Unified Engineering Thermo Quiz
Fall 2005

ID NUMBER__WAITZ SOLUTIONS_____

Unified Quiz: Thermodynamics

October 14, 2005

Calculators allowed.
No books or notes allowed.
A list of equations is provided.

- **Put your ID number on each page of the exam.**
- **Read all questions carefully.**
- **Do all work for each problem on the pages provided.**
- **Show intermediate results.**
- **Explain your work --- don't just write equations.**
- **Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.**
- **Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.**
- **Show appropriate units with your final answers.**
- **Box your final answers.**

Exam Scoring

#1 (10%)	
#2 (15%)	
#3 (35%)	
#4 (20%)	
#5 (20%)	
Total	

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1) (10%, 2 parts * 5% each -- partial credit given)

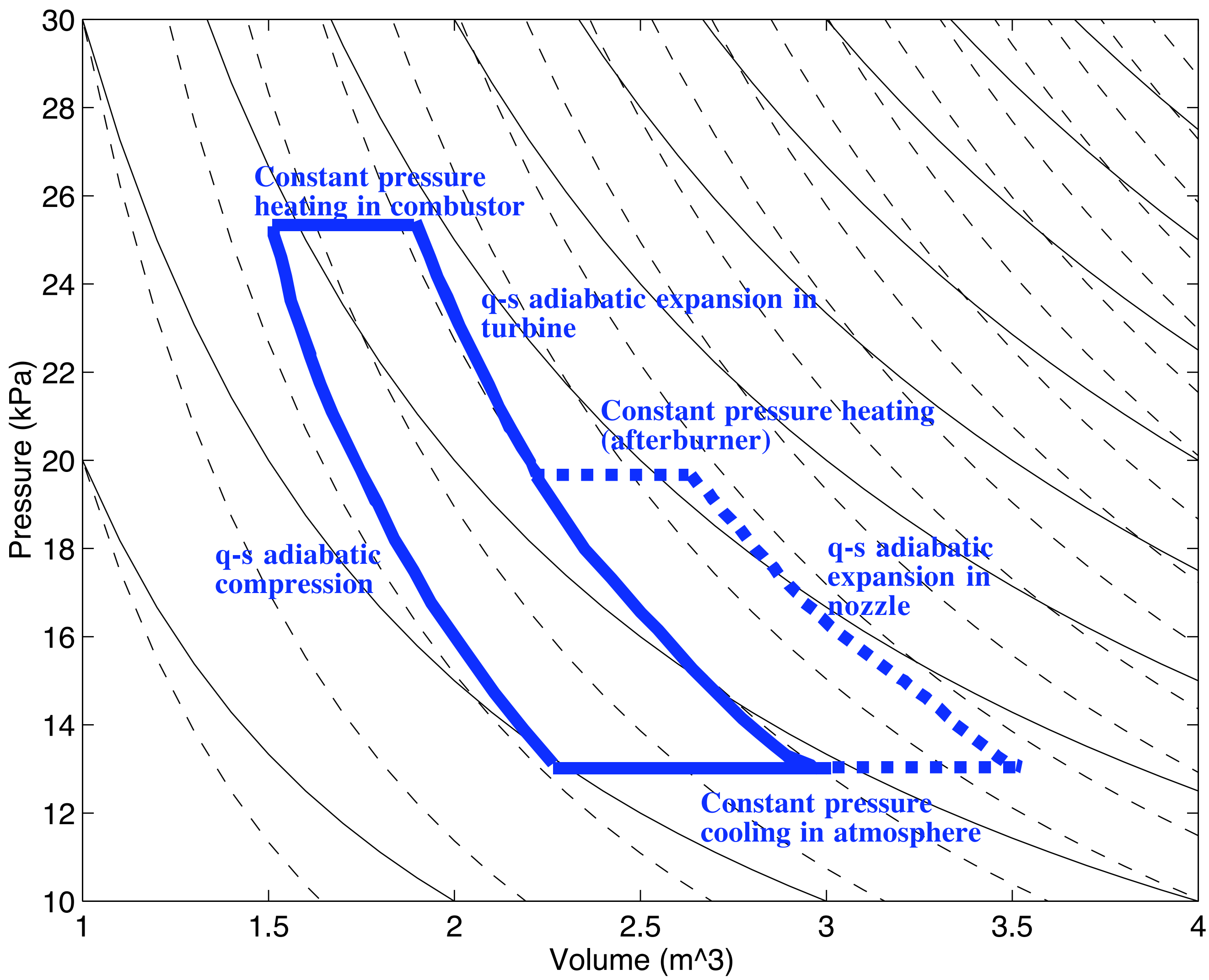
a) What is an irreversible process? (LO #5)

An irreversible process is a process that cannot be reversed (i.e. returning the system to its original state) without a net change in heat and work transfers to the surroundings. That is, for an irreversible process, it is possible to reverse the system to its original state, but the surroundings are changed as a result. In contrast, for a reversible process, after completing the forward and the reverse process, there is no net change in the system or the surroundings.

b) What is the difference between heat and temperature? (LO #1)

Heat is a transfer of energy across a system boundary by virtue of a temperature difference only. It is measured in Joules. Temperature is a thermodynamic property and a function of the state of a system. It is measured in Kelvin.

