \dot{i}_2 - \frac{u}{R_3}$$

Therefore, state eq'ns are

$$\frac{dv_1}{dt} = -\frac{1}{R_3 C_1} v_1 - \frac{1}{C_1} i_2 - \frac{1}{R_3 C_1} u$$

$$\frac{di_2}{dt} = \frac{1}{L_2} v_1 + \frac{1}{L_2} u$$

$$\Rightarrow \quad \dot{\underline{x}} = \begin{bmatrix} -1/R_3 C_1 & -1/C_1 \\ 1/L_2 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} -1/R_3 C_1 \\ 1/L_2 \end{bmatrix} u$$

Finally, $y(t) = u + v_1$

$$\Rightarrow \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \end{bmatrix} u$$