First determine the relative magnitude of $q(x)$ in terms of $P$.

We are given:

$$
\int_0^L q(x) \, dx = P
$$

We need a functional expression for $q(x)$. Assign it a value of $q_0$ at the midpoint $(x = L)$ and work to determine that value. Note two "parts" to $q(x)$:

$0 \leq x < L$; $L < x < 2L$. Consider each of these separately.
For $0 < x < L$, $f(x)$ increases linearly from a value of 0 at $x = 0$ to $f_0$ at $x = L$. So:

$$f(x) = \frac{f_0}{L} x \quad 0 < x < L$$

Check:

$$f(0) = 0 \checkmark$$

$$f(L) = f_0 \checkmark$$

$$\frac{df(x)}{dx} = \frac{f_0}{L} \quad \text{increases linearly}$$

Now for $L < x < 2L$, $f(x)$ decreases linearly from a value of $f_0$ at $x = L$ to a value of 0 at $x = 2L$. So:

$$f(x) = f_0 \left(\frac{2L-x}{L}\right)$$

(Work: can determine by beginning with $f(x) = ax + b$ and using the two conditions at two operations in a and b and solve for a and b)

Check: (just engineers always do when they can)

$$f(L) = f_0 \checkmark$$

$$f(2L) = 0 \checkmark$$

$$\frac{df(x)}{dx} = -\frac{f_0}{L} \quad \text{decreases linearly}$$
Now determine $f_0$. Remember:

$$\int_0^{2L} f(x) \, dx = P$$

$$\Rightarrow \int_0^L f(x) \, dx + \int_L^{2L} f(x) \, dx = P$$

$$\int_0^L \frac{f_0}{2} x \, dx + \int_L^{2L} \frac{f_0}{2} (2L-x) \, dx = P$$

$$\frac{f_0}{2} \left[ \frac{x^2}{2} \right]_0^L + \frac{f_0}{2} \left[ 2Lx - \frac{f_0}{2} x^2 \right]_L^{2L} = P$$

$$\frac{f_0}{2} \left[ 4L^2 - 2L^2 - (2L - \frac{L}{2}) \right] = P$$

$$\Rightarrow \frac{f_0}{2} = P \Rightarrow f_0 = \frac{2P}{L}$$

Finally:

$$f(x) = \begin{cases} \frac{P}{L^2} & 0 < x < L \\ \frac{P}{L^2} (2L-x) & L < x < 2L \end{cases}$$

Proceed to:

(a) First step is to draw the Free Body Diagram.
We equilibrate:

\[ \Sigma F_x = 0 \implies H_B = 0 \]

\[ \Sigma F_z = 0 \implies V_A = V_B + \int_0^{2L} q_0(x) \, dx = 0 \]

\[ \text{Recall this integral has a magnitude of } P \]

\[ \implies V_A + V_B = -P \quad (1) \]

\[ \Sigma M_A = 0 \implies V_B (L) - \int_0^L q_0(x) (L-x) \, dx 
+ \int_L^{2L} q_0(x) (x-L) \, dx = 0 \]

\[ \implies V_B L - \int_0^L \frac{P x}{L^2} (L-x) \, dx + \int_L^{2L} \frac{P x}{L^2} (2L-x) (x-L) \, dx = 0 \]

\[ V_B L - \left( \frac{P x^2}{2L} - \frac{P x^3}{3L^2} \right)_0^L + \int_L^{2L} \frac{P}{L^2} \left( -2L^2 + 3Lx - x^2 \right) \, dx = 0 \]

\[ V_B L - \frac{PL}{2} + \frac{PL}{3} + \frac{P}{L^2} \left[ -2L^2 x + \frac{3}{2} L x^2 - \frac{x^3}{3} \right]_L^{2L} = 0 \]
Continuing:

\[ V_B L - \frac{P L}{6} + \frac{P}{L^2} \left[ -4L^3 + 6L^3 - \frac{4}{3} L^3 \right. \]
\[ \left. - (-2L^3 + \frac{3}{2} L^3 - \frac{L^3}{3}) \right] = 0 \]

\[ \Rightarrow V_B L - PL \left( \frac{1}{6} - 4 + 6 - \frac{8}{3} + 2 - \frac{3}{2} + \frac{1}{3} \right) = 0 \]

\[ V_B = P \left( 4 - \frac{1}{6} - \frac{14}{6} - \frac{9}{6} \right) \]

\[ \Rightarrow \frac{V_B}{P} = 0 \]

**Using (1):** \[ V_A = -P \]

**Summarizing the reactions are:**

\[
\begin{array}{c}
H_B = 0 \\
V_A = -P \\
V_B = 0 \\
\end{array}
\]

**Note 1:** The integral in the moment for \( q(x) \) could have been found to begin about \( A \) by inspection due to the symmetry of \( f(x) \) about \( x = c \). The \( q(x) \) to the left counteracts that to the right.
Note 2: It appears that the beam "feeters" on the roller support. However, the pin provides a reaction if there is any shift in $q(x)$.

(b) This needs to be done in parts since there is a point load (reaction) along the beam at $x=L$ and there is also a change in $q(x)$ also at $x=L$.

$\Rightarrow \Phi$, for $0 < x < L$:

\[ F(x) = 0 \]

There is no load in $x$ \[ F(x) = 0 \]

Found: $q(x) = \frac{P}{L^2}$

Use: $\frac{ds}{dx} = q(x)$

$\Rightarrow s(x) = \int q(x)dx = \int \frac{P}{L^2} dx$

$\Rightarrow s(x) = \frac{Px^2}{2L^2} + C,$

Use a boundary condition to set the constant of integration.

Look at $x = 0^+$:
\[ s(0)^+ \quad s_0 : \sum F_x = 0 \quad F_+ \Rightarrow 0 = s(0^+) \]

\[ \text{no other loads} \]

\[ \text{giving} \quad s(0^+) = C_1 \quad \Rightarrow \quad C_1 = 0 \]

\[ \text{finally yielding} \quad s(x) = \frac{P x^2}{2L^2} \]

Proceeding to the moment...

\[ \frac{dM}{dx} = s \]

\[ \Rightarrow M(x) = \int s(x) \, dx = \int \frac{P x^2}{2L^2} \, dx \]

\[ = \frac{P x^3}{6L^2} + C_2 \]

Again, use a boundary condition. At \( x = 0 \),

then in an applied moment \( M(0^+) = 0 \)

\[ \text{giving} \quad C_2 = 0 \]

\[ \Rightarrow M(x) = \frac{P x^3}{6L^2} \]

Summary:

For \( 0 < x < L \)

\[ f(x) = \frac{P x}{L^2} \]
\[ F(x) = 0 \]
\[ s(x) = \frac{P x^2}{2L^2} \]
\[ M(x) = \frac{P x^3}{6L^2} \]
Move on to $L < x < 2L$:

There is still no loading in $x$, so $\{ F(x) = 0 \}$

Using $\frac{dF(x)}{dx} = q(x)$ with $q(x) = \frac{P}{L^2} (2L-x)$

we again get:

$$S(x) = \int q(x) \, dx = \int \frac{P}{L^2} (2L-x) \, dx$$

$$\Rightarrow S(x) = \frac{2Px}{L} - \frac{Px^2}{2L^2} + C_3$$

There are differences in the boundary conditions in this sector, but one can go to the top ($x = 2L$) and take a cut giving a "negative" force. There is no reaction at:

$$\left. \begin{array}{c} \Sigma F(x) = 0 \\ \Phi = 0 \rightarrow S(2L) = 0 \end{array} \right\}$$

