**Problem 1**

Consider a situation as sketched. The wall is made of steel, with a thermal conductivity $K = 40 \, \text{W/m/K}$. The gas is air at a pressure of 50 atm, and flows at a velocity of 700 m/s. The liquid is water, flowing at 60 m/s. The friction coefficients are estimated as $c_f = 0.0011$ on the gas side, and $c_f = 0.0033$ on the liquid side.

Calculate the steady state heat flux $q$ and the temperatures $T_{wh}$, $T_{wc}$.

**Problem 2**

The earth’s surface is exposed to yearly variations of air temperature, of the form

$$T_s = \bar{T} + \Delta T \sin \omega t$$

(1)

where $\bar{T}$ is the mean temperature, $\Delta T$ is the peak seasonal variation, say 20K in temperate latitudes, and

$$\omega = \frac{2\pi}{1 \, \text{yr}} = \frac{2\pi}{3.15 \times 10^7 \, \text{s}} \, \text{s}^{-1}.$$

The soil has a thermal conductivity $k = 1.6 \, \text{W/m/K}$, a density $\rho = 2000 \, \text{Kg/m}^3$ and a specific heat $c = 2000 \, \frac{J}{\text{KgK}}$.

(a) Assume the temperature distribution is of the form $T - \bar{T} = \text{Re} \left[ A \, e^{i(\alpha - ky)} \right]$ where $A$ is a complex amplitude factor, and $\text{Re} \left[ \right]$ means the real part of a complex number. Substitute into the transient heat conduction equation and calculate $k$ as $k = \pm \sqrt{-i \frac{\alpha}{\omega}} = \pm (1 - i) \sqrt{\frac{\omega}{2\alpha}}$. Show that the lower sign leads to
exponential divergence with \( y \), so select the upper sign. Choose the constant \( A \) so as to reproduce the variation \( T_i(t) \) on the surface. Your solution should be

\[
T(y,t) = \bar{T} + \Delta T \, e^{-\frac{\omega y}{2\alpha}} \sin \left( \frac{\omega t - \frac{\omega y}{\sqrt{2\alpha}}}{2} \right)
\]

(2)

(b) At what depth is the amplitude of the temperature oscillation reduced to \( \pm 1 K \)? If peak surface temperature happens on August 1, when does it happen at that depth?