2) (15%, partial credit given) Draw (and label) an ideal thermodynamic cycle representing a gas turbine engine with an afterburner. Show graphically the difference in work with and without the afterburner lit. (LO's #2, 3, 4, 6)



(dashed lines = adiabats, solid lines = isotherms)

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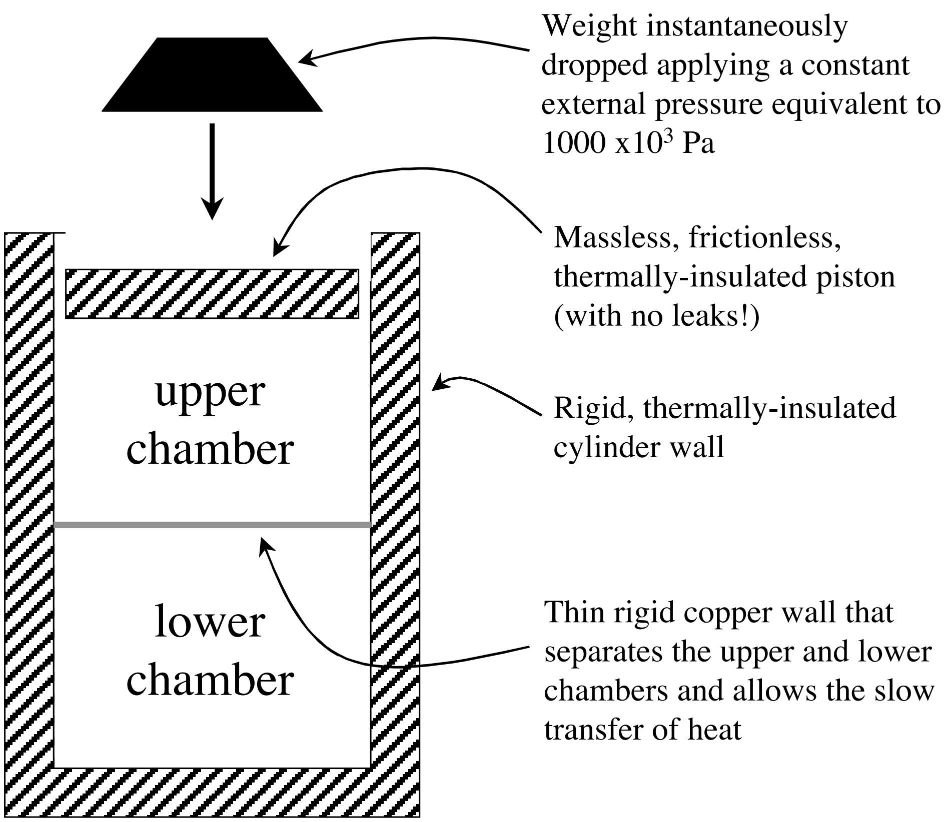
3) (35%, partial credit given) Below is a piston-cylinder arrangement where the piston has two chambers. Although the cylinder is thermally-insulated from the surroundings, the lower chamber is isolated by a thin copper barrier, which is rigid but which allows the slow transfer of heat between the two chambers. The cylinder walls are rigid. The chambers are filled with air which behaves as an ideal gas with R=287 J/kg-K and you can assume the specific heats are constant at c_v=716.5 J/kg-K, c_p=1003.5 J/kg-K.

Both chambers start in thermodynamic equilibrium with T=300K, p=100x10^3 Pa with a volume of 0.1 m^3. A weight equivalent to an external pressure of 1000x10^3 Pa is instantaneously dropped on the upper cylinder. You can assume that the piston itself is massless and free to move without friction. Your objective is to devise a simplified thermodynamic model of this system and to use it to estimate the temperature the gas in the lower chamber would come to when the two-chamber system eventually comes to thermodynamic equilibrium.



a) (7%, partial credit given) Describe the energy exchange processes in the device in terms of heat, work and various forms of energy. (LO's #1, #2)

External work is done on the upper chamber by the weight. The potential energy of the weight is reduced and the internal energy of the gas in the upper chamber is increased. The process is not quasi-static. Then heat is gradually transferred from the upper chamber to the lower chamber. During this process the internal energy of the upper chamber is decreased and the internal energy of the lower chamber is increased. As the heat is transferred from the upper chamber, work continues to be done on the upper chamber since the piston is free to move (and the potential energy of the weight continues to decrease). When the processes are over, the surroundings have provided energy (from the change in potential energy of the weight). The two chambers have received the energy, and it appears as increased internal energy of each of the chambers.

