Unified Quiz: Materials & Structures

November 4, 2005

Calculators allowed

No books or notes allowed

A list of equations will be provided if necessary

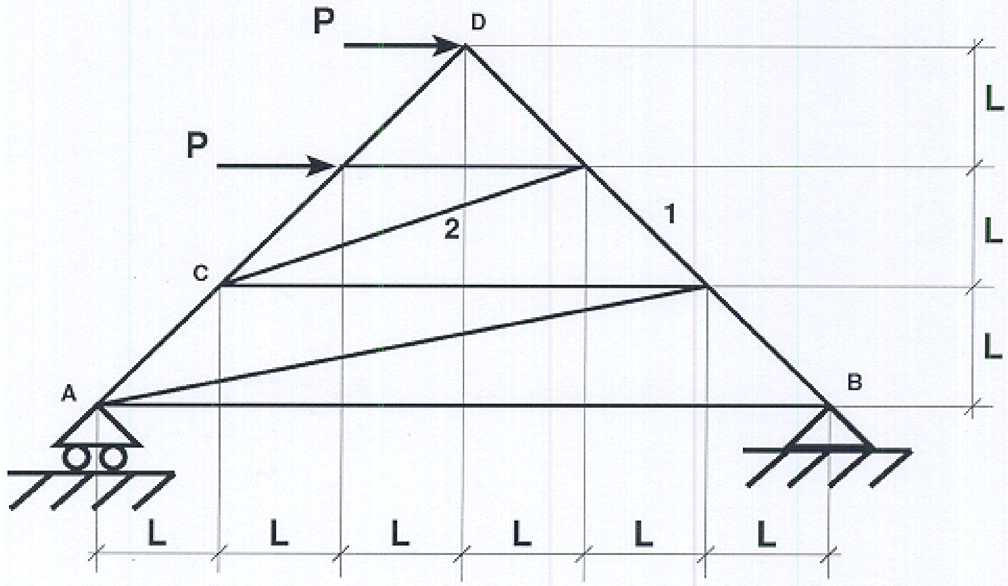
ID Number: SOLUTION

Question	Grade
1) 25 %	
2) 25 %	
3) 25 %	
4) 25 %	
Total:	

- Put your ID number on each page of the exam.
- Read all questions carefully.
- Do all work for each problem on the pages provided.
- Show intermediate results.
- Explain your work — don t just write equations.
- Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers. Box your final answers.

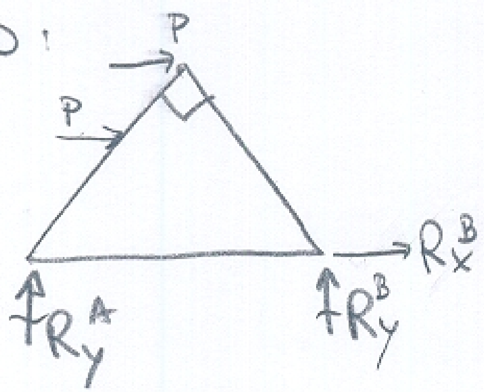
(Questions start on the next page)

1. (25%) For the truss structure shown in the figure, $L = 1m$ and $P = 1kN$:



(a) (5%) Compute the reactions at the supports

FBD:



$\Sigma F_x = 0 \quad R_x^B + 2P = 0 \Rightarrow \boxed{R_x^B = -2P}$

$\boxed{R_x^B = -2kN}$

$\Sigma M_z^A = 0 \quad R_y^B \times 6\cancel{L} - P \times 2\cancel{L} - P \times 3\cancel{L} = 0$

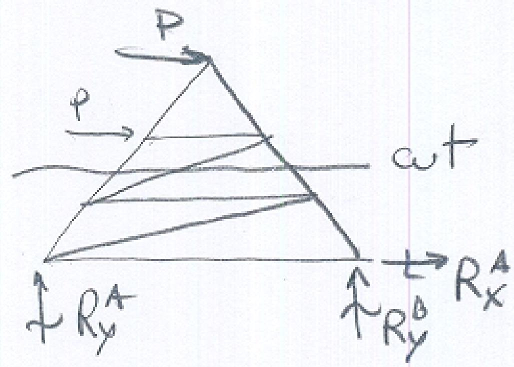
$\Rightarrow \boxed{R_y^B = \frac{5}{6} P = 0.83\ kN}$