Providing $S(2L) = 0 = \Phi - 2\Phi + C_3$

$\Rightarrow S(x) = \frac{P}{L} \left( 2x - \frac{x^2}{2L} - 2L \right)$

$\Rightarrow C_3 = -2P$

Proceed again to $\frac{dM}{dx} = S(x)$

$$\Rightarrow M(x) = \int S(x) \, dx = \int \frac{P}{L} \left( 2x - \frac{x^2}{2L} - 2L \right) \, dx$$
\[ M(x) = \frac{Px^2}{L} - \frac{Px^3}{6L^2} - 2Px + C_4 \]

Again for \( x = 2L \), there are no applied or reactive moments so

\[ M(2L) = 0 \]

\[ 0 = 4PL - \frac{4}{3} 4PL + C_4 \]

\[ C_4 = \frac{4}{3} 4PL \]

Finally:

\[ M(x) = P \left( \frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4}{3} L \right) \]

Summary:

For \( L < x < 2L \):

- \( q(x) = \frac{P}{L} (2L-x) \)
- \( f(x) = 0 \)
- \( s(x) = \frac{P}{L} (2x - \frac{x^2}{2L} - 2L) \)
- \( M(x) = P \left( \frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4}{3} L \right) \)

\[ \frac{PL}{6} = P \left( L - \frac{L}{6} - 2L + \frac{4}{3} L \right) \]
\[ \frac{PL}{6} = P \left( -\frac{L}{6} - \frac{5L}{6} \right) \]
\[ \frac{PL}{6} = \frac{PL}{6} \quad \checkmark \quad \text{YES} \]

→ Now draw these. In sketching, use the relations of the derivatives to fit a shape. Calculate end point values to begin. And recall that point loads cause equal jumps in shear (account for proper direction and sign).

\[ F(x) = 0 \text{ everywhere... we need to plot} \]

→ next page...
Loading
\[ \frac{q(x)}{p} \]

Point force:
\[ V_A = -P \quad \text{at } x = L \]

\[ \sigma(x) = \frac{P}{2} \]

Boundary value
\[ \sigma(L) = \frac{P}{2} \]

\[ V_A = -P \quad \text{at } x = L \]

\[ \sigma(L) = P(2 - \frac{L}{2} - 2) = -\frac{P}{2} \]

\[ M(x) = \frac{P}{6} \]

Boundary value
\[ M(L) = M(L^-) \]

Calculate \[ M(L^+) = M(L^-) \]

Boundary value
\[ M = 0 \]
(c) Cut the beam first at \( x = \frac{y_2}{2} \)

\[
\begin{align*}
\frac{P_x}{L^2} &\rightarrow M\left( \frac{y_2}{2} \right) \\
\sum \tau &\rightarrow S\left( \frac{y_2}{2} \right) \\
\end{align*}
\]

Use equilibrium:

\[
\begin{align*}
\sum \tau_x &= 0 \Rightarrow \phi \left( \frac{y_2}{2} \right) = 0 \checkmark \\
\sum \tau_x &= 0 \Rightarrow \int_{0}^{y_2} \frac{P_x}{L^2} \, dx - S\left( \frac{y_2}{2} \right) = 0 \\
\Rightarrow & S\left( \frac{y_2}{2} \right) = \frac{P_x^2}{2L^2} \left| \int_{0}^{y_2} \right. \\
S\left( \frac{y_2}{2} \right) &= \frac{P}{8} \checkmark \\
\text{Check:} & S(x) = \frac{P_x^2}{2L^2} \Rightarrow S\left( \frac{y_2}{2} \right) = \frac{P}{8} \checkmark
\end{align*}
\]

