Problem 1.

In general $u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ $\quad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

or $\vec{V} = \nabla \phi = \nabla \times (\psi \hat{k})$

a) Irrotational flow has $\nabla \times \vec{V} = 0$ or $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

1: $\nabla \times \vec{V} = \nabla \times (\nabla \phi) = 0$ irrotational

2: $u = 2y$, $v = -2x$, $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2 - 2 = -4 \neq 0$ rotational

3: $u = e^{x^2+y^2} 2y$, $v = -e^{x^2+y^2} 2x$

$\frac{\partial u}{\partial y} = 2e^{x^2+y^2} + 4y^2 e^{x^2+y^2}$, $\frac{\partial v}{\partial x} = -2e^{x^2+y^2} - 4x^2 e^{x^2+y^2}$

$\rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = (-2 - 4x^2 - 2 - 4y^2) e^{x^2+y^2} \neq 0$

rotational

b) Mass conservation requires $\nabla \cdot \vec{V} = 0$ or $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

1: $\nabla \cdot \vec{V} = \nabla^2 \phi = 2 + 2 = 4 \neq 0$ does not conserve mass

2: $\nabla \cdot \vec{V} = \nabla \cdot [\nabla \times (\psi \hat{k})] = 0$ conserves mass

3: $\nabla \cdot \vec{V} = \nabla \cdot [\nabla \times (\psi \hat{k})] = 0$ conserves mass

c) Simplest to use Stokes' Theorem:

$\Gamma = -\oint \vec{V} \cdot d\vec{s} = -\iint \zeta \, dA$

For flow 2: $\zeta = -\nabla^2 \psi = -4$

$$\Gamma = -\iint (-4)\, dA = 4 \iint dA = 4 \cdot \text{Area}$$

$$\boxed{\Gamma = 4}$$

Can also evaluate $\int \vec{V} \cdot d\vec{s}$ on each of the four sides.

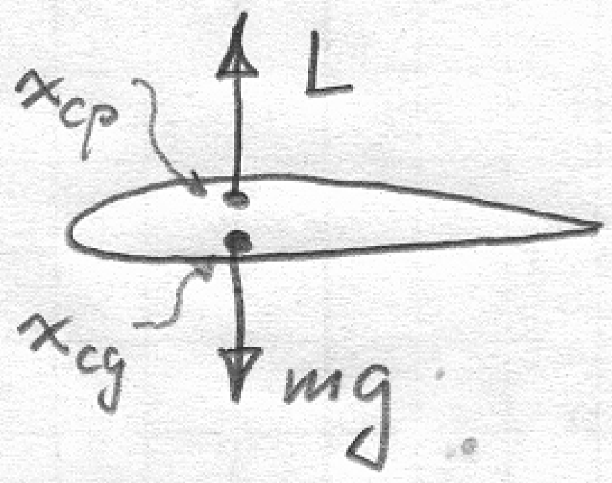
a) $\left(M'_{ref} = M'_{LE} + L' x_{ref}\right) b$, note: $M' b = M$, $L' b = L$

$$M_{ref} = M_{LE} + L\, x_{ref}$$

for this case we set $x_{ref} = 0.25c$

$$M_{c/4} = q_\infty S c\,(0.025 - 1.25\alpha) + q_\infty S \overbrace{5\alpha \cdot 0.25c}^{1.25\alpha}$$

$$\boxed{M_{c/4} = q_\infty S c \cdot 0.025}$$ (independent of α)



b) In equilibrium flight, must have $x_{cg} = x_{cp}$ to get zero net moment.

Or, we can simply start by requiring $M_{0.2c} = 0$

$$M_{0.2c} = M_{LE} + L \cdot 0.2c = 0$$

$$q_\infty S c \cdot (0.025 - 1.25\alpha) + q_\infty S\, 5\alpha \cdot 0.2c = 0$$

$$q_\infty S c\,(0.025 - 1.25\alpha + 1.0\alpha) = 0$$

$$0.025 - 0.25\alpha = 0$$

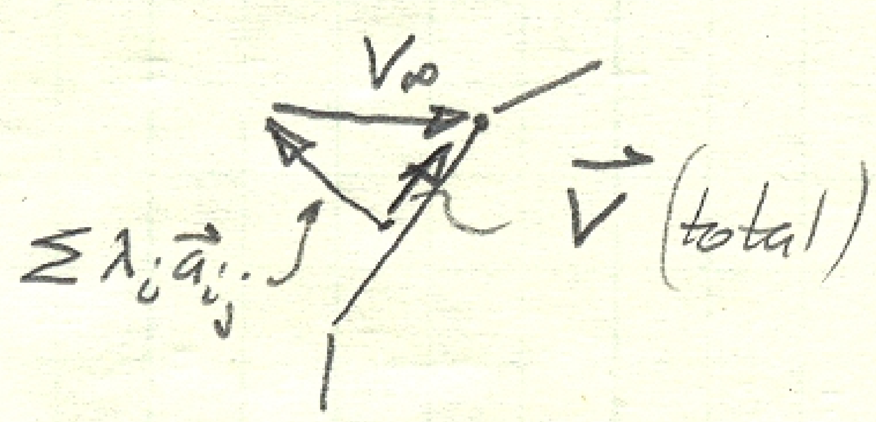
$$\boxed{\alpha = 0.1 \text{ radians}}$$