$\Sigma F_y = 0 \quad R_y^A + R_y^B = 0$

$\Rightarrow R_y^A = -R_y^B = -\frac{5}{6} P$

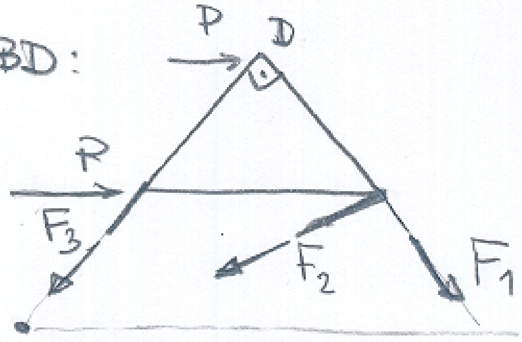
$R_y^A = -0.83\ kN$

(b) (10%) Compute the force in bar 1.



Using method of sections:

FBD:



$$\sum M_z^C = 0$$

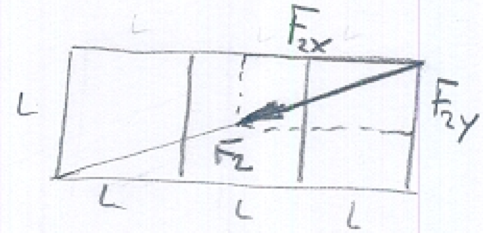
$$-P\times 2\cancel{L} - P\times \cancel{L} - F_1\times \sqrt{2}\cancel{L}\times 2 = 0$$

$$\boxed{F_1 = \frac{-3P}{2\sqrt{2}L}}$$

$$\boxed{F_1 = -1.06\,kN}$$

(c) (10%) Compute the force in bar **2**.

Use same cut.



$$\sum M_z^D = 0$$

$$-F_{2x} \times \cancel{L} - F_{2y} \times \cancel{L} + P \times \cancel{L} = 0$$

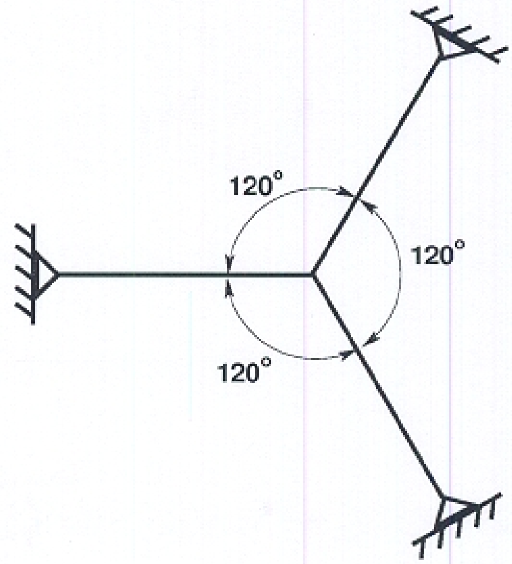
$$\frac{F_{2x}}{F_2} = \frac{3L}{\sqrt{L^2+(3L)^2}} = \frac{3}{\sqrt{10}}$$

$$\frac{F_{2y}}{F_2} = \frac{L}{\sqrt{L^2+(3L)^2}} = \frac{1}{\sqrt{10}}$$

$$-\frac{3}{\sqrt{10}} F_2 - \frac{1}{\sqrt{10}} F_2 + P = 0$$

$$\boxed{F_2 = \frac{\sqrt{10}}{4} P = 0.79 \text{ kN}}$$

2. (25%) The truss system of the figure is subjected to a uniform temperature change ΔT from its assembled, stress-free condition. All three bars have the same geometric and material properties A, L, E, α and form angles of 120^o with the other two bars.