Finally:

\[
\begin{align*}
\sum \tau_y &= 0 \Rightarrow M\left( \frac{y_2}{2} \right) - \int_{0}^{y_2} \frac{P_x}{L^2} \left( \frac{y_2}{2} - x \right) \, dx = 0 \\
\Rightarrow & M\left( \frac{y_2}{2} \right) = \frac{P_x^2}{4L} - \frac{P_x^3}{3L^2} \left| \int_{0}^{y_2} \right. \\
&= \frac{PL}{c_6} - \frac{PL}{24} = PL \left[ \frac{3}{48} - \frac{2}{48} \right] = \frac{PL}{48}
\end{align*}
\]
Check: $M(\frac{L}{2}) = \frac{PL}{48} \checkmark$

$\rightarrow$ Now cut the beam at $x = \frac{3L}{2}$. Make it simpler by taking a negative face cut:

$M(\frac{3L}{2})$

$M(\frac{L}{2}) \rightarrow \frac{3L}{2}$

Again use equilibrium:

$\sum F_x = 0 \Rightarrow \frac{P}{2} \cdot \frac{3L}{2} = 0 \checkmark$

$\sum F_y = 0 \Rightarrow M(\frac{3L}{2}) + \int_{\frac{3L}{2}}^{2L} \frac{P}{L} (2x - x) dx = 0$

$\Rightarrow M(\frac{3L}{2}) = - \frac{9P}{8}$

$f_{\text{right}} M(\frac{3L}{2}) = -\frac{P}{8}$

Check: $M(x) = \frac{P}{L} (2x - \frac{x^2}{2L} - 2x)$

$\Rightarrow M(\frac{3L}{2}) = P(3 - \frac{9}{8} - 2) = -\frac{P}{8} \checkmark$
\[ \sum M_{3y_2} = 0 \iff -M \left( \frac{3L}{2} \right) + \int_{3y_2}^{2L} \frac{P}{L^2} (2L-x)(x-\frac{3L}{2}) \, dx = 0 \]

\[ \implies M \left( \frac{3L}{2} \right) = \int_{3y_2}^{2L} \frac{P}{L^2} (-3L^2 + \frac{7}{2}Lx - x^2) \, dx \]

\[ M \left( \frac{3L}{2} \right) = -3PL + \frac{7P}{4L} x^2 - \frac{P}{3L^2} x^3 \bigg|_{3y_2}^{2L} \]

\[ = PL \left( -6 + 7 - \frac{8}{3} + \frac{9}{3} - \frac{63}{18} + \frac{9}{4} \right) \]

\[ = PL \left( \frac{48}{48} - \frac{126}{48} + \frac{216}{48} - \frac{63}{48} + \frac{9}{48} \right) \]

\[ \implies M \left( \frac{3L}{2} \right) = \frac{PL}{48} \]

Check:

\[ M(x) = P \left( \frac{x^2}{L} - \frac{x^3}{6L^2} - 2x + \frac{4L}{3} \right) \]

\[ \implies M \left( \frac{3L}{2} \right) = PL \left( \frac{9}{4} - \frac{9}{16} - 3 + \frac{4}{3} \right) \]

\[ = PL \left( \frac{36}{48} - \frac{27}{48} - \frac{144}{48} + \frac{64}{48} \right) \]

\[ \therefore M \left( \frac{3L}{2} \right) = \frac{PL}{48} \checkmark \]

\[ \text{All Checks OK} \]
M 3.2

Model for Case 1: Lift constant along wing span

(a) Note that this can be done irrespective of the model used. Consider the reason...

The model is the free body diagram. There are no reaction forces since the wing has no internal supports that can carry the load.

NOTE: It is not dynamic because of symmetry of the lift (for all models) resulting in a moment balance and because of the special condition that the integrated lift force equals total plane weight in stable flight (for all cases).
Further Note: If the lift is not symmetric, we get rotation about the fuselage -- a way to maneuver the aircraft.

(b) We now must consider this case by case.

