M10.1 *(15 points)* Let’s further explore the concept of instability via the configuration considered in lecture of the rod of length \( L \) pinned with a torsional spring of stiffness \( k_T \) and loads in each of the two directions. This is represented in the accompanying figure:

We extend the model in this case from the rod being infinitely rigid/stiff along its length to it being able to deform based on the basic rod model. This can be considered as being a linear spring with a stiffness constant proportional to the properties of the rod of the area (\( A \)), the modulus (\( E \)), and the length (\( L \)), resulting in the expression: \( k_L = EA/L \). The length of the rod will thus change as it is loaded along its length. (NOTE that buckling of the rod as a column is not considered here).

(a) Determine the basic expression governing the rotational angle, \( \theta \), of the configuration as a function of the two applied loads and the parameters as defined.

(b) Determine an expression for the critical value of the load applied along the length of the rod, \( P_{2\text{cr}} \), defined as where instability occurs.

(c) Plot this critical value of the load as a function of the value of the rod stiffness, \( EA \), normalized by the ratio of the torsional spring stiffness to the rod length: \( n = EA/(k_T/L) \). Explain the behavior that is observed.
M10.2 (15 points) A column of length $L$ is clamped at one end and has no support at the other end. That end is loaded via a pressure of magnitude $p$ and the overall load thus changes with and is proportional to the overall area of the cross-section. The column is made of a material with a modulus of $E$.

Different cross-sections are to be considered for the column. For each, determine the expressions to find the buckling load and the buckling mode.

(a) Cross-section A is square with side lengths equal to $a$.

(b) Cross-section B is circular with a diameter equal to $a$. 