Problem 1.

In general \( u = \frac{\partial \phi}{\partial x} = \frac{\partial^2}{\partial x^2}, \quad v = \frac{\partial \phi}{\partial y} = -\frac{\partial^2}{\partial y^2} \)

or \( \vec{V} = \nabla \phi = \nabla \times (\vec{r} \hat{k}) \)

a) Irrotational flow has \( \nabla \times \vec{V} = 0 \) or \( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \)

1: \( \nabla \times \vec{V} = \nabla \times (\vec{r} \hat{k}) = 0 \) irrotational

2: \( u = 2y, \quad v = -2x, \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2 - 2 = -4 \neq 0 \) rotational

3: \( u = e^{x+y} 2y, \quad v = -e^{x+y} 2x \)
\[ \frac{\partial u}{\partial y} = 2e^{x+y} + 4y^2 e^{x+y}, \quad \frac{\partial v}{\partial x} = -2e^{x+y} - 4x^2 e^{x+y} \]
\[ \Rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = (-2 - 4x^2 - 2 - 4y^2)e^{x+y} \neq 0 \) rotational

b) Mass conservation requires \( \nabla \cdot \vec{V} = 0 \) or \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \)

1: \( \nabla \cdot \vec{V} = \nabla^2 \phi = 2 + 2 = 4 \neq 0 \) does not conserve mass

2: \( \nabla \cdot \vec{V} = \nabla \cdot (\nabla \times (\vec{r} \hat{k})) = 0 \) conserves mass

3: \( \nabla \cdot \vec{V} = \nabla \cdot (\nabla \times (\vec{r} \hat{k})) = 0 \) conserves mass

c) Simplest to use Stokes' Theorem:

\[ \Gamma = -\oint \vec{V} \cdot d\vec{s} = -\int \int \frac{\partial^2 \phi}{\partial s^2} \, dA \]

For flow 2: \( \frac{\partial^2 \phi}{\partial s^2} = -4 \)

\[ \Gamma = -\int \int (-4) \, dA = 4 \int \int dA = 4 \cdot \text{Area} \]

\[ \Gamma = 4 \]

Can also evaluate \( \int \vec{V} \cdot d\vec{s} \) on each of the four sides.
a) \( M'_{\text{ref}} = M'_{LE} + \dot{L} x_{\text{ref}} \)  
   \( M'_{\text{ref}} = M'_{LE} + L x_{\text{ref}} \)

For this case, we set \( x_{\text{ref}} = 0.25c \)

\[ M_{c/4} = \rho_0 Sc \left( 0.025 - 1.25\alpha \right) + \rho_0 S \tilde{5} \alpha \cdot 0.25c \]

\[ M_{c/4} = \rho_0 Sc \cdot 0.025 \quad \text{(independent of } \alpha) \]

b) In equilibrium flight, must have \( x_{cg} = x_{cp} \) 
   to get zero net moment.

Or, we can simply start by requiring \( M_{0.2c} = 0 \)

\[ M_{0.2c} = M_{LE} + L \cdot 0.2c = 0 \]

\[ \rho_0 Sc \left( 0.025 - 1.25\alpha \right) + \rho_0 S \tilde{5} \alpha \cdot 0.2c = 0 \]

\[ \rho_0 Sc \left( 0.025 - 1.25\alpha + 1.0\alpha \right) = 0 \]

\[ 0.025 - 0.25\alpha = 0 \]

\[ \alpha = 0.1 \text{ radians} \]
Problem 3.

a) \( y_i \xi_i = \sum_{i=1}^{N} \Lambda_{\text{panel}} \rightarrow \Lambda = \sum_{i=1}^{N} \Lambda_{\text{panel}} = \sum_{i=1}^{N} y_i \xi_i \)

b) Must have \( \Lambda = 0 \) for a closed body, else the dividing streamline will not close.

\[ \Lambda = 0 \quad , \quad \Lambda > 0 \]

c) Front panel:
\[ \sum_{i=1}^{N} y_i \xi_i \]
\[ \vec{V}_{\text{total}} \]
\[ a, b) \]

Increased normal velocity needed to offset additional horizontal velocity from vortex.
\[ \lambda_a \text{ must increase } \uparrow \quad (\text{more positive}) \]

Rear panel:
\[ \vec{V}_0 \]
\[ \sum_{i=1}^{N} y_i \xi_i \]
\[ a, b) \]

Same as for front panel, but opposite sign:
\[ \lambda_b \text{ must decrease } \downarrow \quad (\text{more negative}) \]

d) Argument in b) still remains: must have \( \Lambda = 0 \)