1. (35 %) If we ignore the variation of wind speed with altitude, wind distribution in a hurricane is in effect a 2D velocity field. The wind speeds of a hurricane vary with radius r from the center roughly as sketched. In the "eye" of the hurricane, the velocity very nearly has the simple form

$$V_\theta = C_0 \, r \qquad\qquad V_r = 0$$

while outside the eye, the velocity nearly has the form

$$V_\theta = C_1 / r \qquad\qquad V_r = 0.$$

Typical constants for a medium-size hurricane are

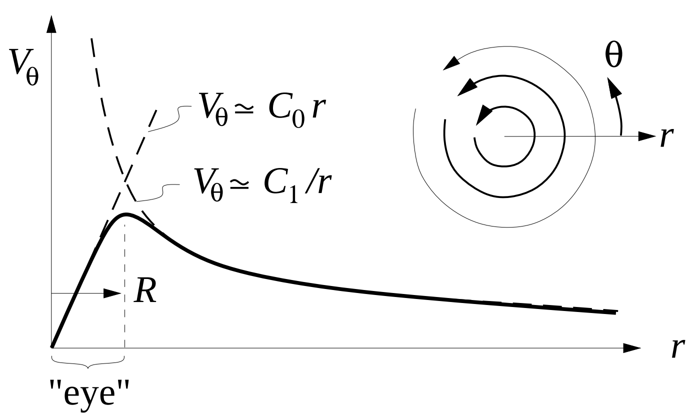
$$C_0 = 1 \, \text{(m/s)/km} \qquad\qquad C_1 = 2500 \, \text{(m/s)km}$$

a) Estimate the radius R of the eye.

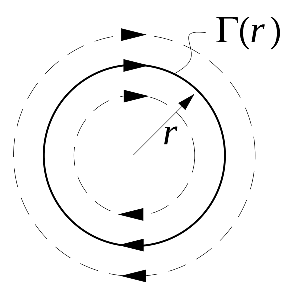
b) Consider a circular circuit of radius r around the center of hurricane. Determine and sketch the circulation $\Gamma(r)$ versus the circuit radius over the entire hurricane. Be sure to specify the units.

c) Determine and sketch the vorticity versus radius $\xi(r)$ over the entire hurricane (i.e. inside and outside the eye). Be sure to specify the units.

a)



b)



2. (40 %) A potential flow consists of a superposition of a uniform flow and doublet.

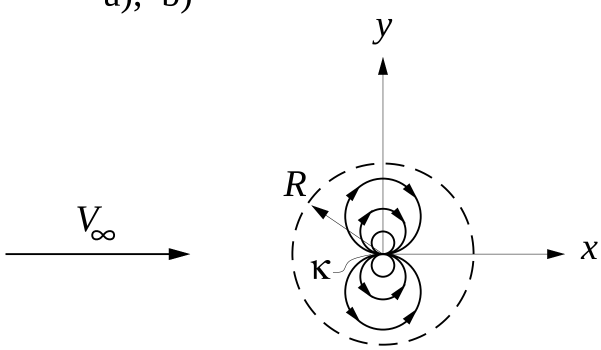
$$\phi \;=\; V_\infty \, r \, \cos\theta \;+\; \frac{\kappa}{2\pi}\frac{\cos\theta}{r}$$

a) Determine the doublet strength κ required to make this be the potential flow about a circular cylinder of radius R.

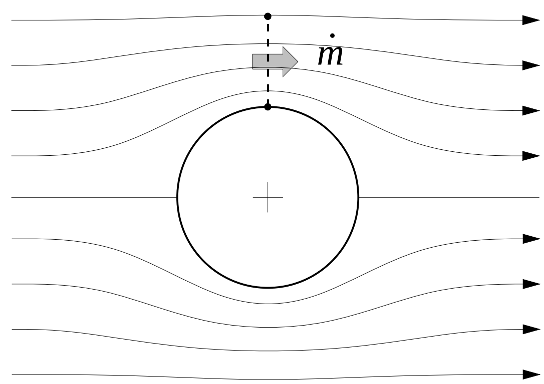
b) Show that with the κ value from a), the surface of the cylinder is a streamline.