b) (8%, partial credit given) What processes will you use to model this system? Why? (LO's #2, #4, #5)

The first process is not quasi-static but it is adiabatic because we expect pressure to equilibrate faster than the time it takes for appreciable heat transfer to occur. Therefore the upper chamber should be modeled as adiabatic with the work determined by considering the external pressure times the change in volume. After this first relatively fast process, we can assume that the heat transfer takes place more slowly. It is no longer adiabatic (both chambers have heat transfer with one anther). But the upper chamber undergoes constant pressure cooling since the piston is free to move, but the weight remains on it. Since the cooling is relatively slow we can assume that the process in the upper chamber is quasi-static in terms of evaluating the work as pdv. The process in the lower chamber is constant volume heating (no work).

c) (10%, partial credit given) What is the temperature the gas in the upper chamber comes to shortly after the instantaneous dropping of the weight? (LO #4)

This problem is similar to homework T4 part (b)—an impulsive expansion—except in this case it is an impulsive compression. You are given the initial state. You are told it is an adiabatic process, and you know the external pressure that is applied. The first law with q=0 becomes:

$$\Delta u = -w = -p_{ext}(v_2 - v_1) \quad \text{or} \quad c_v(T_2 - T_1) = -p_{ext}(v_2 - v_1)$$

You know T_1, v_1, but not T_2 and v_2. However you know from the ideal gas law that

$$v_2 = \frac{RT_2}{p_2} = \frac{RT_2}{p_{ext}}$$

since $p_2 = p_{ext}$ when the system comes to pressure equilibrium. Therefore,

$$c_v(T_2 - T_1) = -p_{ext}\left(\frac{RT_2}{p_{ext}} - v_1\right) = -p_{ext}\left(\frac{RT_2}{p_{ext}} - \frac{RT_1}{p_1}\right) = p_{ext}\frac{RT_1}{p_1} - RT_2$$

so

$$c_vT_2 - c_vT_1 = p_{ext}\frac{RT_1}{p_1} - RT_2 \quad \text{or} \quad (c_v + R)T_2 = p_{ext}\frac{RT_1}{p_1} + c_vT_1$$

$$(c_v + c_p - c_v)T_2 = T_1\left(\frac{Rp_{ext}}{p_1} + c_v\right) \quad \text{or} \quad T_2 = \frac{T_1}{c_p}\left(\frac{Rp_{ext}}{p_1} + c_v\right)$$

$$T_2 = \frac{300\text{K}}{1003.5\text{J/kgK}}\left(\frac{287\text{J/kgK}(1000\text{x}10^3\text{Pa})}{100\text{x}10^3\text{Pa}} + 716.5\text{J/kgK}\right) = 1072\text{K}$$

d) (10%, partial credit given) What is the temperature the gas in the lower chamber comes to when the whole system eventually reaches thermodynamic equilibrium? (LO #4)

The upper chamber slowly cools in a constant pressure cooling process. The lower chamber slowly heats in a constant volume heating process. Other than the rigid copper wall separating the two chambers everything else is thermally-insulated, so the heat transferred from the upper chamber is equal to the heat transferred to the lower chamber.

The easiest way to do this is to use two different forms of the first law for the two chambers.

$du = \delta q - pdv$ is convenient for the constant volume process in the lower chamber

$dh = \delta q + vdp$ is convenient for the constant pressure process in the upper chamber

So for the lower chamber $c_v\left(T_{3_{lower}} - T_{2_{lower}}\right) = \delta q$ since volume is constant

and for the upper chamber $c_p\left(T_{3_{upper}} - T_{2_{upper}}\right) = \delta q$ since pressure is constant

Here I have assigned state 2 as the state right after the upper chamber comes to pressure equilibrium. For the lower chamber, this is the same as state 1 (for my model, I assumed that no appreciable heat is transferred to the lower chamber during the first process).

The heat that leaves the upper chamber is the same magnitude, but opposite in sign from that which is added to the lower chamber. So with the addition of a negative sign, we can equate the two first law expressions.

$$c_p\left(T_{3_{upper}} - T_{2_{upper}}\right) = -c_v\left(T_{3_{lower}} - T_{2_{lower}}\right)$$