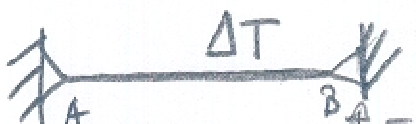


(a) (10%) What is the deformed position of the joint? Provide arguments in words to support your answer.

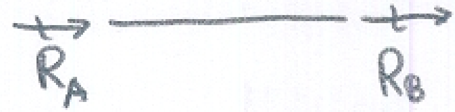
The joint will not move due to the axial symmetry of the problem.

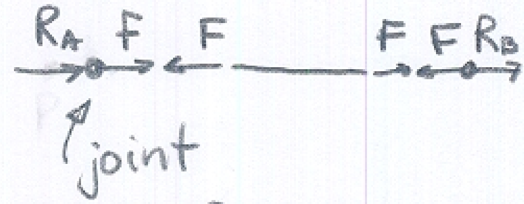
(b) (15%) Compute the forces in all three bars.

Each bar can be analyzed as follows



Equilibrium:

FBD 



$\Sigma F_x = 0$: (1) $R_A + R_B = 0$ (2) $F + R_A = 0$

Constitutive response

(3) $\delta = \dfrac{FL}{AE} + \alpha \Delta T L$

Compatibility

(4) $\delta = 0$

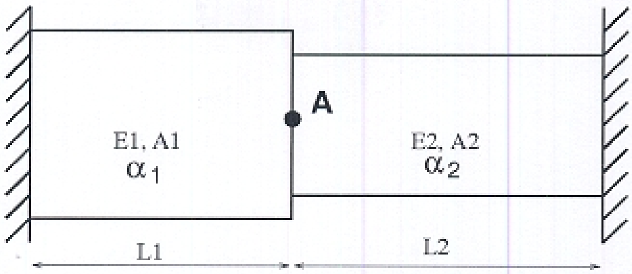
Solve: (4) in (3) $0 = \dfrac{F\cancel{L}}{AE} + \alpha \Delta T \cancel{L}$

$\boxed{F = -\alpha \Delta T A E}$ This is the load on all three bars

(2) $R_A = -F = \alpha \Delta T A E$

(1) $R_B = -R_A = F = \alpha \Delta T A E$

3. (25%) The structure of the figure is subjected to a uniform temperature change ΔT from its assembled, stress-free condition. The geometric and material properties of the two bars are A_1, L_1, E_1, α_1 and A_2, L_2, E_2, α_2, respectively. Assuming uniaxial behavior,

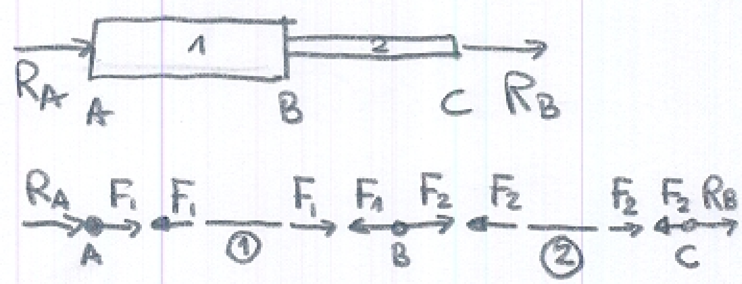


(a) (5%) Enumerate the principle(s) you need to use to analyze this system and explain why?

Equilibrium, constitutive behavior, compatibility
All three principles are needed to analyze and solve the system as it cannot be solved exclusively from equilibrium considerations (statically indeterminate).

(b) (20%) Compute the force in each bar and the displacement of point A.