Case 1 -- lift constant along span from root term:

\[ q(x) = q_0 \]

There are no axial forces so \( F(x) = 0 \)

we have two sections of the wing:

\[ 0 < x < \frac{1}{2} \]

\[ -\frac{1}{2} < x < 0 \]

There is symmetry, so our results should be the same, but let's be sure:

For: \[ 0 < x < \frac{1}{2} \]

\[ q(x) = q_0 \]
\[
\frac{dc}{dx} = g(x) \Rightarrow c(x) = \int g(x) \, dx = q_0 x + C,
\]

Go to the tip and see \( s = 0 \) at \( x = \frac{L}{2} \)
\[
\Rightarrow s \left( \frac{L}{2} \right) = 0 = \frac{q_0 L}{2} + C \Rightarrow C, = -\frac{q_0 L}{2}
\]

\[
\Rightarrow s(x) = q_0 \left( x - \frac{L}{2} \right)
\]

Proceed to:
\[
\frac{dm}{dx} = s
\]
\[
\Rightarrow m(x) = \int q_0 \left( x - \frac{L}{2} \right) \, dx = q_0 \frac{x^2}{2} - q_0 \frac{Lx}{2} + C_2
\]

Again, at the tip: \( M = 0 \) at \( x = \frac{L}{2} \)
\[
\Rightarrow M \left( \frac{L}{2} \right) = q_0 \frac{L^2}{8} - q_0 \frac{L^2}{4} + C_2
\]
\[
\Rightarrow C_2 = \frac{q_0 L^2}{8}
\]

Finally:
\[
M(x) = \frac{q_0}{2} \left( x^2 - Lx + \frac{L^2}{4} \right)
\]

Now for: \(-\frac{L}{2} < x < 0\)

Again: \( g(x) = q_0 \)
Using: \( \frac{ds}{dx} = q(x) \Rightarrow s(x) = \int q \, dx \)
\[ = q_0 \, x + C_3 \]

Go to that tip \((x = -\frac{L}{2})\) where \(s = 0\) and:
\[
s(-\frac{L}{2}) = -q_0 \, \frac{L}{2} + C_3 \quad \Rightarrow \quad C_3 = \frac{q_0 \, L}{2}
\]
\[ \Rightarrow \quad s(x) = q_0 \, (x + \frac{L}{2}) \]

Note that the value must change by the concentrated load of the weight \(P\) at the root \((\frac{ds}{dx}(x = 0) = -P)\) and it does as \(P = q_0 \, L\)!

And finally with: \( \frac{dM}{dx} = s(x) \)
\[ \Rightarrow M(x) = \int q_0 \, (x + \frac{L}{2}) \, dx \]
\[ = q_0 \, \frac{x^2}{2} + q_0 \, \frac{L}{2} \, x + C_4 \]

Again, at this tip \((x = -\frac{L}{2})\), the moment is zero. So:
\[
M(-\frac{L}{2}) = 0 = \frac{q_0 \, L^2}{8} - q_0 \, \frac{L^2}{4} + C_4
\]
\[ \Rightarrow \quad C_4 = \frac{q_0 \, L^2}{8} \]
\[ M(x) = \frac{g_0}{2} \left( x^2 + Lx + \frac{L^2}{4} \right) \]

This is symmetric about the root of \( x \), switches from + to - in value.

**Summarizing for Case 1**

\[
\begin{align*}
F(x) &= 0 \quad \text{everywhere} \\
S(x) &= g_0 \left( x - \frac{L}{2} \right) \quad 0 < x < \frac{L}{2} \\
    &= g_0 \left( x + \frac{L}{2} \right) \quad -\frac{L}{2} < x < 0 \\
M(x) &= \frac{g_0}{2} \left( x^2 - Lx + \frac{L^2}{4} \right) \quad 0 < x < \frac{L}{2} \\
    &= \frac{g_0}{2} \left( x^2 + Lx + \frac{L^2}{4} \right) \quad -\frac{L}{2} < x < 0 \\
\end{align*}
\]

with: \( g_0 = 2750 \, \text{lb/ft} \), \( L = 200 \, \text{ft} \)