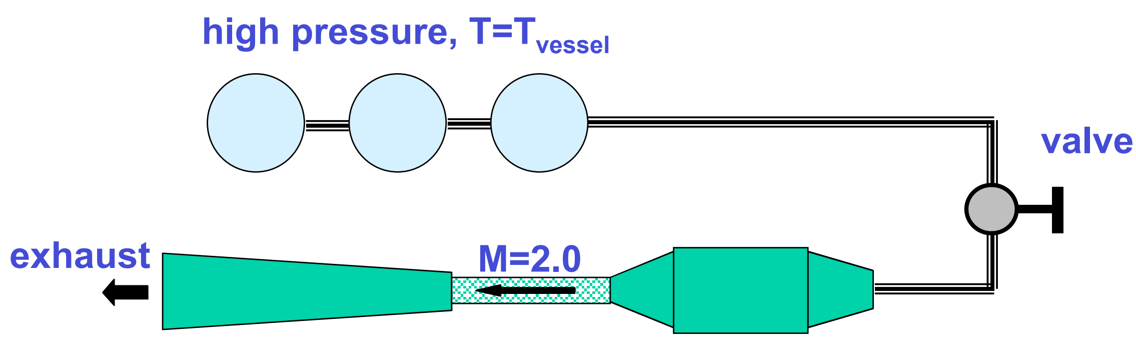
and we know that $T_{2lower} = T_1$=300K, and T_{2upper}=1072K from above, and $T_{3upper}=T_{3lower}$ since the system comes to thermal equilibrium!

$$1003.5\,J/kgK\left(T_{3_{lower/upper}} - 1072K\right) = -716.5\,J/kgK\left(T_{3_{lower/upper}} - 300K\right)$$

So $T_{3_{upper}} = T_{3_{lower}} = 750.5K$

4) (20%, partial credit given) A wind-tunnel is being designed to test an aircraft that flies at M=2 at 11km (T_{atm} = 217K, p_{atm} = 22.6kPa, γ = 1.4). The tunnel will be a blowdown facility like that shown below. High pressure air will be metered through a valve so that it flows through the wind-tunnel test section at Mach = 2 relative to the stationary lab frame. Assume the high pressure air passes through the system with no heat transfer and no external work.

What temperature would it be necessary to set the pressure vessels to so that the static and stagnation temperature in the test section match those experienced in the reference frame of the airplane during high altitude flight? (LO #4)



Conceptually, if we start from an atmospheric temperature of 217K and fly through it at M=2 we expect the skin temperature of the vehicle to be higher than the atmospheric temperature. We can determine what this elevated temperature is using the steady flow energy equation for the condition of an adiabatic process with no external work (the stagnation temperature).

$$T_T = T\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$\therefore T = 217K, \quad M = 2 \quad \Rightarrow T_T = 390.6K$$

Similarly if we start from an atmospheric temperature of 300K in the high pressure vessels for the wind-tunnel, and then accelerate the flow adiabatically with no external work, we expect the temperature to drop. But then if it is brought to rest again on the model (in the same reference frame as the vessels where its flow speed was zero), via an adiabatic process with no external work it would return to the stagnation temperature of the vessels.

So we need to set the stagnation pressure of the vessels to a temperature equal to the stagnation temperature seen in flight.

If we set T_{vessel} = 390.6K, then the M=2 static temperature in the wind-tunnel would be 217K and the stagnation temperature on the model would equal 390.6K, just as in flight.

5) (20%, partial credit given) The Gas Turbine Laboratory uses a compressor to provide high pressure air for various experiments. The compressor takes in 0.3kg/s of air from the atmosphere (100kPa, 300K) and delivers it at a stagnation pressure of 500 kPa. Assume the air behaves as an ideal gas with R=287 J/kg-K, c_v=716.5 J/kg-K, c_p=1003.5 J/kg-K. If the compressor process is quasi-static and adiabatic, what is the stagnation temperature of the air at the exit of the compressor and what is the minimum amount of power that would be required to operate this compressor? (LO# 4)

The stagnation temperature at the inlet is T_{T1}= 300K,

The stagnation pressure at the inlet is p_{T1}= 100×10^3 Pa.

At the outlet p_{T2}= 500×10^3 Pa.

For quasi-static adiabatic processes

$$\left(\frac{T_{T2}}{T_{T1}}\right) = \left(\frac{p_{T2}}{p_{T1}}\right)^{\gamma-1/\gamma} \quad \text{so} \quad T_{T2} = T_{T1}\left(\frac{p_{T2}}{p_{T1}}\right)^{\gamma-1/\gamma} = 300\text{K}(5)^{0.4/1.4} = 475\text{K}$$

The minimum power required is the shaft power for an ideal (quasi-static, adiabatic) compression process.

$$q_{1-2} - w_{s1-2} = h_{T2} - h_{T1} = c_p\left(T_{T2} - T_{T1}\right)$$

$$\text{Power} = \dot{m}\left(-w_{s1-2}\right) = \dot{m}c_p\left(T_{T2} - T_{T1}\right)$$

$$\text{Power input} = 0.3\text{kg/s}\left(1003.5\text{J/kgK}\right)\left(475\text{K} - 300\text{K}\right) = 52.7\text{kW}$$

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