c) Determine the mass flow rate $\dot{m}$ through the vertical line connecting the points $(x, y) = (0, R)$ and $(0, 2R)$. You result will be in terms of V_∞, R, and some constant density ρ.

a), b)



c)

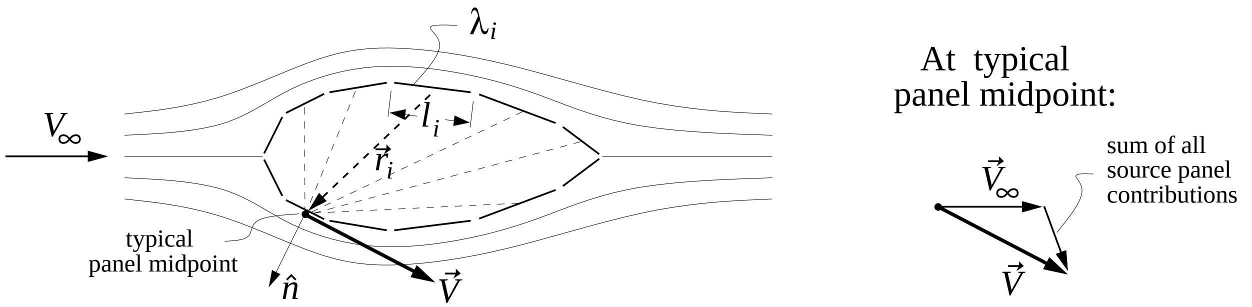


3. (25 %) The flow around a nonlifting body is represented in a panel method by a superposition of N source panels and a uniform freestream.

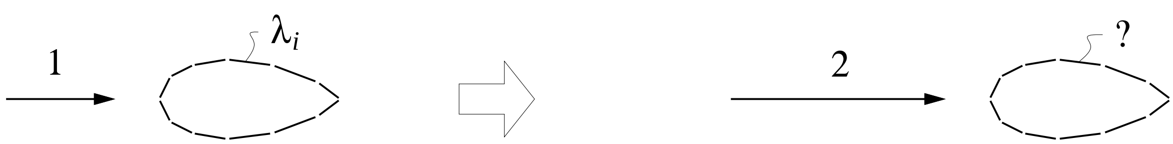
$$\vec{V}(x,y) \;=\; \vec{V}_\infty \;+\; \sum_{i=1}^{N} \lambda_i \int_0^{\ell_i} \frac{\vec{r}_i}{2\pi r_i^2}\, ds$$

The i'th panel has a length ℓ_i, and a constant strength λ_i. All the λ_i are determined by the panel method program so as to obtain flow tangency on each panel midpoint, as shown in the Figure below.

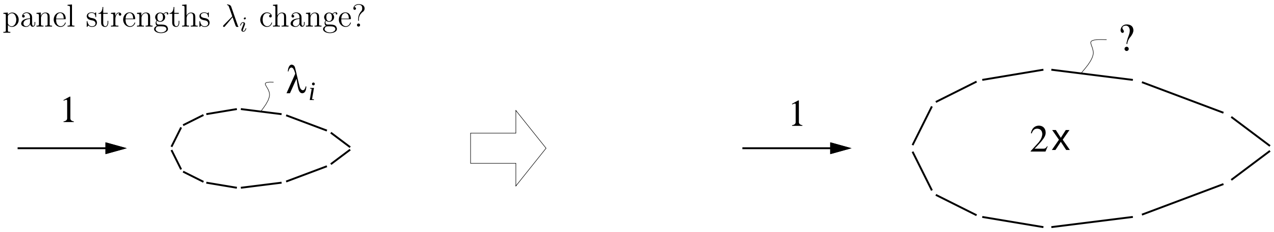
$$\vec{V} \cdot \hat{n} \;=\; 0 \qquad\qquad \text{on all panel midpoints}$$



a) The freestream speed is now doubled, from $V_\infty = 1$ to $V_\infty = 2$, and the panel method is run again. By how much will all the panel strengths λ_i change?



b) The freestream speed is set back to $V_\infty = 1$, but the body and all the panels are doubled in size, keeping the shape the same. The panel method is then run again. How will all the panel strengths λ_i change?



Hint: For each case, consider what must or must not happen to the direction of each $\vec{V}(x,y)$ vector at the panel midpoints when V_∞ or the geometry size is changed.