S1) A LTI system has input $x(t)$ and output $y(t)$.

The input and output of the LTI system satisfy the following differential equation

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

The input is a periodic function of time that produces the following periodic output

$$y(t) = 2\cos(t - \pi/4)$$

a) [2pts] Determine $H(j\omega)$, the input/output frequency response, of the LTI system

b) [1pt] What is $T$, the period of the output $y(t)$?

c) [1pt] What is $\omega_0$, the fundamental frequency of the output $y(t)$?

d) [4pts] Show that the Fourier coefficients of the output $y(t)$ are-

$$b_1 = e^{-j\pi/4}$$
$$b_{-1} = e^{j\pi/4}$$
$$b_k = 0 \quad k \neq \pm 1$$

e) [2pts] What is the period of the input $x(t)$?

f) [6pts] Determine the Fourier coefficients of the input $x(t)$

g) [4pts] Write the input $x(t)$ as a real function of time (i.e., not as a complex variable)
Signals and Systems Question S1
Student ID Number __________________________