Consider the 2-D velocity field $\vec{V} = u \hat{i} + v \hat{j}$

$$u = y \quad \quad v = x$$

whose hyperbolic-shape streamlines are sketched below. The density $\rho$ is everywhere constant.

a) Evaluate the mass-flow integral

$$I = \oint \rho (\vec{V} \cdot \hat{n}) \, dA$$

for the square control volume of side length $\ell$ shown in the figure.

Hint: First express $I$ as four separate integrals $I_1, I_2, I_3, I_4$ over each of the four segments of the C.V. boundary. Note that $dA$ is a length in 2-D.

b) Does this flow satisfy the mass conservation law everywhere? Explain.