Equilibrium : FBD



External: $R_A + R_B = 0$ (E)

(A) $F_1 + R_A = 0$

(B) $F_2 - F_1 = 0$

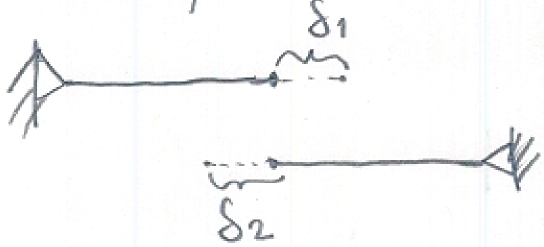
(C) $R_B - F_2 = 0$

constitutive response

$$\delta_i = F_i \underbrace{\frac{L_i}{A_i E_i}}_{\frac{1}{k_i}} + \alpha_i \Delta T L_i \qquad i = 1, 2$$

(C1) (C2)

Compatibility



$$\delta_1 = -\delta_2 \quad \text{comp}$$

Solve C_1, C_2 in comp

$$\frac{F_1}{k_1} + \alpha_1 L_1 \Delta T = -\frac{F_2}{k_2} - \alpha_2 L_2 \Delta T$$

using B
$$F_1\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = -(\alpha_1 L_1 + \alpha_2 L_2)\Delta T$$

$$\boxed{F_1 = -(\alpha_1 L_1 + \alpha_2 L_2)\frac{k_1 k_2}{k_1 + k_2}\Delta T = F_2}$$

$$\delta = \delta_1 = \frac{F_1}{k_1} + \alpha_1 \Delta T L_1$$

$$= -\frac{(\alpha_1 L_1 + \alpha_2 L_2) k_2 \Delta T}{k_1 + k_2} + \alpha_1 L_1 \Delta T$$

$$= \frac{\Delta T}{k_1 + k_2}\left[\alpha_1 L_1 (k_1 + \cancel{k_2} - \cancel{k_2}) - \alpha_2 L_2 k_2\right]$$

$$\boxed{\delta = \frac{\Delta T}{k_1 + k_2}(\alpha_1 L_1 k_1 - \alpha_2 L_2 k_2)}$$

4. (25%) Plane stress.

(a) (5%) Define in words and mathematically the notion of "plane stress"

It is the case in which all the components of the stress field are in the plane (say x_1-x_2 plane) Therefore $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$. By symmetry of the stress tensor σ_{31} and σ_{32} are also 0.

$\Rightarrow$ the stress components may be written using greek indices $\sigma_{\alpha\beta}$

(b) (20%) The stress field in a plane stress problem is given in terms of the two scalar functions $\phi(x_1, x_2)$ and $\Omega(x_1, x_2)$ as follows:

$$\sigma_{11} = \frac{\partial^2 \phi}{\partial x_2^2} - \Omega \qquad \sigma_{22} = \frac{\partial^2 \phi}{\partial x_1^2} - \Omega \qquad \sigma_{12} = \sigma_{21} = -\frac{\partial^2 \phi}{\partial x_1 \partial x_2}$$

Show the conditions under which the stress field satisfies equilibrium.

For equilibrium $\frac{\partial \sigma_{\alpha\beta}}{\partial x_\alpha} + f_\beta = 0$

$\underline{x_1:}$ $\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + f_1 = 0$

$$\cancel{\frac{\partial^3 \phi}{\partial x_2^2 \partial x_1}} - \frac{\partial \Omega}{\partial x_1} + \cancel{\left(-\frac{\partial^3 \phi}{\partial x_1 \partial x_2^2}\right)} + f_1 = 0 \rightarrow f_1 = \frac{\partial \Omega}{\partial x_1}$$

$$x_2: \quad \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + f_2 = 0$$

$$\cancel{-\frac{\partial^3 \phi}{\partial x_1^2 \partial x_2}} + \cancel{\frac{\partial^3 \phi}{\partial x_1^2 \partial x_2}} - \frac{\partial \Omega}{\partial x_2} + f_2 = 0$$

$$f_2 = \frac{\partial \Omega}{\partial x_2}$$

$$\Rightarrow \underline{f} = \underline{\nabla} \Omega$$

The body forces need to derive from a potential, i.e. they need to be conservative, which is a very common situation.