Problem S6.1: Look-back to Lectures 9 and 10 (10 points)

Consider the following spring-mass-damper system:

where \( d_1 \) is the displacement of mass \( m_1 \) and \( d_2 \) is the displacement of mass \( m_2 \), both measured relative to the mass rest positions. \( k \) is the spring constant, \( c \) is the damping coefficient, and \( f \) is the force applied to \( m_2 \) as shown in the diagram.

(a) Derive the differential equations governing this system.

(b) If the input is the applied force \( f \) and the output of interest is \( d_2 \), what are the state-space matrices \( A, B, C, \) and \( D \)?

(c) Sketch a block diagram of your state-space system in part (b).

(d) If the input is the applied force \( f \), and the outputs of interest are \( d_1 \) and \( d_2 \), what are the state-space matrices \( A, B, C, \) and \( D \)?

(e) For the following parameter values: \( m_1 = 1, m_2 = 0.1, k = 0.1, c = 2 \), compute the eigenvalues and eigenvectors of your matrix \( A \) from part (b). You can do this computationally using Matlab as follows. First, define the matrix \( A \) in Matlab. Then the command \([V,D] = \text{eig}(A);\) returns the eigenvectors of \( A \) as columns of the matrix \( V \) and the corresponding eigenvalues of \( A \) on the diagonal entries of the matrix \( D \). Type \texttt{help eig} for more help on how to use this function.