Note concerning symmetry: We showed the symmetry in this case. For subsequent cases we take this symmetry and calculate for only one wing and determine the expression for the other wing by:

1. changing \( S(x) \) by \( P \) at the fuselage (+ or - if proceeding from + \( x \) to - \( x \)) and changing the sign of even powers of \( x \).
(2) changing odd powers of $x$ in the moment expression in theirsign

\[ q(x) = q_0 \left(1 - \frac{2x}{l}\right) \quad 0 < x < \frac{l}{2} \]
\[ = q_0 \left(1 + \frac{2x}{l}\right) - \frac{q_0}{l} x < 0 \]

For the 787: $q_0 = 5400 \text{ lbs/ft}$
$L = 150 \text{ ft}$

Again, there are no axial terms so $F(x) = 0$

Use $\frac{dS}{dx} = q(x) \Rightarrow S(x) = \int q(x) dx \quad \text{for} \quad 0 < x < \frac{l}{2}$:

\[ S(x) = \int q_0 \left(1 - \frac{2x}{l}\right) dx \]
\[ = q_0 x - \frac{q_0 x^2}{c} + C \]

In all cases, we have at the tip $S(\frac{l}{2}) = 0$
\[ S_0: \quad q_0 \frac{L}{2} - q_0 \frac{L}{4} + C_x = 0 \]
\[ \Rightarrow C_x = -\frac{q_0 L}{4} \]
\[ \Rightarrow S(x) = q_0 \left( x - \frac{x^2}{L} - \frac{L}{4} \right) \]

Progress to:
\[ \frac{dm}{dx} = S \]
\[ \Rightarrow M(x) = \int q_0 \left( x - \frac{x^2}{L} - \frac{L}{4} \right) dx \]
\[ = q_0 \frac{x^2}{2} - q_0 \frac{x^3}{3L} - q_0 \frac{Lx}{4} + C_2 \]

Again at the top \( x = \frac{L}{2} \), \( M = 0 \)
\[ \Rightarrow M \left( \frac{L}{2} \right) = 0 = q_0 L^2 \left( \frac{1}{8} - \frac{1}{24} - \frac{3}{8} \right) + C_2 \]
\[ \Rightarrow C_2 = q_0 L^2 \left( \frac{6}{48} - \frac{1}{24} - \frac{3}{48} \right) \]
\[ \text{finally: } C_2 = \frac{q_0 L^2}{24} \]

Finally:
\[ M(x) = q_0 \left( \frac{x^2}{2} - \frac{x^3}{3L} - \frac{Lx}{4} + \frac{L^2}{24} \right) \]
Summarizing for Case 2

\[ F(x) = 0 \quad \text{everywhere} \]
\[ S(x) = \begin{cases} \frac{q_0}{2} \left( x - \frac{x^2}{L} - \frac{c}{4} \right) & 0 < x < \frac{L}{2} \\
\frac{q_0}{2} \left( x + \frac{x^2}{L} - \frac{c}{4} \right) + P & -\frac{L}{2} < x < 0 \end{cases} \]
\[ M(x) = \begin{cases} \frac{q_0}{2} \left( \frac{x^2}{2} - \frac{x^3}{3L} - \frac{c^2}{4} + \frac{L^2}{24} \right) & 0 < x < \frac{L}{2} \\
\frac{q_0}{2} \left( \frac{x^2}{2} + \frac{x^3}{3L} + \frac{c^2}{4} + \frac{L^2}{24} \right) & -\frac{L}{2} < x < 0 \end{cases} \]

with \( q_0 = 5700 \text{ kN/m} \), \( L = 200 \text{ m} \), \( P = 570,000 \text{ kN} \)

\[ \rightarrow \text{Case 3: lift linear variation along span} \]
\[ \text{with maximum value at root down to} \]
\[ \text{minimum value at tip} \]