Problem 3.

a) $\gamma_i l_i = \Lambda_{panel\, i} \quad \rightarrow \quad \Lambda = \sum_{i=1}^{N} \Lambda_{panel\, i} = \sum_{i=1}^{N} \gamma_i l_i$

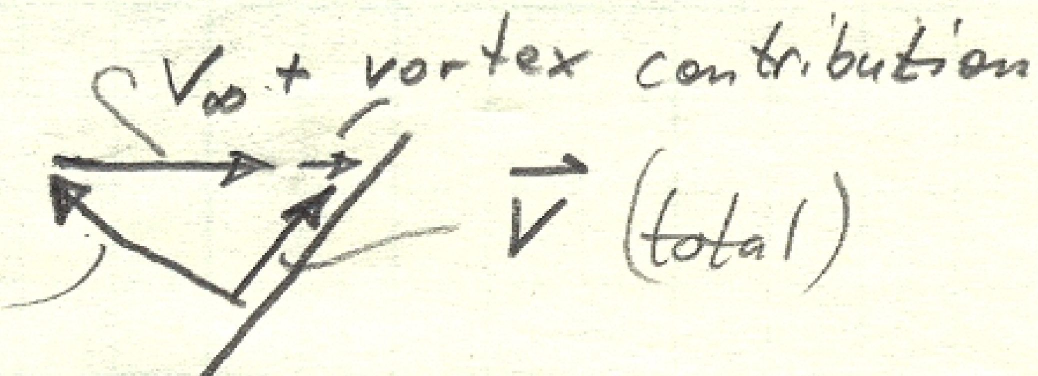
b) Must have $\Lambda = 0$ for a closed body, else the dividing streamline will not close.

c) Front panel:



a,b)

c,d)

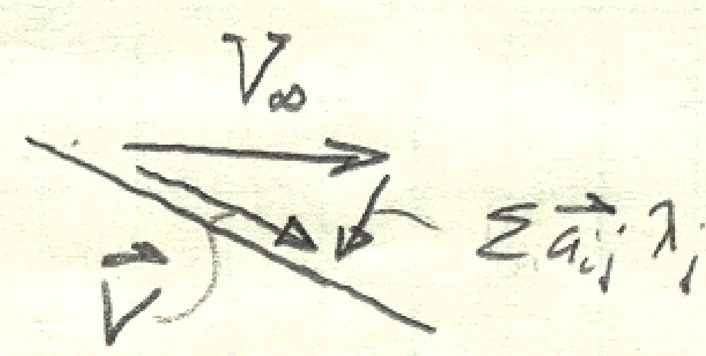


increased normal velocity needed to offset additional horizontal velocity from vortex.

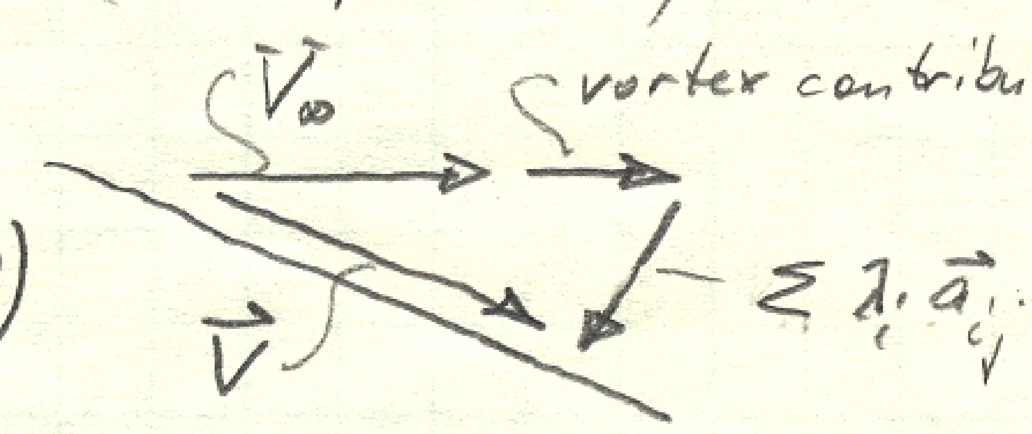
λ_a must increase ↑ (more positive)

Rear panel:

a,b)



c,d)



Same as for front panel, but opposite sign:

λ_b must decrease ↓ (more negative)

d) Argument in b) still remains: must have $\Lambda = 0$