1. (35%) If we ignore the variation of wind speed with altitude, wind distribution in a hurricane is in effect a 2D velocity field. The wind speeds of a hurricane vary with radius \( r \) from the center roughly as sketched. In the “eye” of the hurricane, the velocity very nearly has the simple form

\[
V_\theta = C_0 r \quad V_r = 0
\]

while outside the eye, the velocity nearly has the form

\[
V_\theta = C_1 /r \quad V_r = 0.
\]

Typical constants for a medium-size hurricane are

\[
C_0 = 1 \text{ (m/s)/km} \quad C_1 = 2500 \text{ (m/s)km}
\]

a) Estimate the radius \( R \) of the eye.

b) Consider a circular circuit of radius \( r \) around the center of hurricane. Determine and sketch the circulation \( \Gamma(r) \) versus the circuit radius over the entire hurricane. Be sure to specify the units.

c) Determine and sketch the vorticity versus radius \( \xi(r) \) over the entire hurricane (i.e. inside and outside the eye). Be sure to specify the units.
2. (40 %) A potential flow consists of a superposition of a uniform flow and doublet.

\[ \phi = V_\infty r \cos \theta + \frac{\kappa \cos \theta}{2\pi r} \]

a) Determine the doublet strength \( \kappa \) required to make this be the potential flow about a circular cylinder of radius \( R \).

b) Show that with the \( \kappa \) value from a), the surface of the cylinder is a streamline.

c) Determine the mass flow rate \( \dot{m} \) through the vertical line connecting the points \((x, y) = (0, R) \) and \((0, 2R) \). Your result will be in terms of \( V_\infty, R \), and some constant density \( \rho \).
3. (25 %) The flow around a nonlifting body is represented in a panel method by a superposition of \( N \) source panels and a uniform freestream.

\[
\vec{V}(x, y) = \vec{V}_\infty + \sum_{i=1}^{N} \lambda_i \int_{0}^{\ell_i} \frac{\vec{r}_i}{2\pi r_i^2} ds
\]

The \( i \)’th panel has a length \( \ell_i \), and a constant strength \( \lambda_i \). All the \( \lambda_i \) are determined by the panel method program so as to obtain flow tangency on each panel midpoint, as shown in the Figure below.

\[
\vec{V} \cdot \hat{n} = 0 \quad \text{on all panel midpoints}
\]

a) The freestream speed is now doubled, from \( V_\infty = 1 \) to \( V_\infty = 2 \), and the panel method is run again. By how much will all the panel strengths \( \lambda_i \) change?

b) The freestream speed is set back to \( V_\infty = 1 \), but the body and all the panels are doubled in size, keeping the shape the same. The panel method is then run again. How will all the panel strengths \( \lambda_i \) change?

Hint: For each case, consider what must or must not happen to the direction of each \( \vec{V}(x, y) \) vector at the panel midpoints when \( V_\infty \) or the geometry size is changed.