From last term:

\[ \mathcal{Z} \]

\[ \mathcal{Q} \]

\[ \mathcal{M} \]

\[ P \]

\[ x \]
\[ q(x) = \begin{cases} \frac{q_0}{L} (1 - \frac{x}{L}) & 0 < x < \frac{L}{2} \\ \frac{q_0}{L} (1 + \frac{x}{L}) & -\frac{L}{2} < x < 0 \end{cases} \]

For the given \( q_0 = 3000 \text{ N/m}, L = 200 \text{ m} \)

Again, there are no axial forces so \( F(x) = 0 \)

Proceeding to \( \frac{dF}{dx} = q(x) \Rightarrow F(x) = \int q(x) \)

for \( 0 < x < L/2 \)

\[ F(x) = \int_{-L/2}^{x} \frac{q_0}{L} (1 - \frac{x}{L}) \, dx \]

\[ = q_0 x - q_0 \frac{x^2}{2L} + C \]

Applying the condition of \( F = 0 \) at the tip \( (x = L/2) \):

\[ q_0 \frac{L}{2} - q_0 \frac{L}{2} + C = 0 \]

\[ \Rightarrow C = -\frac{q_0 L}{8} \]

\[ \Rightarrow F(x) = q_0 (x - \frac{x^2}{2L} - \frac{3L}{8}) \]

Proceeding to:

\[ \frac{dM}{dx} = F \]

\[ \Rightarrow M(x) = \int q_0 (x - \frac{x^2}{2L} - \frac{3L}{8}) \, dx \]
Given:
\[ M(x) = \int_{\frac{x}{2}}^{0} \left( x - \frac{x^2}{2L} - \frac{3L}{8} \right) dx \]
\[ = \left[ \frac{x^2}{2} - \frac{x^3}{6L} - \frac{3Lx}{8} \right]_{\frac{x}{2}}^{0} + C_2 \]

Applying the condition of \( M = 0 \) at the +y (\( x = \frac{L}{2} \)):
\[ M\left( \frac{L}{2} \right) = 0 = \frac{q_0 L^2}{4} \left( \frac{1}{8} - \frac{1}{4} - \frac{3}{16} \right) + C_2 \]
\[ = C_2 = -\frac{q_0 L^2}{4} \left( \frac{1}{8} - \frac{1}{4} - \frac{3}{16} \right) \]
\[ \text{Solving: } C_2 = \frac{q_0 L^2}{4} \]

Finally:
\[ M(x) = \frac{q_0}{80} \left( \frac{x^2}{2} - \frac{x^3}{6L} - \frac{3Lx}{8} + \frac{L^2}{12} \right) \]

**Summarizing for Case 3**

\[ f(x) = 0 \text{ everywhere} \]
\[ S(x) = \frac{q_0}{80} \left( x - \frac{x^2}{2L} - \frac{3L}{8} \right) \quad 0 < x < \frac{L}{2} \]
\[ = \frac{q_0}{80} \left( x + \frac{x^2}{2L} - \frac{3L}{8} \right) + \rho \quad -\frac{L}{2} < x < 0 \]
\[ M(x) = \frac{q_0}{80} \left( \frac{x^2}{2} - \frac{x^3}{6L} - \frac{3Lx}{8} + \frac{L^2}{12} \right) \quad 0 < x < \frac{L}{2} \]
\[ = \frac{q_0}{80} \left( \frac{x^2}{2} + \frac{x^3}{6L} + \frac{3Lx + L^2}{12} \right) \quad -\frac{L}{2} < x < 0 \]

with \( q_0 = 3600 \text{ lb/ft} \), \( L = 200 \text{ ft} \), \( \rho = 5700 \text{ lb/ft} \), \( \beta = 570 \text{ lb sec/ft} \).
Case 4: Lift quadratically open with maximum at root to zero at tip

From last term:

\[ q(x) = q_0 \left( 1 - \frac{4x^2}{L^2} \right) \]

valid throughout

For the 787, \[ q_0 = 4048 \text{ kips}, L = 200 \text{ ft} \]

There being no axial forces, \( f(x) = 0 \)

Then with \( \frac{dS}{dx} = f(x) \)

\[ s(x) = \int q_0 \left( 1 - \frac{4x^2}{L^2} \right) \, dx \]

\[ = q_0 \left( x - \frac{4x^3}{3L^2} \right) + C_1 \]

Using the condition of no shear at the tip:

\[ s(\frac{L}{2}) = 0 = q_0 \left( \frac{L}{2} - \frac{L}{6} \right) + C_1 \]

\[ \Rightarrow C_1 = -\frac{q_0 L}{3} \]

\[ \Rightarrow s(x) = q_0 \left( x - \frac{4x^3}{3L^2} - \frac{L}{3} \right) \]
And then to:  \[ \frac{dM}{dx} = S \]

\[ \Rightarrow M(x) = \int_0^x \left( x - \frac{4x^3}{3L^2} - \frac{L}{3} \right) dx \]

\[ = q_0 \left( \frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3} \right) + C_2 \]

And again using the condition of no moment at the tip:

\[ M(\frac{L}{2}) = 0 = q_0 \left( \frac{L^2}{8} - \frac{L^2}{48} - \frac{L^2}{6} \right) + C_2 \]

\[ \Rightarrow C_2 = -q_0 L^2 \left( \frac{L^2}{48} - \frac{L^2}{48} - \frac{L^2}{6} \right) \]

\[ L_0: C_2 = \frac{-q_0 L^2}{16} \]

Finally:

\[ M(x) = q_0 \left( \frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3} + \frac{L^2}{16} \right) \]

Summarizing for Case 4:

\[ F(x) = 0 \text{ everywhere} \]
\[ S(x) = q_0 \left( x - \frac{4x^3}{3L^2} - \frac{L}{3} \right) \quad 0 < x < \frac{L}{2} \]

\[ = q_0 \left( x - \frac{4x^3}{3L^2} - \frac{L}{3} \right) + P \quad -\frac{L}{2} < x < 0 \]

\[ M(x) = q_0 \left( \frac{x^2}{2} - \frac{x^4}{3L^2} - \frac{Lx}{3} + \frac{L^2}{16} \right) \quad 0 < x < \frac{L}{2} \]

\[ = q_0 \left( \frac{x^2}{2} - \frac{x^4}{3L^2} + \frac{Lx}{3} + \frac{L^2}{16} \right) \quad -\frac{L}{2} < x < 0 \]

with \( q_0 = 40.48 \text{ lb/ft}, \quad L = 200 \text{ ft}, \quad P = 540,000 \text{ lb}. \)
(c) Now compare via plotting

**Axial Force** -- zero everywhere in all cases
(no need to plot)

**Shear Force**

\[ S(x) \, [\text{lb}] \]

\[ \begin{align*}
  & \cdots 1 \\
  & \cdots 2 \\
  & \cdots 3 \\
  & \cdots 4
\end{align*} \]

\[ (\, \text{ft} \,) \]

**NOTE:** All models have the same value at the root. Reason -- each wing carries the same integrated lift = \( \frac{\pi}{2} \).

All models change by concentrated weight \( P \), at the root.

**Moment**

\[ M(x) \, [\text{lb} \cdot \text{ft}] \]

\[ \begin{align*}
  & \cdots 1 \\
  & \cdots 2 \\
  & \cdots 3 \\
  & \cdots 4
\end{align*} \]

\[ (\, \text{ft} \,) \]
NOTE: The moment at the root is maximum in all cases but varies a great deal in value. Thus the lift distribution plays a considerable role in the moment carried by the beam.

(d) The highest moment is at the root so that is the location of greatest loading as in beam for all cases. The load is transferred at the attachment to the friselage.